STUDY MATERIAL

X Maths

CONTENTS

CHAPTER 1

REAL NUMBERS

KEY POINTS

1. **Euclid's division lemma :**

   For given positive integers 'a' and 'b' there exist unique whole numbers 'q' and 'r' tisfying the relation $a = bq + r$, $0 \leq < b$.

2. **Euclid's division algorithms :**

   HCF of any two positive integers a and b. With $a > b$ is obtained as follows:

   Step 1 : Apply Euclid's division lemma to a and b to find q and r such that $a = bq + r$. $0 \leq r < b$.

   Step 2 : If $r = 0$, HCF (a, b), $= b$ if $r \neq 0$, apply Euclid's lemma to b and r.

3. **The Fundamental Theorem of Arithmetic :**

   Every composite number can be expressed (ctorized) as a product of primes and this ctorization is unique, apart from the order in which the prime ctors occur.

4. Let $x = \frac{p}{q}, q \neq 0$ to be a rational number, such that the prime ctorization of 'q' is of the form $2^m 5^n$, where m, n are non-negative integers. Then x has a decimal expansion which is terminating.

5. Let $x = \frac{p}{q}, q \neq 0$ be a rational number, such that the prime ctorization

   of q is not of the form $2^m 5^n$, where m, n are non-negative integers. Then x has a decimal expansion which is non-terminating repeating.

6. $\sqrt{p}$ is irrational, which p is a prime. A number is called irrational if it cannot be written in the form $\frac{p}{q}$ where p and q are integers and $q \neq 0$.

MULTIPLE CHOICE QUESTIONS

1. $7 \times 11 \times 13 + 7$ is *a*

   (a) prime number (b) composite number

   (c) odd number (d) none

2. Which of these numbers always ends with the digit 6.

   (a) 4^n (b) 2^n

   (c) 6^n (d) 8^n

   where *n* is a natural number.

3. For *a, b* ($a \neq b$) positive rational numbers $\left(\sqrt{a}+\sqrt{b}\right)\left(\sqrt{a}-\sqrt{b}\right)$ is a ____

   (a) Rational number (b) irrational number

   (c) $\left(\sqrt{a}-\sqrt{b}\right)^2$ (d) 0

4. If *p* is a positive rational number which is not a perfect square then $-3\sqrt{p}$ is

   (a) integer (b) rational number

   (c) irrational number (d) none of the above.

5. All decimal numbers are–

   (a) rational numbers (b) irrational numbers

   (c) real numbers (d) integers

6. In Euclid Division Lemma, when $a = bq + r$, where *a, b* are positive integers which one is correct.

   (a) $0 < r \leq b$ (b) $0 \leq r < b$

   (c) $0 < r < b$ (d) $0 \leq r \leq b$

7. Which of the following numbers is irrational number

   (a) 3.131131113... (b) 4.46363636...

   (c) 2.35 (d) *b* and *c* both

8. The decimal expansion of the rational number $\frac{51}{2^3 5^4}$ will terminate after ___ decimal places.

(a) 3 (b) 4

(c) 5 (d) never

9. HCF is always

(a) multiple of L.C.M. (b) ctor of L.C.M.

(c) divisible by L.C.M. (d) *a* and *c* both

10. Which one is not the ctor(s) of 255

(a) 5 (b) 25

(c) 3 (d) 17

11. Which of the following is an irrational number between 0 and 1

(a) 0.11011011... (b) 0.90990999...

(c) 1.010110111... (d) 0.3030303...

12. $p^n = (a \times 5)^n$. For p^n to end with the digit zero a = __ for natural no. n

(a) any natural number (b) even number

(c) odd number (d) none.

13. After how many places the decimal expansion of $\frac{51}{1500}$ will terminate.

(a) 2 places (b) 3 places

(c) 4 places (d) 5 places

SHORT ANSWER TYPE QUESTIONS

14. What will be the value of $0.\overline{3} + 0.\overline{4}$?

15. If unit's digit of 7^3 is 3 then what will be the unit's digit of 7^{11}.

16. Given that HCF (135, 225) = 45. Find LCM (135, 225).

17. Solve $\sqrt{18} \times \sqrt{50}$. What type of number is it, rational or irrational.

18. What type of decimal expansion will $\frac{69}{60}$ represent? After how many places will the decimal expansion terminate?

19. Find the H.C.F. of the smallest composite number and the smallest prime number.

20. If $a = 4q + r$ then what are the conditions for a and q. What are the values that r can take?

21. What is the smallest number by which $\sqrt{5} - \sqrt{3}$ be multiplied to make it a rational no? Also find the no. so obtained.

22. What is the digit at unit's place of 9^n?

23. Find one rational and one irrational no. between $\sqrt{3}$ and $\sqrt{5}$.

24. If the no. p^n ever to end with the digit 0 then what are the possible value (s) of p?

25. State Euclid's Division Lemma and hence find HCF of 16 and 28.

26. State fundamental theorem of Arithmetic and hence find the unique fraternization of 120.

27. Prove that $\frac{1}{2 - \sqrt{5}}$ is irrational number.

28. Prove that $5 - \frac{2}{7}\sqrt{3}$ is irrational number.

29. Prove that $\sqrt{2} + \sqrt{7}$ is not rational number.

30. Find HCF and LCM of 56 and 112 by prime ctorition method.

31. Why $17 + 11 \times 13 \times 17 \times 19$ is a composite number? Explain.

32. Check whether $5 \times 7 \times 11 + 7$ is a composite number.

33. Check whether $7 \times 6 \times 3 \times 5 + 5$ is a composite number.

34. Check whether 14^n can end with the digit zero for any natural number, n.

35. Show that 9^n can never ends with the digit zero.

36. If the HCF of 210 and 55 is expressible in the form $210 \times 5 + 55y$ then find y.

LONG ANSWER TYPE QUESTIONS

37. Find HCF of 56, 96 and 324 by Euclid's algorithm.

38. Show that the square of any positive integer is either of the form $3m$ or $3m + 1$ for some integer m.

39. Show that any positive odd integer is of the form $6q + 1$, $6q + 5$ where q is some integer.

40. Prove that the square of any positive integer is of the form $5q$, $5q + 1$, $5q + 4$ for some integer, q.

41. Prove that the product of three consecutive positive integers is divisible by 6.

42. Show that one and only one of n, $n + 2$, $n + 4$ is divisible by 3.

43. Two milk containers contains 398 l and 436 l of milk the milk is to be transferred to another container with the help of a drum. While transferring to another container $7l$ and $11l$ of milk is left in both the containers respectively. What will be the maximum capacity of the drum.

ANSWERS

1. *b*	2. *c*
3. *a*	4. *c*
5. *c*	6. *b*
7. *a*	8. *b*
9. *b*	10. *b*
11. *b*	12. *b*
13. *b*	14. $\frac{7}{9}$

15. 3

16. 675

17. 30, rational

18. Terminate after two places

19. 2

20. *a*-positive integer, *r, q* whole number $0 \le r < 4$

21. $\left(\sqrt{5} + \sqrt{3}\right)$, 2

22. Even power = 1, Odd power = 9

23. –

24. Multiples of 10

25. 4

26. $2 \times 2 \times 2 \times 3 \times 5$

27. ____

28. ____

29. –

30. H.C.F. = 28, L.C.M. = 336

31. –

32. Yes

33. Yes

34. No

35. –

36. Find HCF (210, 55) = 5, as $5 = 210 \times 5 + 55y \Rightarrow y = -19$

37. 4

38. Take $a = 3q + r$

39. Take $a = 6q + r$

40. –

41. –

42. Take $n = 3q + r$

43. 17

CHAPTER 2

POLYNOMIALS

KEY POINTS

1. Polynomials of degrees 1, 2 and 3 are called linear, quadratic and cubic polynomials respectively.
2. A quadratic polynomial in x with real coefficient is of the form $ax^2 + bx + c$, where a, b, c are real number with $a \neq 0$.
3. The zeroes of a polynomial $p(x)$ are precisely the x–coordinates of the points where the graph of $y = p(x)$ intersects of the x-axis *i.e.* $x = a$ is a zero of polynomial $p(x)$ if $p(a) = 0$.
4. A polynomial can have at most the me number zeroes as the degree of polynomial.
5. For quadratic polynomial $ax^2 + bx + c$ (a $\neq$ 0)

   Sum of zeroes $= -\dfrac{b}{a}$

   Product of zeroes $= \dfrac{c}{a}$.
6. The division algorithm states that given any polynomial $p(x)$ and polynomial $g(x)$, there are polynomials $q(x)$ and $r(x)$ such that :

   $$p(x) = g(x).q\ (x) + r(x),\ g(x) \neq 0$$

   wether $r(x) = 0$ or degree of $r(x) <$ degree of $g(x)$.

MULTIPLE CHOICE QUESTIONS

1. *A* real no. α is a zero of the polynomial $f(x)$ if

   (a) $f(\alpha) > 0$ (b) $f(\alpha) = 0$

   (c) $f(\alpha) < 0$ (d) none

2. The zeroes of a polynomial $f(x)$ are the coordinates of the points where the graph of $y = f(x)$ intersects

(a) x-axis
(b) y-axis
(c) origin
(d) (x, y)

3. If β is 0 zero of $f(x)$ then _____ is one of the ctors of $f(x)$

(a) $(x - \beta)$
(b) $(x - 2\beta)$
(c) $(x + \beta)$
(d) $(2x - \beta)$

4. If $(y - a)$ is ctor of $f(y)$ then ___ is a zero of $f(y)$

(a) y
(b) a
(c) $2a$
(d) $2y$

5. Which of the following is not correct for : A quadratic polynomial may have

(a) no real zeroes
(b) two equal real zeroes
(c) two distinct zeroes
(d) three real zeros.

6. Cubic poly $x = f(y)$ cuts y-axis at almost

(a) one point
(b) two points
(c) three points
(d) four points

7. Polynomial $x^2 + 1$ has ___ zeroes

(a) only one real
(b) no real
(c) only two real
(d) one real and the other non-real.

8. Zeroes of the polynomial $4x^2 - 1$ are

(a) equal
(b) unequal with the me sign
(c) equal in magnitude but opposite in sign
(d) Unequal with different sign.

9. If P is the sum of the zeroes and s is the product then quadratic polynomial can be obtained as follows.

(a) $x^2 - sx + p$ (b) $x^2 - px + s$

(c) $x^2 + sx - p$ (d) $x^2 + px - s$

10. If 2 is a zero of both the polynomial, $3x^2 + ax - 14$ and $2x - b$ then $a - 2b =$ ___

(a) –2 (b) 7

(c) –8 (d) –7

11. If zeroes of the polynomial $ax^2 + bx + c$ are reciprocal of each other than

(a) $a = c$ (b) $a = b$

(c) $b = c$ (d) $a = -c$

12. Three zeroes of $(x + 4)(x^2 - 6x + 8)$ are

(a) 4, –4, 2 (b) 4, 4, –2

(c) –4, –4, 2 (d) –4, –4, –2

13. Graph of $y = ax^2 + 6x + c$ intersects x-axis at 2 distinct points if

(a) $b^2 - 4ac > 0$ (b) $b^2 - 4ac < 0$

(c) $b^2 - 4ac = 0$ (d) none

SHORT ANSWER TYPE QUESTIONS

14. If α and β are the zeroes of the polynomial $2x^2 - 7x + 3$. Find the sum of the reciprocal of its zeroes.

15. If $\frac{1}{3}$ is a zero of he polynomial $3x^3 - 4x^2 - 17x - k$ then find value of k.

16. If the polynomial $6x^3 + 16x^2 + px - 5$ is exactly divisible by $3x + 5$, then find the value of p.

17. If $(x + a)$ is a ctor of the polynomial $2x^2 + 2ax + 5x + 10$ find a.

18. Find a quadratic polynomial whose zeroes are $\left(5 - 3\sqrt{2}\right)$ and $\left(5 + 3\sqrt{2}\right)$.

19. If $\frac{1}{5}$ and -2 are respectively product and sum of the zeroes of a quadratic polynomial. Find the polynomial.

20. Find zeroes of $\sqrt{3}x^2 - 8x + 4\sqrt{3}$.

21. If $(x + k)$ is a ctor of the polynomial $x^2 - 2x - 15$ and $x^3 + a$. Find k and a.

22. Find zeroes of $2x^2 - 5x + 3$.

23. If sum of the zeroes of $kx^2 + 3k + 2x$ is equal to their product. Find k.

24. If one zero of $4x^2 - 9 - 8kx$ is negative of the other find k.

LONG ANSWER TYPE QUESTIONS

25. Find the zeroes of $5x^2 - 4 - 8x$. Verify the relationship between he zeroes and coefficients.

26. If one zero of he polynomial $(a^2 + a)\ x^2 + 13x + 6a$ is reciprocal of he other, find value (s) of a.

27. If -5 is one of the zeroes of $2x^2 + px - 15$. Quadratic polynomial $p(x^2 + x) + k$ has both the zeros equal to each other. Then find k.

28. Find the value of k such that $3x^2 + 2kx + x - k - 5$ has the sum of the zeroes as half of their product.

29. If $f(x) = 2x^4 - 5x^3 + x^2 + 3x - 2$ is divided by $g(x)$ the quotient $q(x) = 2x^2 - 5x + 3$ and $r(x) = -2x + 1$ find $g(x)$.

30. If $(x - 2)$ is one of the ctors of $x^3 - 3x^2 - 4x + 12$ find the other zeroes.

31. If α and β the zeroes of he polynomial $x^2 - 5x + k$ such that $\alpha - \beta = 1$, find the value of k.

32. Find he zeroes of the polynomial $3x^2 - x - 4$ and verify the relationship between the zeros and the coefficients.

33. Obtain all zeroes of $x^4 - x^3 - 7x^2 + x + 6$ if 3 and 1 are zeros.

34. Find all the zeroes of he polynomial $4x^4 - 20x^3 + 23x^2 + 5x - 6$ if two of its zeros are 2 and 3.

35. If $\left(2+\sqrt{3}\right)$ and $\left(2-\sqrt{3}\right)$ are two zeroes of $x^4 - 4x^3 - 8x^2 + 36x - 9$ find the other two zeroes.

36. What must be subtracted from $8x^4 + 14x^3 - 4x^2 + 7x - 8$ so that the resulting polynomial is exactly divisible by $4x^2 + 3x - 2$.

37. When we add $p(x)$ to $4x^4 + 2x^3 - 2x^2 + x - 1$ the resulting polynomial is divisible by $x^2 + 2x - 3$ find $p(x)$.

38. Find a and f if $(x^4 + x^3 + 8x^2 + ax + f)$ is a multiple of $(x^2 + 1)$.

39. If the polynomial $6x^4 + 8x^3 + 17x^2 + 21x + 7$ is divided by $3x^2 + 1 + 4x$ then $r(x) = (ax + b)$ find a and b.

40. Obtain all the zeroes of $2x^4 - 2x^3 - 7x^2 + 3x + 6$ if $\left(x \pm \sqrt{\frac{3}{2}}\right)$ are two ctors of this polynomial.

41. Find all the zeroes of $x^4 - 3x^3 - x^2 + 9x - 6$ if $-\sqrt{3}$ and $\sqrt{3}$ are two of its zeros.

42. If $(x^3 - 3x + 1)$ is one of the ctors of the polynomial $x^5 - 4x^3 + x^2 + 3x + 1$, find the other two ctors.

ANSWERS

1. *b*
2. *a*
3. *a*
4. *b*
5. *a*
6. *c*
7. *b*
8. *c*
9. *b*
10. *d*
11. *a*
12. *a*
13. *a*
14. $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{7}{3}$

15. -6

16. $p = 7$

17. $a = 2$

18. $x^2 - 10x + 7$

19. $x^2 + 2x + \frac{1}{5}$

20. $2\sqrt{3}, \frac{2}{3}\sqrt{3}$

21. $k = -5, 3$ and $a = -125, +27$

22. $1, \frac{3}{2}$

23. $-\frac{2}{3}$

24. 0

25. $2, -\frac{2}{5}$

26. 5

27. $p = 7, k = \frac{7}{4}$

28. $k = 1$

29. $g(x) = x^2 - 1$

30. $-2, 3$

31. $k = 6$

32. $\frac{4}{3}, -1$

33. $-2, -1$

34. $-\frac{1}{2}, +\frac{1}{2}$

35. ± 3

36. $14x - 10$

37. $61x + 65$

38. $r(x) = 0$

$$\left.\begin{array}{l} \Rightarrow (a-1)x + (f-7) = 0 \\ \Rightarrow a = 1 \text{ and } f = 7 \end{array}\right]$$

39. $r(x) = x + 2 = ax + f \Rightarrow a = 1$ and $f = 2$

40. $2, -1 \pm \sqrt{\frac{3}{2}}$

41. $\pm\sqrt{3}, 1, 2$

42. $(x - 1), (x + 1)$

CHAPTER 3

PAIR OF LINEAR EQUATION IN TWO VARIABLE

KEY POINTS

1. The most general form of a pair of linear equations is :

   $a_1x + b_1y + c_1 = 0$

   $a_2x + b_2y + c_2 = 0$

   Where a_1, a_2, b_1, b_2, c_1, c_2 are real numbers and $a_1^2 + b_1^2 \neq 0$, $a_2^2 + b_2^2 \neq 0$.

2. The graph of a pair of linear equations in two variables is represented by two lines;

   (i) If the lines intersect at a point, the pair of equations is consistent. The point of intersection gives the unique solution of the equation.

   (ii) If the lines coincide, then there are infinitely many solutions. The pair of equations is consistent. Each point on the line will be a solution.

   (iii) If the lines are parallel, the pair of the linear equations has no solution. The pair of linear equations is inconsistent.

3. If a pair of linear equations is given by $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$

   (i) $\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \Rightarrow$ the pair of linear equations is consistent. (Unique solution).

   (ii) $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2} \Rightarrow$ the pair of linear equations is inconsistent (No solution).

(iii) $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \Rightarrow$ the pair of linear equations is dependent and consistent (infinitely many solutions).

MULTIPLE CHOICE QUESTIONS

1. Every linear equation in two variables has ___ solution(s).

   (a) no (b) one

   (c) two (d) infinitely many

2. $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ is the condition for

   (a) intersecting lines (b) parallel lines

   (c) coincident lines (d) none

3. For a pair to be consistent and dependent the pair must have

   (a) no solution (b) unique solution

   (c) infinitely many solutions (d) none of these

4. Graph of every linear equation in two variables represent a ___

   (a) point (b) straight line

   (c) curve (d) triangle

5. Each point on the graph of pair of two lines is a common solution of he lines in case of ___

   (a) Infinitely many solutions (b) only one solution

   (c) no solution (d) none of these

6. Which of he following is the solution of the pair of linear equations $3x - 2y = 0$, $5y - x = 0$

   (a) (5, 1) (b) (2, 3)

   (c) (1, 5) (d) (0, 0)

7. One of the common solution of $ax + by = c$ and y-axis is _____

(a) $\left(0, \frac{c}{b}\right)$ (b) $\left(0, \frac{b}{c}\right)$

(c) $\left(\frac{c}{b}, 0\right)$ (d) $\left(0, -\frac{c}{b}\right)$

8. If the value of x in the equation $2x - 8y = 12$ is 2 then the corresponding value of y will be

(a) –1 (b) +1

(c) 0 (d) 2

9. The pair of linear equations is id to be inconsistent if they have

(a) only one solution (b) no solution

(c) infinitely many solutions. (d) both a and c

10. On representing $x = a$ and $y = b$ graphically we get ____

(a) parallel lines (b) coincident lines

(c) intersecting lines at (a, b) (d) intersecting lines at (b, a)

11. How many real solutions of $2x + 3y = 5$ are possible

(a) no (b) one

(c) two (d) infinitely many

12. The value of k for which the system of equation $3x + 2y = -5$, $x - ky = 2$ has a unique solutions.

(a) $k = \frac{2}{3}$ (b) $k \neq \frac{2}{3}$

(c) $k = -\frac{2}{3}$ (d) $k \neq -\frac{2}{3}$

13. If the lines represented by the pair of linear equations $2x + 5y = 3$, $2(k + 2)\ y + (k + 1)\ x = 2k$ are coincident then the value of k is ____

(a) –3 (b) 3

(c) 1 (d) –2

14. The coordinates of the point where x-axis and the line represented by $\frac{x}{2} + \frac{y}{3} = 1$ intersect, are

(a) (0, 3) (b) (3, 0)

(c) (2, 0) (d) (0, 2)

15. Graphically $x - 2 = 0$ represents a line

(a) parallel to x-axis at a distance 2 units from x-axis.

(b) parallel to y-axis at a distance 2 units from it.

(c) parallel to x-axis at a distance 2 units from y-axis.

(d) parallel to y-axis at a distance 2 units from x-axis.

16. If $ax + by = c$ and $lx + my = n$ has unique solution then the relation between the coefficients will be ____

(a) $am \neq lb$ (b) $am = lb$

(c) $ab = lm$ (d) $ab \neq lm$

SHORT ANSWER TYPE QUESTIONS

17. Form a pair of linear equations for : The sum of the numerator and denominator of fraction is 3 less than twice the denominator. If the numerator and denominator both are decreased by 1, the numerator becomes half the denominator.

18. Amar gives ₹ 9000 to some athletes of a school as scholarship every month. Had there been 20 more athletes each would have got ₹ 160 less. Form a pair of linear equations for this.

19. Find the value of k so that the equations $x + 2y = -7$, $2x + ky + 14 = 0$ will represent concident lines.

20. Give linear equations which is coincident with $2x + 3y - 4 = 0$

21. What is the value of a for which (3, a) lies on $2x - 3y = 5$

22. The sum of two natural nos. is 25 of their difference is 7. Find the nos.

23. Dinesh in walking along the line joining (1, 4) and (0, 6), Naresh is walking along the line joining (3, 4,) and (1,0). Represent on graph and find the point where both of them cross each other.

24. Solve the pair or linear equations

    $x - y = 2$ and $x + y = 2$. Also find p if $p = 2x + 3$

25. For what value of K the following system of equation are parallel.

    $$2x + Ky = 10$$

    $$3x + (k + 3)\ y = 12$$

26. For m a pair of linear equations for the following situation assuming speed of boat in still water as 'x' and speed of stream 'y' : A boat covers 32 km upstream and 36 km downstream in 7 hours/ It also covers 40 km upstream and 48 km downstream in 9 hours.

27. Check graphically whether the pair of linear equations $3x + 5y = 15$, $x - y = 5$ is consistent. Also check whether the pair is dependent.

28. For what value of p the pair of linear equations

    $$(p + 2)\ x - (2\ p + 1)y = 3\ (2p - 1)$$

    $$2x - 3y = 7$$

    has unique solution.

29. Find the value of K so that the pair of linear equations :

    $$(3\ K + 1)\ x + 3y - 2 = 0$$

    $(K^2 + 1)\ x + (k-2)y - 5 = 0$ is inconsistent.

30. Given the linear equation $x + 3y = 4$, write another linear equation in two variables such that the geometrical representation of the pair so formed is (i) intersected lines (ii) parallel lines (iii) coincident lines.

31. Solve $x - y = 4$, $x + y = 10$ and hence find the value of p when $y = 3\ x - p$

32. Determine the value of K for which the given system of linear equations has infinitely many solutions:

$$Kx + 3y = K - 3$$

$$12x + Ky = K$$

33. Find the values of α and β for which and following system of linear equations has infinite no of solutions :

$$2x + 3y = 7$$

$$2\alpha x + (\alpha + \beta)y = 28.$$

34. Solve for x and y :

$$\frac{(x+1)}{2} + \frac{(y-1)}{3} = 8; \quad \frac{(x-1)}{3} + \frac{(y+1)}{2} = 9$$

35. Solve for x and y :

$$2^x + 3^y = 17$$

$$2^{x+2} - 3^{y+1} = 5.$$

36. Solve for x and y

$$\begin{cases} 139x + 56y = 641 \\ 56x + 139y = 724 \end{cases}$$

37. Solve for x and y

$$\frac{5}{x+y} + \frac{1}{x-y} = 2$$

$$\frac{15}{x+y} - \frac{5}{x-y} = -2$$

38. Solve for x and y

$$37x + 43y = 123$$

$$43x + 37y = 117$$

39. Check graphically whether the pair of lines $3x + 2y - 4 = 0$ and $2x - y - 2 = 0$ is consistent. Also find the coordinates of the points where the graphs of the lines of equations meet the y-axis.

LONG ANSWER TYPE QUESTIONS

40. Solve for x and y

$$\frac{1}{2(2x+3y)} + \frac{12}{7(3x-2y)} = \frac{1}{2}$$

$$\frac{7}{(2x+3y)} + \frac{4}{(3x-2y)} = 2 \quad \text{for } 2x + 3y \neq 0 \text{ and } 3x - 2y \neq 0$$

41. Solve for p and q

$$\frac{p+q}{pq} = 2, \frac{p-q}{pq} = 6, \; p \neq 0, \; q \neq 0.$$

42. Solve for x and y

$$\frac{2}{3x+2y} + \frac{3}{3x-2y} = \frac{17}{5} \qquad 3x + 2y \neq 0 \text{ and } 3x - 2y \neq 0$$

$$\frac{5}{3x+2y} + \frac{1}{3x-2y} = 2$$

43. $\dfrac{6}{x+y} = \dfrac{7}{x-y} + 3, \; \dfrac{1}{2(x+y)} = \dfrac{1}{3(x-y)}, \; x + y \neq 0, \; x - y \neq 0$

44. $\dfrac{2}{\sqrt{x}} + \dfrac{3}{\sqrt{y}} = 2, \; \dfrac{4}{\sqrt{x}} - \dfrac{9}{\sqrt{y}} = -1, \qquad x \neq 0, \quad y \neq 0.$

45. $ax + by = 1$

$$bx + ay = \frac{(a+b)^2}{a^2 + b^2} - 1.$$

46. If from twice the greater of two numbers, 20 is subtracted, the result is the other number If from twice the smaller number, 5 is subtracted, the result is the greater number Find the numbers.

47. 27 pencils and 31 rubbers together costs ₹ 85 while 31 pencils and 27 rubbers together costs ₹ 89. Find the cost of 2 pencils and 1 rubber.

48. The area of a rectangle remain the me if its length is increased by 7 cm and the breadth is decreased by 3 cm. The area remains unaffected if length is decreased by 7 cm and the breadth is increased by 5 cm. Find length and breadth.

49. A two digit number is obtained by either multiplying the sum of the digits by 8 and adding 1; or by multiplying the difference of the digits by 13 and adding 2. Find the number. How many such nos. are there.

50. *A* number consists of three digits whose sum is 17. The middle one exceeds the sum of other two by 1. If the digits are reversed, the no. is diminished by 396. Find the number.

51. A boatman rows his boat 35 km upstream and 55 km down stream in 12 hours. He can row 30 km. upstream and 44 km downstream in 10 hours. Find the speed of he stream and that of the boat in still water. Hence find the total time taken by the boat man to row 50 cm upstream and 77 km downstream.

52. In a function if 10 guests are sent from room *A* to *B*, the number of guests in room *A* and *B* are me. If 20 guests are sent from *B* to *A*, the number of guests in *A* is double the numbr of guests in *B*. Find number of guests in both the rooms in the beginning.

53. In a function Madhu wished to give ₹ 21 to each person present and found that she fell short of ₹ 4 so she distributed ₹ 20 to each and found that ₹ 1 were left over. How much money did she gave and how many persons were there.

54. A mobile company charges a fixed amount as monthly rental which includes 100 minutes free per month and charges a fixed amount these after for every additional minute. Abhishek paid Rs. 433 for 370 minutes and Ashish paid Rs. 398 for 300 minutes. Find the bill amount under the me plain, if Usha use for 400 minutes.

55. ther's age is three times the sum of ages of his two childrem. After 5 years his age will be twice the sum of ages of two children. Find the age of the ther.

56. Draw the graphs of the following equations :

$3x - 4y + 6 = 0$ and $3x + y - 9 = 0$. Also find the coordinates of the vertices of the triangle formed by these lines and the x-axis.

57. 90% and 97% pure acid solutions are mixed to obtain 21 litres of 95% pure acid solution. Find the quantity of each type of acid to be mixed to form the mixture.

58. The sum of the numerator and denominator of a fraction is 8. If 3 is added to both the numerator and denominator, the fraction becomes 3/4. Find the fraction.

59. The monthly income of Supriya and Dhruv are in the ratio 5 : 4 and their monthly expenditures are in the ratio 7 : 5. If each ves ₹3000 per month. Find the monthly income of each.

60. Find four angles of a cyclic quadrilateral ABCD in which $\angle A = 2x - 3)^\circ$, $\angle B = (y + 7)^\circ$, $\angle C = (2y + 17)^\circ$ and, $\angle D = (4x - 9)^\circ$.

ANSWERS

1. *d*	2. c
3. *c*	4. *b*
5. *a*	6. *d*
7. *a*	8. *a*
9. *b*	10. *c*
11. *d*	12. *d*
13. *b*	14. *c*
15. *b*	16. *a*

17. If $N = x$ and $D = y$ then $x - y = -3$, $2x - y = 1$

18. No. of athletes $= x$, No. of athletes increased $= y$

$$\begin{cases} y - x = 20 \\ \dfrac{1}{x} - \dfrac{1}{y} = \dfrac{4}{225} \end{cases}$$

19. $k = 4$

20. $k(2x + 3y - 4) = 0$, k is any real $k \neq 0$.

21. $\dfrac{1}{3}$	22. 16, 9

23. (2, 2)

24. (2, 0) $p = 7$

25. $k = 6$

26. Speed of boat = x, speed of stream = y

$$\begin{cases} \dfrac{32}{x-y} + \dfrac{36}{x+y} = 7 \\ \dfrac{40}{x-y} + \dfrac{48}{x+y} = 9 \end{cases}$$

27. Yes, No

28. $p \neq 4$

29. $k = -1, k \neq \dfrac{19}{2}$

30. ___

31. (7, 3), 18

32. $k = 6$

33. (4, 8)

34. (7, 13)

35. (3, 2) [**Hint. :** put $2^x = m$, $3^y = n$]

36. (3, 4)

37. (3, 2)

38. (1, 2)

39. Yes, (0, 2), (0, –2)

40. (2, 1)

41. $\left(-\dfrac{1}{2}, \dfrac{1}{4}\right)$

42. (1, 1)

43. $\left(-\dfrac{5}{4}, -\dfrac{1}{4}\right)$

44. (4, 9)

45. $\left(\dfrac{a}{a^2+b^2}, \dfrac{b}{a^2+b^2}\right)$

46. 15, 10

47. ₹ 5

48. 28m, 15m

49. 41 or 14 (2 numbers possible)

50. $692 \begin{bmatrix} x + y = 8 \\ y - x = 4 \end{bmatrix}$

51. 3 km/hr., 8 km/hr., 17 hr.

52. 100, 80

53. Rs. 101, 5

54. $\left[\text{Rs. 298, Rs. } \dfrac{1}{2}\right]$ Rs. 448

55. 45 years

56. (–2, 0), (2, 3), (3, 0).

57. 6 litre of 90%, 15 litre of 97%.

58. $\dfrac{3}{5}$.

59. ₹ 10000, ₹ 8000

60 60°, 57°, 117°, 123°.

CHAPTER 4

SIMILAR TRIANGLES

KEY POINTS

1. **Similar Triangles :** Two triangles are id to be similar if their corresponding angles are equal and their corresponding sides are proportional.

2. Criteria for Similarity :

   in ΔABC and ΔDEF

   (i) ***AAA* Similarity :** $\Delta ABC \sim \Delta DEF$ when $\angle A = \angle D$, $\angle B = \angle E$ and $\angle C = \angle F$

   (ii) ***S* Similarity :**

   $\Delta ABC \sim \Delta DEF$ when $\frac{AB}{DE} = \frac{AC}{DF}$ and $\angle B = \angle E$

   (iii) **SSS Similarity :** $\Delta ABC \sim \Delta DEF$, $\frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF}$.

3. The proof of the following theorems can be asked in he examination :

   (i) **Basic Proportionality Theorems :** If a line is drawn parallel to one side of a triangle to intersect the other sides in distinct points, the other two sides are divided in the me ratio.

   (ii) The ratio of the area of two similar triangles is equal to the square of the ratio of their corresponding sides.

   (iii) **Pythagoras Theorem :** In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

(iv) **Converse of Pythagoras Theorem :** In a triangle, if the square of one side is equal to the sum of the squares of the other two sides then the angle opposite to the first side is a right angle.

MULTIPLE CHOICE QUESTIONS

1. $\Delta ABC \sim \Delta DEF$. If $DE = 2\ AB$ and $BC = 3$cm then EF is equal to _______.

   (a) 1.5 cm (b) 3 cm

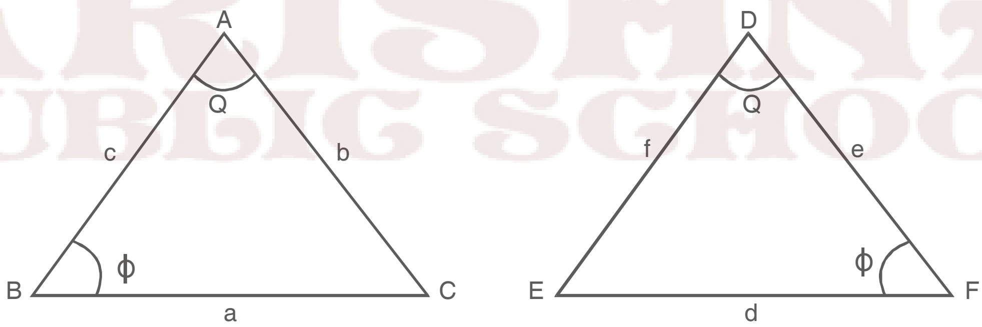
   (c) 6 cm (d) 9 cm

2. In ΔDEW, $AB \parallel EW$ If $AD = 4$ cm, $DE = 12$cm and $DW = 24$ cm then the value of DB = ____

   (a) 4 cm (b) 8 cm

   (c) 12 cm (d) 16 cm

3.



   In the figure the value of cd = ________

   (a) ae (b) af

   (c) bf (d) be

4. If in ΔABC, $AB = 6$ cm, $BC = 12$cm and $CA = 6\sqrt{3}\ cm$ then the measure of $\angle A$ is

   (a) 30° (b) 45°

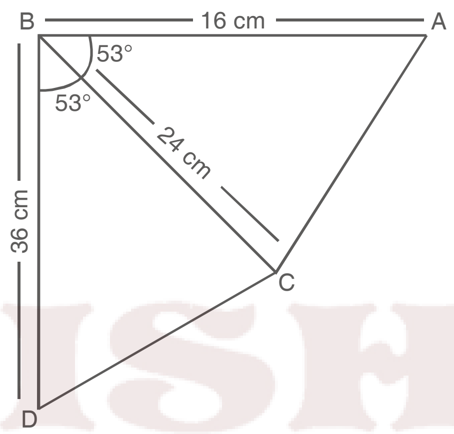
   (c) 60° (d) 90°

5. The area of two similar triangles are in the ratio 9 : 16. The corresponding sides must be in the ratio ______

(a) 9 : 16 (b) 16 : 9

(c) 3 : 4 (d) 4 : 3

6. In the figure, ΔABC is similar to ______



(a) ΔBDC (b) ΔDBC

(c) ΔCDB (d) ΔCBD

7. $\Delta AMB \sim \Delta CMD$. Also $2ar(\Delta AMB) = ar(\Delta CMD)$ the length of MD is

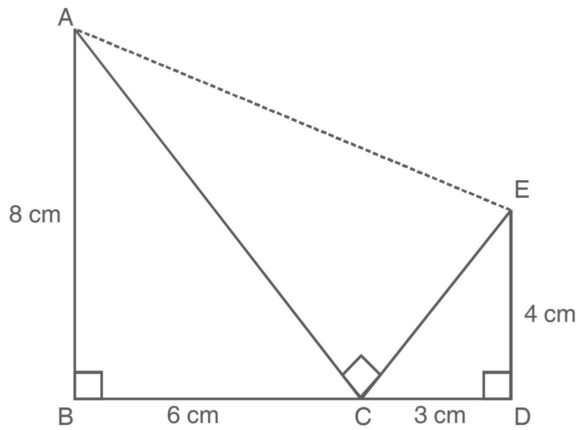
(a) $\sqrt{2}\ MB$ (b) $\sqrt{2}\ MD$

(c) $\frac{\sqrt{2}}{MB}$ (d) $\frac{\sqrt{2}}{MD}$

8. In fig. lenth of AE is

(a) 10 cm (b) 9 cm

(c) $5\sqrt{5}$ cm (d) $\sqrt{5}$ cm

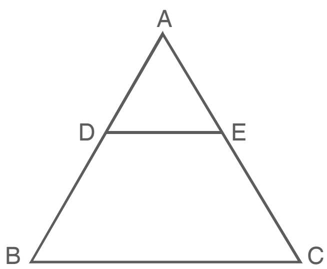


9. In ΔPQR if S and T are the points on the sides PR and QR respectively such that $ST \parallel PQ$ then $\frac{RS}{RT}$ = ______

(a) $\frac{SP}{TQ}$ (b) $\frac{PR}{QR}$

(c) $\frac{SP}{RS}$ (d) $\frac{TQ}{RT}$

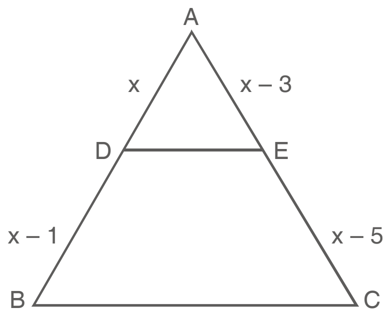
10. In ΔABC, $DE \parallel BC$. If $\frac{AD}{DB} = \frac{3}{5}$ then $\frac{ar(\Delta ADE)}{ar(\Delta ABC)}$ = ______



(a) $\frac{3}{5}$ (b) $\frac{3}{8}$

(c) $\frac{9}{64}$ (d) $\frac{9}{25}$

11. In ΔABC, $DE \parallel BC$. In the figure the value of x is ______



(a) 1 (b) –1

(c) 3 (d) –3

12. In ΔABC, $\angle B = 90°$, BE is the perpendicular bisector of AC then $\dfrac{ar\,(\Delta BEC)}{ar\,(\Delta ABC)} =$ ________

(a) $\dfrac{1}{2}$ (b) $\dfrac{2}{1}$

(c) $\dfrac{4}{1}$ (d) $\dfrac{1}{4}$

13. The altitude of an equilateral triangle, having the length of its side 12cm is

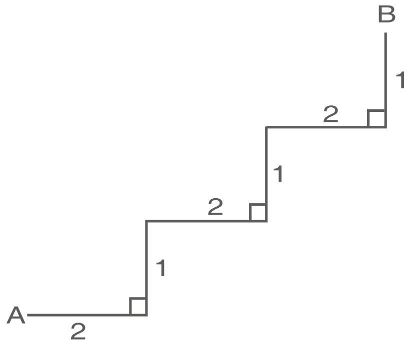
(a) 12 cm (b) $6\sqrt{2}$ cm

(c) 6 cm (d) $6\sqrt{3}$ cm

14. The straight line distance between A and B is

(a) $3\sqrt{5}$ (b) $5\sqrt{3}$

(c) 5 (d) $5\sqrt{2}$



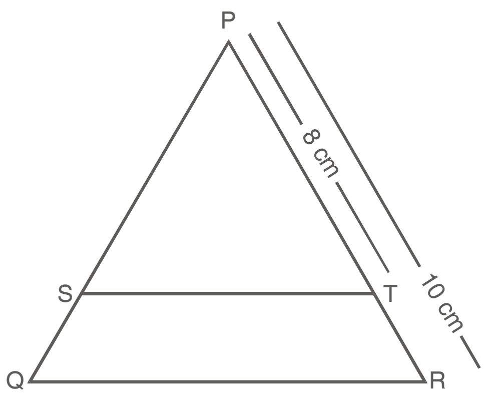
15. If in an isosceles right-angled triangle the length of the hypotenuse is 10 cm then the perimeter of he triangle is

(a) $5\sqrt{2}$ cm (b) $2\sqrt{5}$ cm

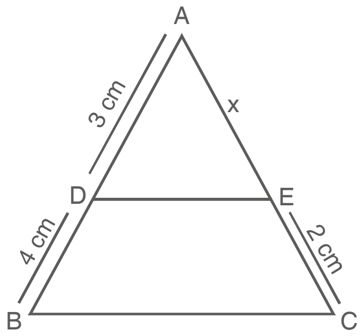
(c) $10\left(\sqrt{2}+1\right)$ cm (d) $10\left(\sqrt{2}-1\right)$ cm

SHORT ANSWER TYPE QUESTIONS

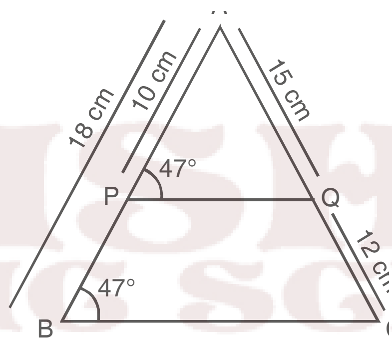
16. In figure if $ST \parallel QR$, $PT = 8$ cm and $PR = 10$ cm then what is the value of $\frac{PS}{SQ}$.



17. In the adjoining figure find *AE* if *DE* || *BC*

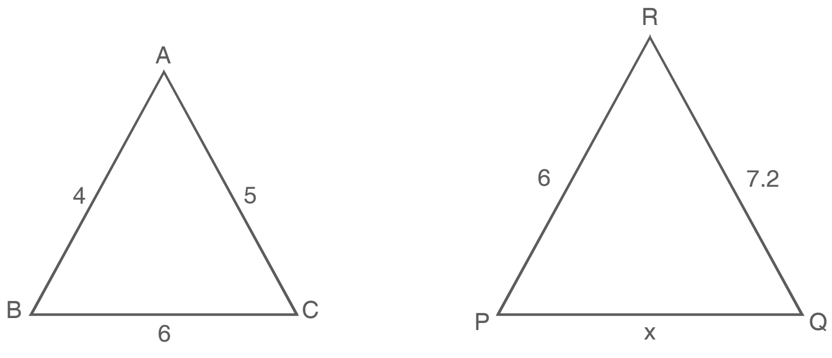


18. In the figure name the similar triangles.

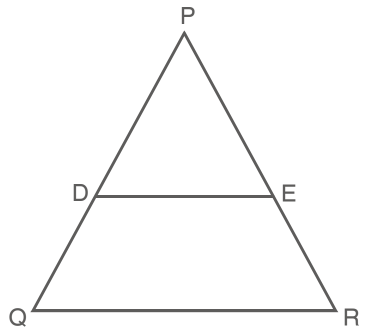


19. An isosecles triangle *ABC* is similar to triangle *PQR*. *AC* = *AB* = 4 cm, *RQ* = 10 cm and *BC* = 6 cm. What is the length of *PR*? Which type of triangle is ΔPQR?

20. In the figure $\Delta ABC \sim \Delta PQR$. What is the value of *x*?



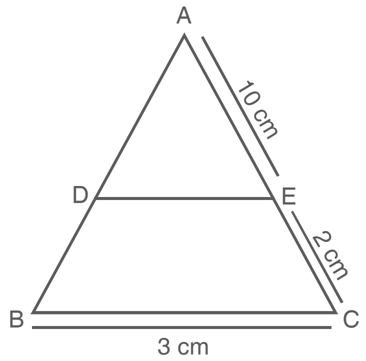
21. In ΔPQR, $DE \parallel QR$ and $DE = \frac{1}{4} QR$. Find $\frac{ar(\Delta PQR)}{ar(\Delta PDE)}$.



22. In triangles ABC and PQR if $\angle B = \angle Q$ and $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{1}{2}$ then what is the value of $\frac{PR}{QR}$?

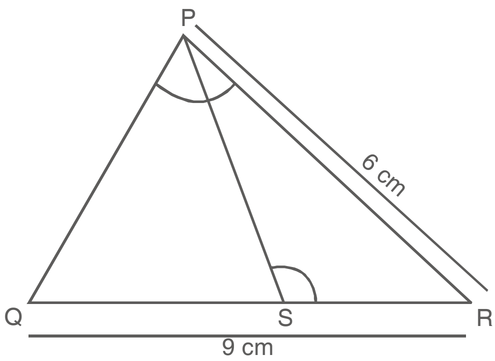
23. The measurement of three sides of a triangle are a, $\sqrt{10}a$, $3a$. What is the measurement of the angle opposite to the longest side?

24. In the adjoining figure $DE \parallel BC$. What is the value of DE.



LONG ANSWER TYPE QUESTIONS

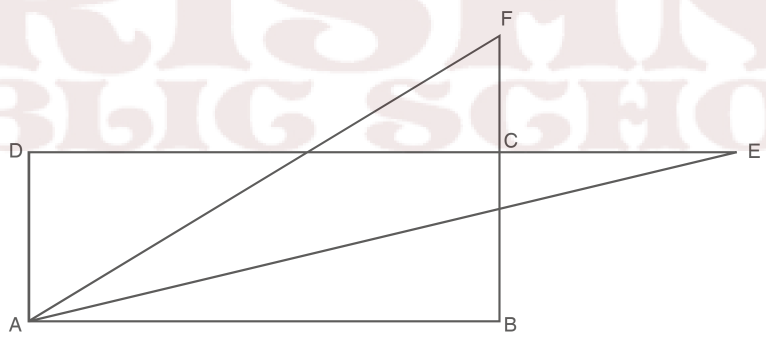
25. In the figure find SR if $\angle QPR = \angle PSR$. $PR = 6$ cm and $QR = 9$ cm



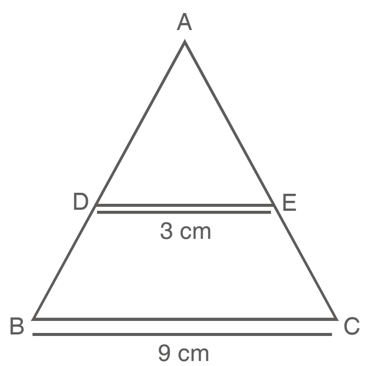
26. In ΔPQR, $RS \perp PQ$, $\angle QRS = \angle P$, $PS = 5$ cm, $SR = 8$ cm. Find PQ.

27. Two similar triangles ABC and PBC are made on opposite sides of the me base BC. Prove that $AB = BP$.

28. In figure $ABCD$ is a rectangle. ΔADE and ΔABF are two triangles such that $\angle E = \angle F$. Prove that $\dfrac{AD}{AE} = \dfrac{AB}{AF}$.



29. In figure $DE \parallel BC$, $DE = 3$ cm, $BC = 9$ cm and or (ΔADE) = 30 cm^2. Find *ar* (trap. $BCED$).

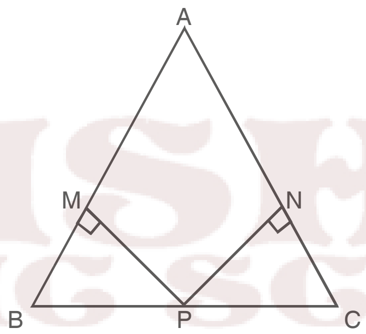


30. Amit is standing at a point on the ground 8m away from a house. A mobile network tower is fixed on the roof of the house. If the top and bottom of the tower are 17m and 10m away from the point. Find the heights of the tower and house.

31. In a right angled triangle *ABC*, right angle at *B*, $\frac{BC}{AB} = \sqrt{3}$. Find $\frac{AB}{AC}$.

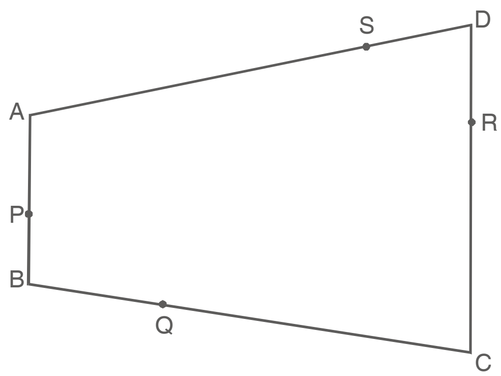
32. In a right angled triangle *PRO*, *PR* is the hypotenuse and the other two sides are of length 6cm and 8cm. *Q* is a point outside the triangle such that *PQ* = 24cm *RQ* = 26cm. What is the measure of $\angle QPR$?

33. In the figure ΔABC is isosceles with *AB* = *AC* *P* is the mid point of *BC*. If $PM \perp AB$ and $PN \perp AC$. Prove that *MP* = *NP*.



34. *PQRS* is a trapezium. *SQ* is a diagonal. *E* and *F* are two points on parallel sides *PQ* and *RS* respectively intersecting *SQ* at *G*. Prove that $SG \times QE = QG \times SF$.

35. In the figure *P, Q, R* and *S* are points on the sides of quadrilateral *ABCD* such that these points divides the sides *AB, CB, CD* and *AD* in the ratio 2 : 1. Prove that *PQRS* is a parallelogram.



36. Prove that if a line is drawn parallel to one side of a triangle, it divides the other two sides in the me ratio.

37. In a rhombus, prove that four times the square of any sides is equal to the sum of squares of its diagonals.

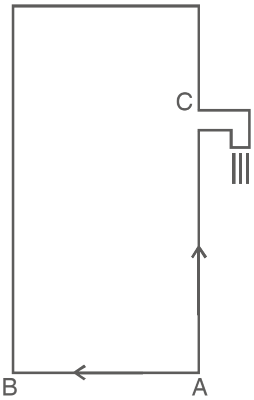
38. Prove that the ratio of the areas of two similar triangles is equal to the ratio of the squares of their corresponding sides.

39. In a triangle, if the square of one side is equal to the sum of the squares on the other two sides, then prove that the angle opposite to the first side is a right triangle.

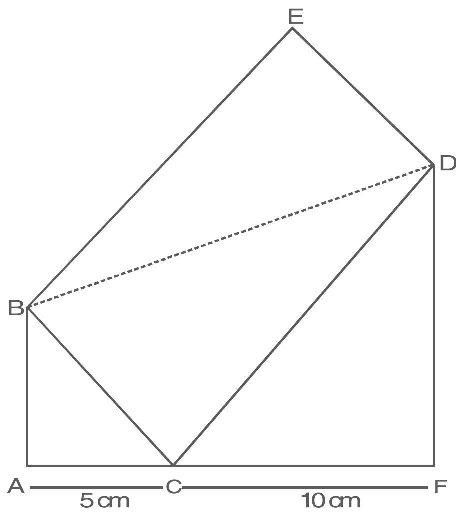
40. Prove that in a right triangle, the square on the hypotenuse is equal to the sum of the squares on the other two sides.

41. *ABCD* is a rectangle in which length is double of its breadth. Two equilateral triangles are drawn one each on length and breadth of rectangle. Find the ratio of their areas.

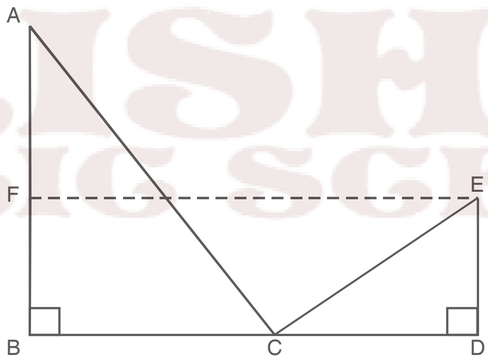
42. Amar and Ashok are two friends standing at a corner of a rectangular garden. They wanted to drink water. Amar goes due north at a speed of 50m/min and Ashok due west at a speed of 60m/min. They travel for 5 minutes. Amar reaches the tap and drink water. How r (minimum distance) is Ashok from the tap now.



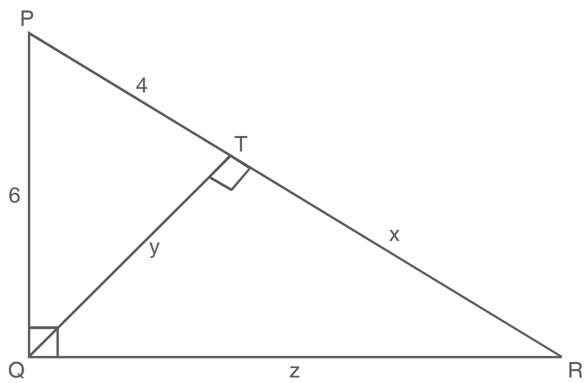
43. In the figure *BCDE* is a rectangle. Also $\angle BCA = \angle DCF$. Find the length of the diagonal *BD* of rectangle.



44. In the figure *BDEF* is a rectangle. *C* is the mid point of *BD. AF* = 7 cm, *DE* = 9 cm and *BD* = 24 cm. If *AE* = 25 cm then prove that $\angle ACE = 90°$.



45. In the figure altitude is drawn to the hypotenuse of a right angled triangle the lengths of different line-segments are marked. Determine *x, y, z*.



ANSWERS

1. *c*
2. *b*
3. *a*
4. *d*
5. *c*
6. *d*
7. *a*
8. *c*
9. *b*
10. *c*
11. *d*
12. *d*
13. *d*
14. *a*
15. *c*
16. (4 : 1)
17. 1.5 cm
18. $\Delta APQ \sim \Delta ABC$
19. $\frac{20}{3}$ cm
20. 4.8 cm
21. 16 : 1
22. $\frac{1}{2}$
23. 90°
24. 2.5 cm
25. 4 cm
26. 17.8 cm

29. 240 cm^2
30. 9m, 6m
31. $\frac{1}{2}$
32. 90°

41. 4 : 1
42. $50\sqrt{61}$ m
43. $5\sqrt{10}$ cm

45. $x = 5,\ y = 2\sqrt{5},\ z = 3\sqrt{5}$

CHAPTER 5

TRIGNOMETRY

KEY POINTS

1. **Trignometrical Ratios :** In ΔABC, $\angle B = 90°$ for angle 'A'

$$\sin A = \frac{\text{Perpendicular}}{\text{Hypotenuse}}$$

$$\cos A = \frac{\text{Base}}{\text{Hypotenuse}}$$

$$\tan A = \frac{\text{Perpendicular}}{\text{Base}}$$

$$\cot A = \frac{\text{Base}}{\text{Perpendicular}}$$

$$\sec .\ A = \frac{\text{Hypotenuse}}{\text{Base}}$$

$$\text{cosec } A = \frac{\text{Hypotenuse}}{\text{Perpendicular}}$$

2. **Reciprocal Relations :**

$$\sin\theta = \frac{1}{\text{cosec }\theta} \quad , \qquad \text{cosec }\theta = \frac{1}{\sin\theta}$$

$$\cos\theta = \frac{1}{\sec\theta} \quad , \qquad \sec\theta = \frac{1}{\cos\theta}$$

$$\tan\theta = \frac{1}{\cot\theta} \quad , \qquad \cot\theta = \frac{1}{\tan\theta}$$

3. **Quotient Relations :**

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \quad , \qquad \cot\theta = \frac{\cos\theta}{\sin\theta'}$$

4. **Indentities :**

$\sin^2\theta + \cos^2\theta = 1 \Rightarrow \sin^2\theta = 1 - \cos^2\theta$ and $\cos^2\theta = 1 - \sin^2\theta$

$1 + \tan^2\theta = \sec^2\theta \Rightarrow \tan^2\theta = \sec^2\theta - 1$ and $\sec^2\theta - \tan^2\theta = 1$

$1 + \cot^2\theta = \mathrm{cosec}^2\theta \Rightarrow \cot^2\theta = \mathrm{cosec}^2\theta - 1$ and $\mathrm{cosec}^2\theta - \cot^2\theta = 1$

5. **Trignometric Ratios of Some Specific Angles :**

$\angle A$	$0°$	$30°$	$45°$	$60°$	$90°$
sin A	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos A	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan A	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined
cosec A	Not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
sec A	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not defined
cot A	Not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

6. **Trignometric Ratios of Complementary Angles**

$$\sin(90° - \theta) = \cos\theta$$
$$\cos(90° - \theta) = \sin\theta$$
$$\tan(90° - \theta) = \cot\theta$$
$$\cot(90° - \theta) = \tan\theta$$
$$\sec(90° - \theta) = \mathrm{cosec}\,\theta$$
$$\mathrm{cosec}(90° - \theta) = \sec\theta$$

MULTIPLE CHOICE QUESTIONS

Note : In the following questions $0° \le \theta \le 90°$

1. If $x = a \sin\theta$ and $y = a\cos\theta$ then the value of $x^2 + y^2$ is ________

   (a) a (b) a^2

   (c) 1 (d) $\frac{1}{a}$

2. The value of cosec 70° – sec 20° is _____

   (a) 0 (b) 1

   (c) 70° (d) 20°

3. If $3 \sec\theta - 5 = 0$ then $\cot\theta$ = _____

   (a) $\frac{5}{3}$ (b) $\frac{4}{5}$

   (c) $\frac{3}{4}$ (d) $\frac{3}{5}$

4. If $\theta = 45°$ then $\sec\theta \cot\theta - \mathrm{cosec}\,\theta \tan\theta$ is

   (a) 0 (b) 1

   (c) $\sqrt{2}$ (d) $2\sqrt{2}$

5. If $\sin (90 - \theta) \cos \theta = 1$ and θ is an acute angle then $\theta =$ ____

(a) 90° (b) 60°

(c) 30° (d) 0°

6. The value of $(1 + \cos \theta)(1 - \cos \theta) \operatorname{cosec}^2\theta =$ _____

(a) 0 (b) 1

(c) $\cos^2 \theta$ (d) $\sin^2 \theta$

7. ΔTRY is a right-angled isosceles triangle then $\cos T + \cos R + \cos Y$ is _____

(a) $\sqrt{2}$ (b) $2\sqrt{2}$

(c) $1 + \sqrt{2}$ (d) $1 + \dfrac{1}{\sqrt{2}}$

8. If $K + 7 \sec^2 62° - 7 \cot^2 28° = 7 \sec 0°$ then the value of K is ______

(a) 1 (b) 0

(c) 7 (d) $\dfrac{1}{7}$

9. The value of $\cot\theta - \sin\left(\dfrac{\pi}{2} - \theta\right)\cos\left(\dfrac{\pi}{2} - \theta\right)$ is _______

(a) $\cot \theta \cos^2 \theta$ (b) $\cot^2\theta$

(c) $\cos^2 \theta$ (d) $\tan^2 \theta$

10. If $\sin \theta - \cos \theta = 0$, $0 \le \theta \le 90°$ then the value of θ is _____

(a) $\cos \theta$ (b) 45°

(c) 90° (d) $\sin \theta$

11. $\dfrac{\sin\theta}{\sqrt{1 - \sin^2\theta}}$ can be written as

(a) $\cot\theta$ (b) $\sqrt{\sin\theta}$

(c) $\dfrac{\sin\theta}{\sqrt{\cos\theta}}$ (d) $\tan\theta$

12. If $\sin\theta = \dfrac{1}{2}$ then the value of $\sin\theta + \operatorname{cosec}\theta$ is ______

(a) 0 (b) 1

(c) $\dfrac{3}{2}$ (d) $\dfrac{5}{2}$

13. In an isosceles right-angled ΔABC, $\angle B = 90°$. The value of $2 \sin A \cos A$ is _____

(a) 1 (b) $\dfrac{1}{2}$

(c) $\dfrac{1}{\sqrt{2}}$ (d) $\sqrt{2}$

14. If $\dfrac{\sin^2 20° + \sin^2 70°}{2\left(\cos^2 69° + \cos^2 21°\right)} = \dfrac{\sec 60°}{K}$ then K is ______

(a) 1 (b) 2

(c) 3 (d) 4

15. $\Delta ABC \sim \Delta PRT$ and $\angle C = \angle R = 90°$. If $\dfrac{AC}{AB} = \dfrac{3}{5}$ then $\sin T$ is _______

(a) $\dfrac{3}{5}$ (b) $\dfrac{5}{3}$

(c) $\dfrac{4}{5}$ (d) $\dfrac{5}{4}$

SHORT ANSWER TYPE QUESTIONS

16. In ΔPQR, $\angle Q = 90°$ and $\sin R = \frac{3}{5}$, write the value of cos P.

17. If A and B are acute angles and sin A = cos B then write the value of $A + B$.

18. If $4 \cot \theta = 3$ then write the value of $\tan \theta + \cot \theta$.

19. Write the value of $\cot^2 30° + \sec^2 45°$.

20. Write the value of $\sin (90 - \theta) \cos \theta + \cos (90 - \theta) \sin \theta$.

21. If $\theta = 30°$ then write the value of $\sin \theta + \cos^2 \theta$.

22. If $1 - \tan^2 \theta = \frac{2}{3}$ then what is the value of θ.

23. What is the value of $2 \operatorname{cosec}^2 \theta + 3 \sec^2 \theta - 10$ if $\theta = 45°$.

24. If θ and ϕ are complementary angles then what is the value of

$$\operatorname{cosec} \theta \sec \phi - \cot \theta \tan \phi$$

25. If $\tan (3x - 15°) = 1$ then what is the value of x.

26. If $8 \cot \theta - 15 = 0$ then what is the value of $\frac{1 + \sin \theta}{\cos \theta}$.

LONG ANSWER TYPE QUESTIONS

27. Simplify :

$$\tan^2 60° + 4 \cos^2 45° + 3 (\sec^2 30° + \cos^2 90°)$$

28. Find the value of

$$\frac{4}{5} \frac{\sin 65°}{\cos 25°} - \frac{13 \cos 53°.\operatorname{cosec} 37°}{5\left(7 \sec^2 32° - 7 \cot^2 58\right)}.$$

29. Prove that

$$\operatorname{cosec}^4 \theta - \operatorname{cosec}^2 \theta = \cot^2 \theta + \cot^4 \theta.$$

30. If sin θ + $\sin^2 \theta = 1$ then find the value of $\cos^2 \theta + \cos^4 \theta$.

31. If $\sin 2\theta = \cos(\theta - 36°)$, 2θ and $\theta - 26°$ are acute angles then find the value of θ.

32. If $\sin(3x + 2y) = 1$ and $\cos(3x - 2y) = \frac{\sqrt{3}}{2}$, where $0 \le (3x + 2y) \le 90°$ then find the value of x and y.

33. If $\sin(A + B) = \sin A \cos B + \cos A \sin B$ then find the value of

(a) sin 75°

(b) cos 15°

34. Prove that $\frac{\cos A}{1 - \tan A} + \frac{\cos A}{1 - \cot A} = \cos A$, $A \ne 45°$.

35. Prove that $\sqrt{\frac{\sec\theta - 1}{\sec\theta + 1}} + \sqrt{\frac{\sec\theta + 1}{\sec\theta - 1}} = 2\operatorname{cosec}\theta$

36. Find the value of

$$\sin^2 5° + \sin^2 10° + \sin^2 15° + ... + \sin^2 85°$$

37. Prove that

$$\frac{\tan\theta + \sec\theta - 1}{\tan\theta - \sec\theta + 1} = \frac{\cos\theta}{1 - \sin\theta}.$$

38. If $2\sin(3x - 15) = \sqrt{3}$ then find the value of

$$\sin^2(2x + 10) + \tan^2(x + 5).$$

39. Find the value of sin 60° geometrically.

40. Let $p = \tan\theta + \sec\theta$ then find the value of $p + \frac{1}{p}$.

41. In right angled ΔOPQ, right angle at P. $OP = 7$ cm and $\angle Q = \alpha$. If $\sec(90 - \alpha) - \tan(90 - \alpha) = \frac{1}{7}$ then what is the value of $OQ - PQ$.

42. If $\sin \alpha = a \sin \beta$ and $\tan \alpha = b \tan \beta$ then prove that $\cos^2 \alpha = \dfrac{a^2 - 1}{b^2 - 1}$.

43. If θ is acute angle and $5 \sin^2 \theta + \cos^2 \theta = 4$ then find the value of θ.

44. In an acute angled ΔABC, if $\sin(A + B - C) = \dfrac{1}{2}$ and $\cos(B + C - A) = \dfrac{1}{\sqrt{2}}$ then find angles A, B and C.

45. If A, B, C are the interior angles of a triangle ABC, show that

$$\sin\left(\frac{B + C}{2}\right)\cos\frac{A}{2} + \cos\left(\frac{B + C}{2}\right)\sin\frac{A}{2} = 1.$$

ANSWERS

1. *b*
2. *a*
3. *c*
4. *a*
5. *d*
6. *b*
7. *a*
8. *b*
9. *a*
10. *b*
11. *d*
12. *d*
13. *a*
14. *d*
15. *a*
16. $\cos P = \dfrac{3}{5}$
17. 90°
18. $\dfrac{25}{12}$
19. 5
20. 1
21. $\dfrac{5}{4}$
22. 30°

23. 0

24. 1

25. $x = 20$.

26. $\frac{5}{3}$

27. 9

28. $\frac{3}{7}$

30. 1

31. $42°$

32. $x = 20, y = 15$

33. $\frac{\sqrt{3} + 1}{2\sqrt{2}}, \frac{\sqrt{3} + 1}{2\sqrt{2}}$, take $A = 45°, B = 30°$

34. –

35. –

36. $\frac{17}{2}$

37. –

38. $\frac{13}{12}$

39. –

40. $2 \sec \theta$

41. 1

42. –

43. $60°$

44. $\angle A = 67.5°, \angle B = 37.5°, \angle C = 75°$

CHAPTER 6

STATISTICS

KEY POINTS

1. The mean for grouped data can be found by :

   (i) The direct method $= \overline{X} = \dfrac{\sum fixi}{\sum fi}$.

   (ii) The assumed mean method $= \overline{X} = a + \dfrac{\sum fidi}{\sum fi}$, where $d_i = x_i - a$.

   (iii) The step deviation method

   $$= \overline{X} = a + \frac{\sum fiui}{\sum fi} \times h, \text{ where } u_i = \frac{x_i - a}{h}.$$

2. The mode for the grouped data can be found by using the formula :

$$\text{mode} = l + \left[\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right] \times h$$

l = lower limit of the model class.

f_1 = frequency of the model class.

f_0 = frequency of the proceeding class of the model class.

f_2 = frequency of the succeeding class of the model class.

h = size of the class interval.

Model class - class interval with highest frequency.

3. The median for the grouped data can be found by using the formula :

$$\text{median} = l + \left[\frac{n/2 - Cf}{f}\right] \times h$$

l = lower limit of the median class.

n = number of observations.

Cf = cumulative frequency of class interval preceeding the median class.

f = frequency of median class.

h = class size.

4. Empirical Formula : Mode = 3 median - 2 mean.

5. Cumulative frequency curve or an Ogive :

   (i) Ogive is the graphical representation of the cumulative frequency distribution.

   (ii) Less than type Ogive :

   - Construct a cumulative frequency table.
   - Mark the upper class limit on the x = axis.

   (iii) More than type Ogive :

   - Construct a frequency table.
   - Mark the lower class limit on the x-axis.

   (iv) To obtain the median of frequency distribution from the graph :

   - Locate point of intersection of less than type Ogive and more than type Ogive :

   Draw a perpendicular from this point of x-axis.

   - The point at which it cuts the x-axis gives us the median.

MULTIPLE CHOICE QUESTIONS

1. Mean of first 10 natural numbers is

   (a) 5 (b) 6

   (c) 5.5 (d) 6.5

2. If mean of 4, 6, 8, 10, x, 14, 16 is 10 then the value of 'x' is

   (a) 11 (b) 12

   (c) 13 (d) 9

3. The mean of x, $x + 1$, $x + 2$, $x + 3$, $x + 4$, $x + 5$ and $x + 6$ is

   (a) x (b) $x + 3$

   (c) $x + 4$ (d) 3

4. The median of 2, 3, 2, 5, 6, 9, 10, 12, 16, 18 and 20 is

   (a) 9 (b) 20

   (c) 10 (d) 9.5

5. The median of 2, 3, 6, 0, 1, 4, 8, 2, 5 is

   (a) 1 (b) 3

   (c) 4 (d) 2

6. Mode of 1, 0, 2, 2, 3, 1, 4, 5, 1, 0 is

   (a) 5 (b) 0

   (c) 1 (d) 2

7. If the mode of 2, 3, 5, 4, 2, 6, 3, 5, 5, 2 and x is 2 then the value of 'x' is

   (a) 2 (b) 3

   (c) 4 (d) 5

8. The model class of the following distribution is

Class Interval	10–15	15–20	20–25	25–30	30–35
Frequency	4	7	12	8	2

(a) 30–35 (b) 20–25

(c) 25–30 (d) 15–20

9. A teacher ask the student to find the average marks obtained by the class students in Maths the student will find

(a) mean (b) median

(c) mode (d) sum

10. The empirical relationship between the three measures of central tendency is

(a) 3 mean = mode + 2 median (b) 3 median = mode + 2 mean

(c) 3 mode = mean + 2 median (d) median = 3 mode – 2 mean

11. Class mark of the class 19.5 – 29.5 is

(a) 10 (b) 49

(c) 24.5 (d) 25

12. Measure of central tendency is represented by the abscis of the point where the 'less than ogive' and 'more than ogive' intersect is

(a) mean (b) median

(c) mode (d) None of these

13. The median class of the following distribution is

Class Interval :	0–10	10–20	20–30	30–40	40–50	50–60	60–70
Frequency :	4	4	8	10	12	8	4

(a) 20–30 (b) 40–50

(c) 30–40 (d) 50–60

14. The mean of 20 numbers is 17, if 3 is added to each number, then the new mean is

(a) 20 (b) 21

(c) 22 (d) 24

15. The mean of 5 number is 18. If one number is excluded then their mean is 16, then the excluded number is

(a) 23 (b) 24

(c) 25 (d) 26

16. The mean of first 5 prime numbers is

(a) 5.5 (b) 5.6

(c) 5.7 (d) 5

17. The sum of deviations of the values 3, 4, 6, 8, 14 from their mean is

(a) 0 (b) 1

(c) 2 (d) 3

18. If median = 15 and mean = 16, then mode is

(a) 10 (b) 11

(c) 12 (d) 13

19. The mean of 11 observations is 50. If the mean of first six observations is 49 and that of last six observations is 52, then the sixth observation is

(a) 56 (b) 55

(c) 54 (d) 53

20. The mean of the following distribution is 2.6, then the value of 'x' is

Variable	1	2	3	4	5
Frequency	4	5	x	1	2

(a) 24 (b) 3

(c) 8 (d) 13

LONG ANSWER TYPE QUESTIONS

21. The mean of 40 observations was 160. It was detected on rechecking that the value of 165 was wrongly copied as 125 for computing the mean. Find the correct mean.

22. Find 'x' if the median of the observations in ascending order 24, 25, 26, $x + 2$, $x + 3$, 30, 31, 34 is 27.5.

23. Find the median of the following data.

x :	10	12	14	16	18	20
f :	3	5	6	4	4	3

24. Find the value of 'p', if mean of the following distribution is 7.5

Variable :	3	5	7	9	11	13
Frequency :	6	8	15	p	8	4

25. Find the mean of the following distribution.

x :	12	16	20	24	28	32
f :	5	7	8	5	3	2

26. Find the mean of the following distribution.

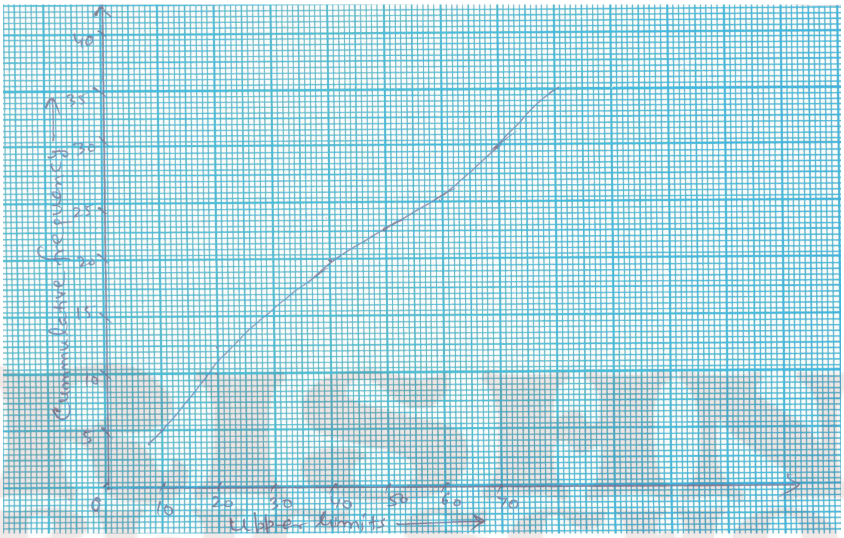
Class :	0–10	10–20	20–30	30–40	40–50
Frequency :	8	12	10	11	9

27. From the cummulative frequency table, write the frequency of the class 20–30.

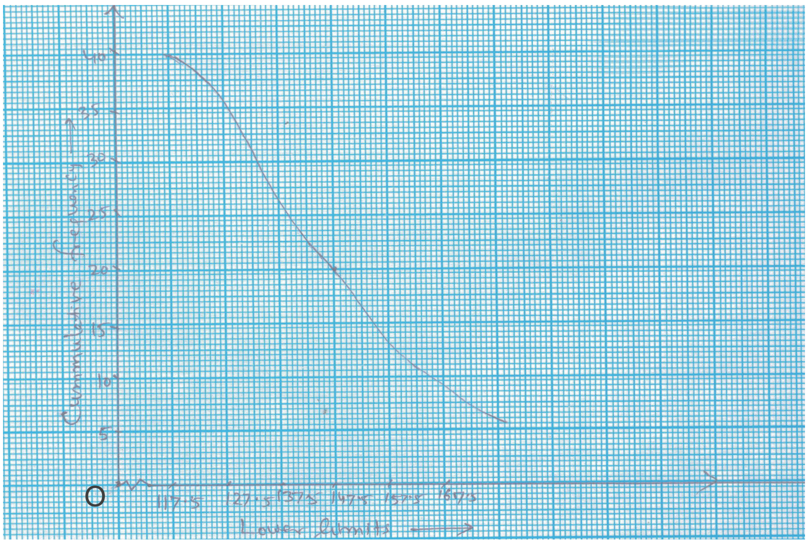
Marks	***Number of Student***
Less than 10	1
Less than 20	14

Less then 30	36
Less than 40	59
Less than 50	60

28. Following is a commulative frequency curve for the marks obtained by 40 students as show in figure. Find the median marks obtained by the student.



29. The following 'more than ogive'. Shows the weight of 40 students of a class. What is the lower limit of the median class.



30. The mean of the following frequency distribution is 62.8 and the sum of all the frequencies is 50. Find the values of x and y.

Class Interval :	0–20	20–40	40–60	60–80	80–100	100–120
Frequency :	5	x	10	y	7	8

31. The following frequency distribution gives the daily wage of a worker of a ctory. Find mean daily wage of a worker.

Daily Wage (in ₹)	***Number of Workers***
More than 300	0
More than 250	12
More than 200	21
More than 150	44
More than 100	53
More than 50	59
More than 0	60

32. The median of the following frequency distribution is 28.5 and sum of all the frequencies is 60. Find the values of x and y.

Class Interval :	0–10	10–20	20–30	30–40	40–50	50–60
Frequency :	5	x	20	15	y	5

33. Find the mean, median and mode of the following :

Class Interval :	0–10	10–20	20–30	30–40	40–50	50–60	60–70
Frequency :	6	8	10	15	5	4	2

34. The following frequency distribution shows the marks obtained by 100 students in a school. Find the mode.

Marks	*Number of Students*
Less than 10	10
Less than 20	15

Less than 30	30
Less than 40	50
Less than 50	72
Less than 60	85
Less than 70	90
Less than 80	95
Less than 90	100

35. Draw 'less than' and 'more than' ogives for the following distribution

Marks :	0–10	10–20	20–30	30–40	40–50	50–60	60–70	70–80	80–90	90–100
No. of Students :	5	6	8	10	15	9	8	7	7	5

Also find median from graph.

36. A survey regarding the height (in cm) of 50 students of class x of a school was conducted and regarding the following data was obtained.

Height (in cm) :	120–130	130–140	140–150	150–160	160–170	Total
No. of Students :	2	8	12	20	8	50

Find the mean, median and mode of the above data.

37. The mode of the following distribution is 65. Find the values of x and y, if sum of the frequencies is 50.

Class Interval :	0–20	20–40	40–60	60–80	80–100	100–120	120–140
Frequency :	6	8	x	12	6	y	3

38. During the medical checkup of 35 students of class '*X*' their weights recorded as follows :

Weight (in kg.) :	38–40	40–42	42–44	44–46	46–48	48–50	50–52
Number Students :	3	2	4	5	14	4	3

find mean median and mode of the above data.

39. The weekly observations on cost of living index is a city for the year 2008-2009 are given below:

Cost of Living Index :	140–150	150–160	160–170	170–180	180–190	190–200	Total
No. of Weeks :	5	10	20	9	6	2	52

Find the mean weekly cost of living index.

40. Find the mode of the following distribution

Class:	3–6	6–9	9–12	12–15	15–18	18–21	21–24
Frequency :	2	5	10	023	21	12	3

ANSWERS

1.

1.	*c*	2.	*b*
3.	*b*	4.	*a*
5.	*b*	6.	*c*
7.	*a*	8.	*b*
9.	*a*	10.	*b*
11.	*c*	12.	*b*
13.	*c*	14.	*a*
15.	*d*	16.	*b*
17.	*a*	18.	*d*
19.	*a*	20.	*c*
21.	161	22.	$x = 25$
23.	14.8	24.	$p = 3$
25.	20	26.	25.2

27. 22

28. 40

29. 147.5

30. $x = 8$, $y = 12$

31. ₹ 182.50

32. $x = 8$, $y = 7$

33. Mean = 30, Median = 30.67, Mode = 33.33

34. 41.82

35. 47.3 (Approx)

36. Mean = 149.8 cm, Median = 151.5 cm, Mode = 154 cm

37. $x = 10$, $y = 5$.

38. Mean = 45.8, Median = 46.5, Mode = 47.9

39. 166.3

40. 14.6

QUESTION PAPER

MATHEMATICS, - 1

Time allowed : 3 hours

Maximum Marks : 80

General Instructions

1. All question are compulsory.
2. The question paper consists of 34 questions divided into four sections A, B, C and D. Section A comprises of 10 questions of 1 mark each. Section B comprises of 8 questions of 2 marks each. Section C comprises of 10 questions of 3 marks each and Section D comprises of 6 questions of 4 marks each.
3. Question numbers 1 to 10 in Section A are multiple choice questions where you are to select one correct option out of the given four.
4. There is no overall choice. How ever, internal choice has been provided in 1 question of 2 marks 3 questions of three marks each and 2 questions of 4 marks each. You have to attempt only one of the alternatives in all such questions.
5. Use of calculators is not permitted.

SECTION A

Question number 1 to 10 are of 1 mark each

1. ΔABC is right angled at A. The value of tan B . tan C is _______

   (a) tan B (b) tan C

   (c) 0 (d) 1

2. In Euclid Division Lemma, when $x = yq + r$, where x and y are positive integers which one is correct.

(a) $0 \leq r < y$ (b) $0 \leq r < y$

(c) $0 < r < y$ (d) $0 \leq r \leq y$

3. If the mean of 2, 4, 6, 8, 10, x, 14, 16 is 9 then the value of x is

(a) 10 (b) 11

(c) 12 (d) 13

4. Graph of $y = ax^2 + bx + c$ intersects x-axis at 2 distinct points if

(a) $b^2 - 4ac = 0$ (b) $b^2 - 4ac > 0$

(c) $b^2 - 4ac < 0$ (d) $b^2 - 4ac \geq 0$

5. If $\sin 3\theta = \frac{\sqrt{3}}{2}$, $0° < \theta < 90°$ then the value of θ is _____

(a) 0° (b) 20°

(c) 30° (d) 60°

6. The modal class of the following distribution is

Class Interval :	10–20	20–30	30–40	40–50	50–60	60–70	70–80
Frequency :	3	5	8	10	9	4	3

(a) 70–80 (b) 40–50

(c) 50–60 (d) 30–40

7. If product of the zeroes is 5 and sum of the zeroes is –2 then the quadratic polynomial will be–

(a) $x^2 - 5x - 2$ (b) $x^2 + 5x - 2$

(c) $x^2 + 2x - 5$ (d) $x^2 + 2x + 5$

8. The relationship in mean, median and mode is

(a) Mode = 2 median – 3 mean (b) Mode = 2 median - mean

(c) Mode = 3 median + 2 mean (d) Mode = 3 median – 2 mean

9. The coordinates of the point where y-axis and the line represented by $\frac{x}{2} + \frac{y}{3} = 1$ intersect are :

(a) (0, 2) (b) (2, 0)

(c) (0, 3) (d) (3, 0)

10. If $x = \tan 2° \cdot \tan 36° \cdot \tan 54° \cdot \tan 88°$ then the value of x is _______

(a) 45° (b) 1

(c) 2 (d) 90°

SECTION B

Question number 11 to 18 are of 2 marks each

11. State Euclid's Division Lemma and hence find HCF of 15 and 21.

12. Find the mean of the following distribution :

x :	12	16	20	24	28	32
f :	5	7	8	5	3	2

13. In ΔABC, D is the mid point of the side AB and $DE \parallel BC$ meets AC at E. Prove that $AE = \frac{1}{2} AC$.

OR

If $\Delta ABC \sim \Delta DEF$, $BC = 5$ cm, $EF = 4$ cm and $ar(\Delta ABC) = 75$ cm^2. Find the area of ΔDEF.

14. If sum of the zeroes of $kx^2 + 5x + k$ is equal to the product of the zeroes. Find value of k.

15. Draw 'less than ogive' for the following distribution :

Class Interval :	0–10	10–20	20–30	30–40	40–50	50–60
Frequency :	5	8	12	10	7	4

16. Without using trigonometric tables, evaluate

$$3\left(\frac{\sin 54}{\cos 36}\right)^2 + 2\tan 14^\circ \tan 30^\circ \tan 76^\circ.$$

17. For what value of p, the pair of linear equations

$$y - 2x - 5 = 0$$

$px = 2y$ has unique solution.

18. If $\sin\theta = \frac{1}{6}$, $0^\circ < \theta < 90^\circ$ then evaluate $\sec\theta + \tan\theta$.

SECTION C

Question number 19 to 28 carry 3 marks each

19. Check graphically whether the pair of linear equations $x - 2y = 4$ and $x - y = 3$ is consistent. Is this pair dependent also.

20. Prove that $\frac{1}{5 - 2\sqrt{3}}$ is irrational.

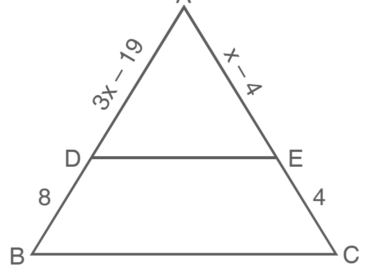
OR

Prove $\sqrt{5} - \sqrt{2}$ that is irrational.

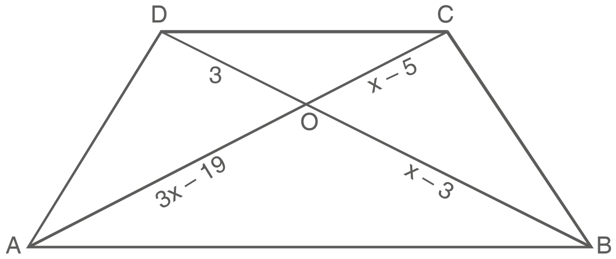
21. In ΔABC, $\angle C = 90^\circ$ points P and Q lies on sides CA and CB respectively prove that

$$AQ^2 + BP^2 = AB^2 + PQ^2$$

22. In figure, find x if $DE \parallel BC$



OR



In the figure *ABCD* is a trapezium. Find the value of *x*.

23. Solve for *x* and *y* : $\frac{2}{x-1} + \frac{3}{y+1} = 2$

$$\frac{3}{x-1} + \frac{2}{y+1} = \frac{13}{6}$$

$x \neq 1,\ y \neq -1$

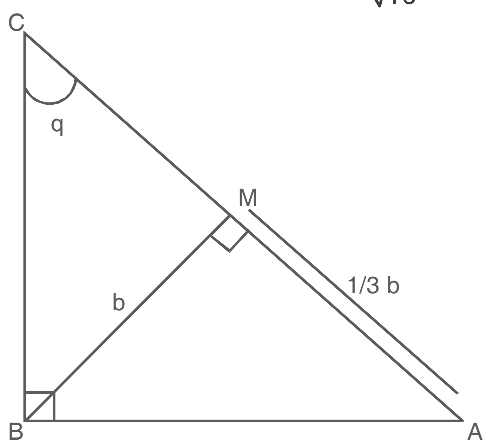
24. Find the other two ctors of $2x^4 - 3x^3 - 3x^2 + 6x - 2$ if two of its ctors are $\left(x - \sqrt{2}\right)$ and $\left(x + \sqrt{2}\right)$.

25. Prove that $(1 + \tan A \tan B)^2 + (\tan A - \tan B)^2 = \sec^2 A \sec^2 B$ where *A* and *B* are acute angles of a triangle.

OR

Prove this $(1 + \cot\theta - \text{cosec}\,\theta)(1 + \tan\theta + \sec\theta) = 2$.

26. In the adjoining figure prove that $\sin\theta = \frac{1}{\sqrt{10}}$



27. Find Geometrically the value of sin 30°.

28. Equiangular triangles are drawn on sides of right angled triangle in which perpendicular is double of the base. Show that the area of the triangle on the hypotenuse is the sum of the areas of the other two triangles.

SECTION D

Question number 29 to 34 carry 4 marks each

29. Show that the square of any positive integer is of the form $5q$, $5q + 1$, $5q + 4$ for some positive integer q.

30. In three digit number, the digit at the hundred's place is three times the digit at one's place. The sum of the digits is 15. If the digits are reversed the number is reduced by 396. Find the original number.

OR

A mily of 4 members is travelling in railways 3 tier coach another mily of 3 members is travelling in 2 tier coach. The combined re of both the milies is Rs. 5100. If first mily had 1 member less and the second had 1 member more, the total re would have been Rs. 300 more. What will be the re for a complex in railways 2-tier coach for the some journey.

31. ΔABC is an acute angled triangle. If tan $(A + B - C) = 1$ and sec $(B + C - A) = 2$ find ∠A, ∠B, and ∠C.

32. If the median of the following distribution is 28.5, than find the values of x and y.

Class Interval :	0–10	10–20	20–30	30–40	40–50	50–60	Total
Frequency :	5	x	20	15	y	5	60

33. Find the mode of the following distribution.

Marks :	0–10	10–20	20–30	30–40	40–50
No. of Students :	5	15	20	8	2

34. Prove that in a right angled triangle, the square of the hypotenuse is equal to the sum of the squares on the other two sides.

ANSWERS

1. *d*

2. *a*

3. *c*

4. *b*

5. *b*

6. *b*

7. *d*

8. *d*

9. *c*

10. *b*

11. 3

12. 20

13. 48 cm ^{2}

14. $k = -5$

15. $3 + \frac{2}{\sqrt{3}}$

17. $p \neq 4$

18. $\frac{7}{\sqrt{35}}$

19. Yes, no

22. $x = 11,\ x = 8\sqrt{x} = 9$

23. $x = 3,\ y = 2$

24. $(2x - 1)\ (x - 1)$

26. –

30. 672 **OR** [Rs. 800, Rs. 900] Rs. 1800

31. $\angle A = 60°,\ \angle B = 52.5°,\ \angle C = 67.5°$

32. $x = 8,\ y = 7$

33. 22.9

34.

QUESTION PAPER

MATHEMATICS, - 1

Time allowed : 3 to 3½ hours
Maximum Marks : 80

General Instructions

1. All question are compulsory.
2. The question paper consists of 34 questions divided into four sections A, B, C and D. Section A comprises of 10 questions of 1 mark each. Section B comprises of 8 questions of 2 marks each. Section C comprises of 10 questions of 3 marks each and Section D comprises of 6 questions of 4 marks each.
3. Question numbers 1 to 10 in Section A are multiple choice questions where you are to select one correct option out of the given four.
4. There is no overall choice. How ever, internal choice has been provided in 1 question of 2 marks 3 questions of three marks each and 2 questions of 4 marks each. You have to attempt only one of the alternatives in all such questions.
5. Use of calculators is not permitted.

SECTION A

Question number 1 to 10 are of 1 mark each

1. Euclid's Division Lemma states that for any two positive integers a and b, there exists unique integers q and r such that $a = bq + r$ where r must tisty :

   (a) $0 < r < b$ (b) $0 \leq r \leq b$

   (c) $0 < r \leq b$ (d) $0 \leq r \leq b$

2. In Fig. 1, the graph of a polynomial $p(x)$ is shown. The number of zeroes of $p(x)$ is:

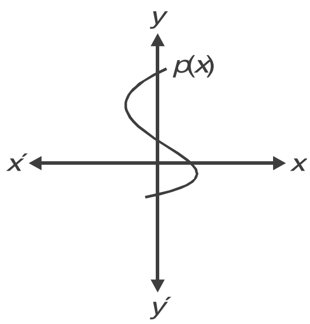


Fig. 1

(a) 1 (b) 2

(c) 3 (d) 4

3. In Fig. 2, if DE || BC, then x equals :

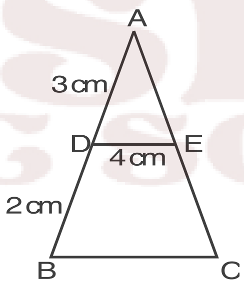


Fig. 2

(a) 3 cm (b) 2 cm

(c) 4 cm (d) $\frac{20}{3}$ cm

4. If $\sin(\theta + 36°) = \cos\theta$ where θ and $\theta + 36°$ are acute angles, then value of θ is

(a) 36° (b) 54°

(c) 27° (d) 90°

5. If $3\cos\theta = 2\sin\theta$ then the value of $\frac{4\sin\theta - 3\cos\theta}{2\sin\theta + 6\cos\theta}$ is :

(a) $\frac{1}{8}$ (b) $\frac{1}{3}$

(c) $\frac{1}{2}$ (d) $\frac{1}{4}$

6. In fig. 3, ΔABC is right angled at B and $\tan A = \frac{4}{3}$. If $AC = 15$ cm the length of BC is :

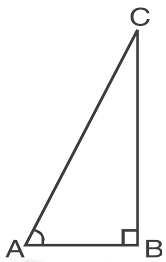


Fig.3

(a) 4 cm (b) 3 cm

(c) 12 cm (d) 9 cm

7. The decimal expansion of $\frac{21}{24}$ will terminate after how many places of decimal?

(a) 1 (b) 2

(c) 3 (d) 4

8. The pair of linear equations $x - 2y = 5$ and $2x - 4y = 10$ have :

(a) Many Solutions (b) No Solution

(c) One Solution (d) Two Solution

9. If $\tan A = \cot = B = \frac{15}{7}$ then $A + B$ is equal to :

(a) zero (b) $90°$

(c) $< 90°$ (d) $> 90°$

10. For a given data with 50 observations 'the less than Ogive' and the 'more than 'Ogive' interesect at (38.5, 34). The median of the data is :

(a) 38.5 (b) 34

(c) 50 (d) 4.5

SECTION B

Question number 11 to 18 are of 2 marks each

11. Is $7 \times 11 \times 13 + 11$ a composite number? Justify your answer.

12. Can $(x + 2)$ be the remainder on division of a polynomial $p(x)$ by $(2x - 5)$. Justify your answer.

13. In Fig. 4, ABCD is a rectangle. Find the value of x and y.

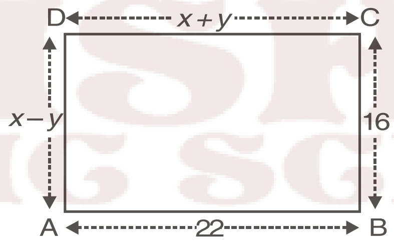


Fig. 4

14. If $\sin(A + B) = 1$ and $\cos(A - B) = 1$, $0° \leq A + B \leq 90°$, find A and B.

OR

If $\cot\theta = \frac{7}{8}$, evaluate $\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$

15. $ABCD$ is a trapezium in which $AB \parallel DC$ and its diagonals intersect each other at O. Prove that $\frac{AO}{BO} = \frac{CO}{DO}$:

16. In Fig. 5, $\angle S = 90°$, $PQ = 10cm$, $QS = 6$ cm and $RQ = 6$ cm. Calculate the length PR.

17. The following table shows the distribution of the heights of a group of 50 ctory workers.

Height (in cm)	150-155	155-160	160-165	165-170	170-175	175-180
No. of Workers	8	14	20	4	3	1

Convert the distribution to a less than type cumulative frequency distribution.

18. Find the mode of the following distribution :

Height (in cm)	30-40	40-50	50-60	60-70	70-80
No. of Plants	4	3	8	11	8

SECTION C

Question number 19 to 28 carry 3 marks each

19. Show that the square of any positive integer is of the form $3q$ or $3q + 1$ for some integer q :

20. Prove that $\frac{3\sqrt{2}}{5}$ is irrational.

OR

Prove $\left(5 + \sqrt{3}\right)$ is irrational.

21. A person starts his job with a certain monthly lary and earns a fixed increment every year. If his lary was Rs. 4500 after 4 years of service and Rs. 5400 after ten years of service, find his initial lary and the annual increment.

OR

After five years the age of Sudama will be three times that of his son. Five years ago Sudama was seven times that of his son. What are their present age?

22. In α, β are the zeroes of the polynomial $3x^2 + 5x - 2$ then form a quadratic polynomials whose zeroes are 2α and 2β.

23. Prove that $\frac{\cot A - \cos A}{\cot A + \cos A} = \frac{\operatorname{cosec} A - 1}{\operatorname{cosec} A + 1}$:

24. If $\cos\theta - \sin\theta = \sqrt{2}\sin\theta$ then prove that $\cos\theta + \sin\theta = \sqrt{2}\cos\theta$:

25. In Fig. $AD \perp BC$. Prove that $AB^2 + CD^2 = BD^2 + AC^2$:

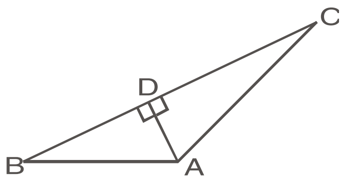


Fig. 6

26. Prove thjat the area of an equilateral triangle on the side of a square is half the area of an equilateral triangle formed on its diagonal.

27. Find mean of the following frequency distribution using step devition method:

Classes	25-30	30-35	35-40	40-45	45-50
Frequency	7	14	22	16	11

OR

The mean of the following frequency distribution is 47. Find the value of p :

Classes	0-20	20-40	40-60	60-80	80-100
Frequency	5	15	20	p	5

28. Find the median of the following data :

Classes	40-45	45-50	50-55	55-60	60-65	65-70
Frequency	2	3	8	6	6	5

SECTION D

Question number 29 to 34 carry 4 marks each

29. Find all the zeroes of $2x^4 + 7x^3 + 19x^2 - 14x + 30$ given that two of its zeroes are $\sqrt{2}$ and $-\sqrt{2}$.

30. Prove that in a right triangle the square of the hypotenuse is equal to the sum of the squares of the other two sides :

OR

Prove that the ratio of the sreas of two similar triangles is equal to the squares of the ratio of their corresponding sides.

31. Prove that $\cos^8 \theta - \sin^8 \theta = (\cos^2 \theta - \sin^2 \theta)(1 - 2\sin^2 \theta \cos^2 \theta)$:

OR

Find the value of :

$$\tan(90^\circ - \theta)\cot\theta - \sec(90 - \theta)\operatorname{cosec}\theta + \frac{3\left(\cot^2 27^\circ - \sec^2 63^\circ\right)}{\cot 26^\circ \cot 41^\circ \cot 45^\circ \cot 49^\circ \cot 64^\circ}$$

32. Prove that : $\dfrac{\cos A}{1 - \tan A} + \dfrac{\sin A}{1 - \cot A} = \sin A + \cos A.$

33. Solve graphically : $4x - y = 4$, $4x + y = 12$.

(*a*) Find the solution nfrom the graph.

(b) Shade the triangle region formed by the lines and the x – axis :

34. The following distribution gives the heights of 100 pupils in a school :

Height (in cm)	120-130	130-140	140-150	150-160	160-170	170-180
No. of Pupils	12	16	30	20	14	8

Change the above distribution to more than type distribution and draw its Ogive.

ANSWERS

1.	B	2.	A
3.	D	4.	C
5.	B	6.	C
7.	C	8.	A
9.	B	10.	A

11. Yes

12. No

13. $x = 19,\ y = 3$

14. $A = 45°,\ B = 45°$ or $\frac{49}{64}$.

16. 17 cm.

18. 65

21. ₹ 3900, ₹ 150 **OR** 40 years, 10 years

22. $3x^2 + 10x - 8$

27. 38.3 or $p = 12$.

28. 58.8

29. $\sqrt{2}, -\sqrt{2}, 5, -3/2$.

33. $x = 2,\ y = 4$.

CONTENTS

CHAPTER 1

QUADRATIC EQUATIONS

KEY POINTS

1. The equation $ax^2 + bx + c = 0$, $a \neq 0$ is the standard form of a quadratic equation, where *a, b* and *c* are real numbers.

2. A real number α is id to be a root of the quadratic equation $ax^2 + bx + c = 0$, $a \neq 0$. If $a\alpha^2 + b\alpha + c = 0$, the zeros of quadratic polynomial $ax^2 + bx + c$ and the roots of the quadratic equation $ax^2 + bx + c = 0$ are the me.

3. If we can ctorise $ax^2 + bx + c = 0$, $a \neq 0$ in to product of two linear ctors, then the roots of the quadratic equation can be found by equating each ctors to zero.

4. The roots of a quadratic equation $ax^2 + bx + c = 0$, $a \neq 0$ are give by $\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, provided that $b^2 - 4ac \geq 0$.

5. A quadratic equation $ax^2 + bx + c = 0$, $a \neq 0$, has ___

   (a) Two distinct and real roots, if $b^2 - 4ac > 0$.

   (b) Two equal and real roots, if $b^2 - 4ac = 0$.

   (c) Two roots are not real, if $b^2 - 4ac < 0$.

6. A quadratic equation can also be solved by the method of completing the square.

   (i) $a^2 + 2ab + b^2 = (a + b)^2$

   (ii) $a^2 - 2ab + b^2 = (a - b)^2$

7. Discriminant of the quadratic equation $ax^2 + bx + c = 0$, $a \neq 0$ is given by $D = b^2 - 4ac$.

MULTIPLE CHOICE QUESTIONS

1. The general form of a quadratic equation is ($a \neq 0$)

   (a) $ax^2 + bx + c$ (b) $ax^2 + bx + c = 0$

   (c) $ax + b$ (d) $ax + b = 0$

2. Number of solutions of a quadratic equation are :

   (a) 0 (b) 1

   (c) 2 (d) 3

3. If one root of $x^2 - 3x + a = 0$, is 1, then value of a is

   (a) 2 (b) – 2

   (c) 2 (d) – 4

4. Discriminant of a quadratic equation $ax^2 + bx + c = 0$ is given by

   (a) $\sqrt{b^2 - 4ac}$ (b) $\sqrt{b^2 + 4ac}$

   (c) $b^2 - 4ac$ (d) $b^2 + 4ac$

5. Which is a quadratic equation?

   (a) $x + \frac{1}{x} = 2$ (b) $x^2 + 1 = (x + 3)^2$

   (c) $x\,(x + 2)$ (d) $x + \frac{1}{x}$.

6. If the roots of a quadratic eqution are 2 and 3, then the equation is

   (a) $x^2 + 5x + 6 = 0$ (b) $x^2 + 5x - 6 = 0$

   (c) $x^2 - 5x - 6 = 0$ (d) $x^2 - 5x + 6 = 0$

7. Roots of the equations $x^2 - 3x + 2 = 0$ are

   (a) 1, –2 (b) –1, 2

   (c) –1, –2 (d) 1, 2

8. If the roots of a quadratic equation are equal, than discriminant is

(a) 1 (b) 0

(c) greater than 0 (d) less than zero.

9. If one root of $2x^2 + kx + 1 = 0$ is $-\frac{1}{2}$, then the value of 'k' is

(a) 3 (b) –3

(c) 5 (d) –5

10. The sum of the roots of the quadratic $5x^2 - 6x + 1 = 0$ is

(a) $\frac{6}{5}$ (b) $\frac{1}{5}$

(c) $-\frac{5}{6}$ (d) $-\frac{1}{5}$

11. The produce of the roots of the quadratic equation $2x^2 + 5x - 7 = 0$ is

(a) $\frac{5}{2}$ (b) $-\frac{7}{2}$

(c) $-\frac{5}{2}$ (d) $\frac{7}{2}$

12. If the roots of the quadratic $2x^2 + kx + 2 = 0$ are equal then the value of 'k' is

(a) 4 (b) –4

(c) ± 4 (d) ± 16

13. If the roots of $4x^2 + 3px + 9 = 0$ are real and distinct then, the value of p is

(a) $p \geq -4$ or $p \leq 4$ (b) $p < -4$ or $p > 4$

(c) $p \leq -4$ or $p \leq 4$ (d) $p \leq -4$ or $p \geq 4$

14. If the sum and product of roots of a quadratic equation are $-\frac{7}{2}$ and $\frac{5}{2}$ respectively, then the equation is

(a) $2x^2 + 7x + 5 = 0$ (b) $2x^2 - 7x + 5 = 0$

(c) $2x^2 - 7x - 5 = 0$ (d) $2x^2 + 7x - 5 = 0$

15. The roots of the equation $3x^2 - 7x + 4 = 0$ are

(a) rationals (b) irrationals

(c) positive intepers (d) negative intepers

SHORT ANSWER TYPE QUESTIONS

16. If one root of the equation $x^2 + 7x + k = 0$ is –2, then finds the value of k and other root.

17. For what value of 'k' the equation $2x^2 + kx + 3 = 0$ has equal roots?

18. For what value of 'p', the equation $3x^2 + px + 3 = 0$ has real roots?

19. The product of two consecutive odd integers is 63. Represent this in form of a quadratic equation.

20. Find the roots of the equation : $x + \frac{1}{x} = 4\frac{1}{4}, x \neq 0.$

21. Find the roots of the equation : $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0.$

22. Divide 51 in to two parts such that their product is 378.

23. Find 'k' so that $(k - 12)\ x^2 + 2\ (k - 12)\ x + 2 = 0$ has equal roots. $(k \neq 12)$.

24. Find the roots of the following, by the method of completing the square.

(a) $2x^2 - 5x + 3 = 0$

(b) $3x^2 + 5x + 1 = 0$

25. Find the roots of the equation

$$\frac{1}{x+2} - \frac{1}{x} = 3,\ x \neq -2,\ x \neq 0.$$

26. Find two consecutive odd positive integers, sum of whose squares is 394.

27. If the roots of the equation $(b - c)x^2 + (c - a)\ x + (a - b) = 0$ are equal, then prove that $2b = a + c$.

28. Find the nature of the roots of the following quadratic equations. If roots are real, find them.

    (a) $5x^2 - 3x + 2 = 0$.

    (b) $2x^2 - 9x + 9 = 0$.

29. Sum of two numbers is 15, if sum of their reciprocal is $\frac{3}{10}$. Find the numbers.

30. Solve the folloiwng quadratic equations

$$x^2 - 8x + 16 = 0$$

31. $a^2x^2 + (a^2 - b^2)\ x - b^2 = 0$.

32. $4\sqrt{3}x^2 + 5x - 2\sqrt{3} = 0$.

33. $ab\ x^2 + (b^2 - ac)\ x - bc = 0$.

34. $\frac{x-1}{x-2} + \frac{x-3}{x-4} = \frac{10}{3},\ x \neq 2,\ x \neq 4$.

35. $\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30},\ x \neq -4,\ x \neq 7$.

36. $3x^2 + 2\sqrt{5}x - 5 = 0$.

37. $\frac{1}{a} + \frac{1}{b} + \frac{1}{x} = \frac{1}{a+b+x},\ a \neq 0,\ b \neq 0,\ x \neq 0,\ x \neq -(a + \mathrm{b})$.

38. A two digit number is ruch that the product of digit is 35, when 18 is added to the number, the digits inter change their places. Find the number.

39. Find two numbers whose sum is 27 and product is 182.

40. A motor boat whose speed is 9 km/h in still water goes 12 km down stream and comes back in a total time 3 hours. Find the speed of the stream.

41. A train travels 360 km at uniform speed. If the speed had been 5 km/hr more it would have taken 1 hour less for the me journey. Find the speed of the train.

42. The hypotenuse of right angled triangle is 6cm more than twice the shortest side. If the third side is 2 cm less than the hypotenuse, find the sides of the triangle.

43. By a reduction of Rs. 2 per kg in the price of sugar. Anita can purchase 2 kg sugar more for Rs. 224. Find the original price of sugar per kg.

44. Rs. 9000 were divided equally among a certain number of students. Had there been 20 more students, each would have got Rs. 160 less. Find the original number of students.

45. An aeroplane takes an hour less for a journey of 1200 km, if the speed is increased by 100 km/hr from its usual speed. Find the usual speed of the aeroplane.

46. 7 years ago age of Aditi was five times the square of the age of rthak. After 3 years, age of rthak will be $\frac{2}{5}$ of the age of Aditi. Find their present ages.

47. Two years ago a man's age was three times the square of his son's age. Three years hence his age will be four times his son's age. Find their present ages.

48. In a cricket match against Sri Lanka, Sehwag took one wicket less than twice the number of wickets taken by Amit Mishra. If the product of the number of wickets takes by these two is 15, find the number of wickets taken by each.

49. A peacock is sitting on the top of a pillar, which is 9 metre high from a point 27 metre away from the bottom of the pillar, a snake is coming to its hole at the base of the pillar. Seeing the snake, the peacock pounces on it. If their speeds are equal, at what distance from the hole is the snake caught?

50. The numerators of a fraction is 1 less than its denominator. If three is added to each of the numerator and denominator, the fraction is increased by $\frac{3}{28}$, find the fraction.

ANSWERS

1. *b*
2. *c*
3. *a*
4. *c*
5. *a*
6. *d*
7. *d*
8. *b*
9. *a*
10. *a*
11. *b*
12. *c*
13. *b*
14. *a*
15. *a*
16. $k = 10$, second root $= -5$
17. $\pm 2\sqrt{6}$
18. $p \geq 6$ or $p \leq -6$
19. $x^2 + 2x - 63 = 0$
20. $4, \frac{1}{4}$
21. $-\frac{5}{\sqrt{2}}, -\sqrt{2}$
22. 9, 42
23. $k = 14$
24. (a) $\frac{3}{2}, 1$ (b) $\frac{-5+\sqrt{13}}{6}, \frac{-5-\sqrt{13}}{6}$.
25. $\frac{-3+\sqrt{3}}{3}, \frac{-3-\sqrt{3}}{3}$.
26. 13, 15
27. **Hint :** For equal roots $D = 0$.
28. (a) Not real roots.
    (b) Roots are real, $3, \frac{3}{2}$.
29. 5, 10
30. 4, 4
31. $-1, \frac{b^2}{a^2}$

32. $\frac{\sqrt{3}}{4}, -\frac{2}{\sqrt{3}}$

33. $\frac{c}{b}, -\frac{b}{a}$

34. $5, \frac{5}{2}$

35. 1, 2

36. $\frac{\sqrt{5}}{3}, -\sqrt{5}$

37. $-a, -b$.

38. 57

39. 13, 14

40. 3 km/hr.

41. 40 km/hr.

42. 26 cm, 24 cm, 10 cm

43. Rs. 16

44. 25 students

45. 300 km/hr.

46. Aditi's age = 27 yrs., rthak's age = 9 yrs.

47. 27 yrs., 5 yrs.

48. Sehwag 5, Amit Mishra 3 wickets.49. 12 m.

50. $\frac{3}{4}$.

CHAPTER 2

ARITHMETIC PROGRESSION

KEY POINTS

1. **Sequence :** A set of numbers arranged in some definite order and formed according to some rules is called a sequence.

2. **Progression :** The sequence that follows a certain pattern is called progression.

3. **Arithmetic Progression :** A sequence in which the difference obtained by substracting from any term its preceeding term is constant throughout, is called on arithmetic sequence or arithmetic progression (A.P.).

   The general form of an A.P. is a, $a + d$, $a + 2d$, ... (a : first term d : common difference).

4. **General Term :** If 'a' is the first term and 'd' is common difference in an A.P., then n^{th} term (general term) is given by $a_n = a + (n - 1)\ d$.

5. **Sum of *n* Terms of An A.P. :** If 'a' is the first term and 'd' is the common difference of an A.P., then sum of first n terms is given by

$$S_n = \frac{n}{2}\left\{2a + (n-1)\,d\right\}$$

   If 'l' is the last term of a finite A.P., then the sum is given by

$$S_n = \frac{n}{2}\left\{a + l\right\}.$$

6. (i) If a_n is given, then common difference $d = a_n - a_{n-1}$.

   (ii) If s_n is given, then n^{th} term is given by $a_n = s_n - s_{n-1}$.

   (iii) If a, b, c are in A.P., then $2b = a + c$.

   (iv) If a sequence has n terms, its r^{th} term from the end = $(n - r + 1)^{th}$ term from the beginning.

MULTIPLE CHOICE QUESTIONS

1. Next term of the A.P. 9, 11, 13, 15, ________ is

   (a) 17 (b) 18

   (c) 19 (d) 20

2. If n^{th} term of on A.P. is $2n + 7$, then 7^{th} term of the A.P. is

   (a) 15 (b) 21

   (c) 28 (d) 25

3. If the sum of n terms of an A.P. is $n^2 + 3n$, then sum of its 15 terms is

   (a) 250 (b) 230

   (c) 225 (d) 270

4. If n^{th} term of the A.P. 4, 7, 10, ________ is 82, then the value of n is

   (a) 29 (b) 27

   (c) 30 (d) 26

5. If a, b and c are A.P. then

   (a) $a = \frac{b + c}{2}$ (b) $b = \frac{a + c}{2}$

   (c) $c = \frac{a + b}{2}$ (d) $b = a + c$

6. 12^{th} term of the A.P. 3, 8, 13, ________ is

   (a) 56 (b) 57

   (c) 58 (d) 59

7. Common difference of A.P. $8\frac{1}{8}, 8\frac{2}{8}, 8\frac{3}{8},$ ________ is

   (a) $\frac{1}{8}$ (b) $1\frac{1}{8}$

(c) $8\frac{1}{8}$ (d) 1

8. n^{th} term of the A.P. –5, –2, 1, ________ is

(a) $3n + 5$ (b) $8 - 3n$

(c) $8n - 5$ (d) $3n - 8$

9. If n^{th} term of an A.P. is $5 - 3n$, than common difference of the A.P. is

(a) 2 (b) –3

(c) –2 (d) 3

10. If 5, $2k - 3$, 9 are in A.P., then the value of 'k' is

(a) 4 (b) 5

(c) 6 (d) –5

11. Sum of first 10 natural numbers is

(a) 50 (b) 55

(c) 60 (d) 65

12. 9^{th} term from the end of the A.P. 7, 11, 15, _______ 147 is

(a) 135 (b) 125

(c) 115 (d) 110

13. If the sum of n terms of on A.P. is n^2, then its n^{th} term is

(a) $2n - 1$ (b) $2n + 1$

(c) $n^2 - 1$ (d) $2n - 3$

14. The sum of 3 numbers in A.P. is 30. If the greatest number is 13, then its common difference is

(a) 4 (b) 3

(c) 2 (d) 5

15. The sum of 6^{th} and 7^{th} terms of an A.P. is 39 and common difference is 3, then the first terms of the A.P. is

(a) 2 (b) –3

(c) 4 (d) 3

LONG ANSWER TYPE QUESTIONS

16. Is $\sqrt{2}, \sqrt{8}, \sqrt{18}, \sqrt{32},$ ______ on A.P.? If yes, then find its next two terms.

17. Find an A.P. whose 2nd term is 10 and the 6th term exceeds the 4th term by 12.

18. Which term of the A.P. 41, 38, 35 _______ is the first negative term? Find the term also.

19. Nidhi ves Rs. 2 on day 1, Rs. 4 on day 2, Rs. 6 on day 3 and so on. How much money she ve in month of Feb. 2011?

20. Find an A.P., whose 3rd term is –13 and 6th term is 2.

21. How many two digits numbers between 6 and 102 are divisible by 6.

22. If s_n the sum of first n terms of an A.P. is given by $s_n = 3n^2 - 4n$, then find its n^{th} term and common difference.

23. The sum of 4th and 8th terms of an A.P. is 24 and sum of 6th and 10th terms is 44. Find A.P.

24. Find the sum of odd positive integers between 1 and 199.

25. How many terms of the A.P. 22, 20, 18, _____ should be taken so that their sum is zero?

26. The angles of a triangle are in A.P. If the smallest angle is one fifth the sum of other two angles. Find the angles.

27. If 11 times of 11th term is equal to 17 times of 17th term of an A.P. find its 28th term.

28. Find an A.P. of 8 terms, whose first term is $\frac{1}{2}$ and last term is $\frac{17}{6}$.

29. The fourth term of an A.P. is equal to 3 times the first term and the seventh term exceeds twice the third term by 1. Find the first term and common difference of the A.P.

30. Find the sum of A.P. 4 + 9 + 14 + _____ + 249.

31. If 2^{nd}, 31^{st} and last terms of on A.P. are $\frac{31}{4}$, $\frac{1}{2}$ and $-\frac{13}{2}$ respectively. Find the number of terms in the A.P.

32. Find the number of terms of the A.P. 57, 54, 51, ______ so that their sum is 570. Explain the double answer.

33. The sum of three numbers in A.P. is 24 and their product is 440. Find the numbers.

34. Find the sum of the first 40 terms of an A.P. whose n^{th} term is $3 - 2n$.

35. If m^{th} and n^{th} terms of an A.P. are $\frac{1}{n}$ and $\frac{1}{m}$ respectively, then find the sum of mn terms.

36. If n^{th} term of on A.P. is 4, common difference is 2 and sum of n terms is –14, then find first term and the number of terms.

37. Find the sum of all the three digits numbers each of which leaves the remainder 3 when divided by 5.

38. The sum of first six terms of an A.P. is 42. The ratio of the 10^{th} term to the 30^{th} term is 1 : 3. Find first term and 11^{th} term of the A.P.

39. The sum of n terms of two A.P.'s are in the ratio $3n + 8 : 7n + 15$. Find the ratio of their 12^{th} terms.

40. If p^{th}, q^{th} and r^{th} terms of an A.P. are l, m and n respectively then prove that $p(m - n) + q(n - l) + r(l - m) = 0$.

41. The sum of first 8 terms of an A.P. is 140 and sum of first 24 terms is 996 find the A.P.

42. The digits of a three digits positive number are in A.P. and the sum of digits is 15. On subtracting 594 from the number the digits are interchanged. Find the number.

43. A picnic group for Shimla consists of students whose ages are in A.P., the common difference being 3 months. If the youngest student Neeraj is just 12 years old and the sum of ages of all students is 375 years. Find the number of students in the group.

44. If the sum of first 20 terms of an A.P. is one third of the sum of next 20 terms. If first term is 1, then find the sum of first 30 terms.

45. The sum of first 16 terms of an A.P. is 528 and sum of next 16 terms is 1552. Find the first term and common difference of the A.P.

46. Kriti, starts a game and scores 200 points in the first attempt and she increases the points by 40 in each attempt. How many points will she score in the 30^{th} attempt?

47. If the roots of the equation $a(b - c)x^2 + b(c - a)x + c(a - b) = 0$ are equal, then show that $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P.

48. If the sum of m terms of an A.P. is n and the sum of n terms is m, then show that the sum of $(m + n)$ terms is $-(m + n)$.

49. The sum of 5^{th} and 9^{th} terms of an A.P. is 8 and their product is 15. Find the sum of first 28 terms of the A.P.

50. Anurag arranged balls in rows to form an equilateral triangle. The first row consists of one ball, the second of two balls, and so on. If 669 more balls are added, then all the balls can be arranged in the shape of a square and each of its sides then contains 8 ball less than each side of the triangle. Determine the initial number of balls, Anurag has.

ANSWERS

1.	*a*	2.	*b*
3.	*d*	4.	*b*
5.	*b*	6.	*c*
7.	*a*	8.	*d*
9.	*b*	10.	*b*
11.	*b*	12.	*c*
13.	*a*	14.	*b*
15.	*d*	16.	Yes, $\sqrt{50}$, $\sqrt{72}$

17. 4, 10, 16, ______

18. 15^{th} term, –1

19. Rs. 812

20. –23, –18, –13, _____

21. 15

22. $6n - 7$, Common difference = 6

23. –13, –8, –3, 2 _____

24. 9800

25. 23

26. 30°, 60°, 90°

27. 0

28. $\frac{1}{2}, \frac{5}{6}, \frac{7}{6}, \_\_\_\_$

29. First term = 3, common difference = 2

30. 6325

31. 59

32. 19 or 20, {20^{th} term is zero}

33. 5, 8, 11

34. –1520

35. $\frac{1}{2}(mn + 1)$

36. First term = – 8, Number of terms = 7

37. 99090

38. First term = 2, 11^{th} term = 22

39. 7 : 16

40. **Hint :** an $= a + (n - 1)\ d$

41. 7, 10, 13, 16, _____

42. 852

43. 25 students

44. 450

45. First term = 3, Common difference = 4

46. 1360

47. **Hint :** In quadratic equation, $D = 0$, for equal roots.

48. **Hint :** $s_n = \frac{n}{2}\{2a + (n - 1)d\}$

49. 115, 45 $\left\{d = \pm\frac{1}{2}\right\}$

50. 1540 balls.

CHAPTER 3

CO-ORDINATE GEOMETRY

KEY POINTS

1. The length of a line segment joining A and B is the distance between two points A (x_1, y_1) and B (x_2, y_2) is $\sqrt{\{(x_2 - x_1)^2 + (y_2 - y)^2\}}$.

2. The distance of a point (x, y) from the origin is $\sqrt{(x^2 + y^2)}$. The distance of P from x-axis is y units and from y-axis is x-units.

3. The co-ordinates of the points $p(x, y)$ which divides the line segment joining the points $A(x_1, y_1)$ and $B(x_2, y_2)$ in the ratio $m_1 : m_2$ are

$$\left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)$$

we can take ratio as $k : 1$, $k = \dfrac{m_1}{m_2}$.

4. The mid-points of the line segment joining the points $P(x_1, y_1)$ and $Q(x_2, y_2)$ is

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

5. The area of the triangle formed by the points (x_1, y_1), (x_2, y_2) and (x_3, y_3) is the numeric value of the expressions

$$\frac{1}{2}\left[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right].$$

6. If three points are collinear then we can not draw a triangle, so the area will be zero *i.e.*

$$x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) = 0$$

MULTIPLE CHOICE QUESTIONS

1. P is a point on x axis at a distance of 3 unit from y axis to its left. The coordinates of P are

   (a) (3, 0) (b) (0, 3)

   (c) (–3, 0) (d) (0, –3)

2. The distance of point P (3, –2) from y-axis is

   (a) 3 units (b) 2 units

   (c) –2 units (d) $\sqrt{13}$ units

3. The coordinates of two points are (6, 0) and (0, –8). The coordinates of the mid point are

   (a) (3, 4) (b) (3, –4)

   (c) (0, 0) (d) (–4, 3)

4. If the distance between (4, 0) and (0, x) is 5 units, the value of x will be

   (a) 2 (b) 3

   (c) 4 (d) 5

5. The coordinates of the point where line $\frac{x}{a}+\frac{y}{b}=7$ intersects y-axis are

   (a) (a, 0) (b) (0, b)

   (c) (0, 2b) (d) (2a, 0)

6. The area of triangle OAB, the coordinates of the points A (4, 0) B (0, –7) and O is origin is

   (a) 11 sq. units (b) 18 sq. units

   (c) 28 sq. units (d) 14 sq. units

7. The distance between the points $P\left(-\frac{11}{3}, 5\right)$ and $Q\left(-\frac{2}{3}, 5\right)$ is

   (a) 6 units (b) 4 units

   (c) 3 units (d) 2 units

8. The line $\frac{x}{2} + \frac{y}{4} = 1$ intersects the axes at P and Q, the coordinates of the mid point of PQ are

(a) (1, 2) (b) (2, 0)

(c) (0, 4) (d) (2, 1)

9. The coordinates of vertex A of ΔABC are (–4, 2) and point D(2, 5), D is mid point of BC. The coordinates of centroid of ΔABC are

(a) (0, 4) (b) $\left(-1, \frac{7}{2}\right)$

(c) $\left(-2, \frac{7}{3}\right)$ (d) (0, 2)

10. The distance between the line $2x + 4 = 0$ and $x - 5 = 0$ is

(a) 9 units (b) 1 unit

(c) 5 units (d) 7 units

11. The distance between the points (5 cos 35°, 0) and (0, 5 cos 55°) is

(a) 10 units (b) 5 units

(c) 1 unit (d) 2 units

12. If a is any positive integer such that the distance between the points $P(a, 2)$ and θ (3, –6) is 10 units then a is

(a) –3 (b) 6

(c) 9 (d) 3

13. The perimeter of triangle formed by the points (0, 0), (2, 0) and (0, 2) is

(a) 4 units (b) 6 units

(c) $6\sqrt{2}$ units (d) $4 + 2\sqrt{2}$ units

14. If the points (1, 2), (–5, 6) and (a, –2) are collinear the value of a is

(a) –3 (b) 7

(c) 2 (d) 5

15. If the centroid of the triangle formed by (9, a), (b, –4) and (7, 8) is (6, 8) then (a, b) is

    (a) (4, 5) (b) (5, 4)

    (c) (5, 2) (d) (3, 2)

SHORT ANSWER TYPE QUESTIONS

16. Find the value of a so that the point (3, a) lies on the line represented by $2x - 3y = 5$.

17. A line is drawn through a point P(3, 2) parallel to x-axis. What is the distance of the line from x-axis?

18. What is the value of a if the points (3, 5) and (7, 1) are equidistant from the point (a, 0)?

19. What is the value of p and q if (2, –3) is the mid point of the line segment joining (2, p) and (q, –1)?

20. AB is diameter of circle with centre at origin. What are the coordinates of B if coordinates of A are (3, –4)?

21. If the mid point of the line segment joining the points p (6, b – 2) and Q (–2, 4) is (2, –3). What is the value of b?

22. For what value of p, are the points (–3, 9), (2, p) and (4, –5) collinear?

23. Find the relation between x and y such that the point (x, y) is equidistant from the points (7, 1) and (3, 5).

24. Find the coordinates of point P if P and Q trisect the line segment joining the points A(1, –2) and B (–3, 4).

25. Find x if the distance between the points (x, 2) and (3, 4) be $\sqrt{8}$ units.

26. Find the area of triangle whose vertices are (1, –1), (–3, 5) and (2, –7).

27. Find a point on y-axis which is equidistant from the points (–2, 5) and (2, –3).

28. The mid point of the line segment joining the points (5, 7) and (3, 9) is also the mid point of the line segment joining the points (8, 6) and (a, b). Find a and b.

29. Find the coordinates of the points which divides the line segment joining the points (1, 3) and (2, 7) in the ratio 3 : 4.

30. P and Q are the points (1, 2) and (2, 3). Find the coordinates of a point R on the line segment PQ such that $\frac{PR}{RQ} = \frac{4}{3}$.

31. The point K (1, 2) lies on the perpendicular bisector of the line segment joining the points E (6, 8) and F (2, 4). Find the distance of the point K from the line segment EF.

32. The vertices of ΔABC are A (–1, 3), B (1, –1) and C (5, 1). Find the length of the median drawn from the vertex A.

33. Find the distance between the points A (a, b) and B (b, a) if $a - b = 4$.

34. Three vertices of a parallelogram taken in order are (–3, 1), (1, 1) and (3, 3). Find the coordinates of fourth vertex.

35. Triangle ABC is an isosceles triangle with $AB = AC$ and vertex A lies on y-axis. If the coordinates of B and C are (–5, –2) and (3, 2) respectively then find the coordinates of vertex A.

36. Point $P(K, 3)$ is the mid point of the line segment AB. If $AB = \sqrt{52}$ If units and coordinates of A are (–3, 5), then find the value of K.

37. Find the coordinates of a point which is $\frac{3}{4}$ of the way (3, 1) to (–2, 5).

38. The area of a triangle with vertices (6, –3), (3, K) and (–7, 7) is 15 sq. unit. Find the value of K.

39. Find the abscis of a point whose ordinate is 4 and which is $a + a$ distance of 5 units from (5, 0).

40. A point P on the x-axis divides the line segment joining the points (4, 5) and (1, –3) in certain ratio. Find the coordinates of point P.

41. In right angled ΔABC, $\angle B = 90°$ and $AB = \sqrt{34}$ unit. The coordinates of points B C are (4, 2) and (–1, y) respectively. If ar (ΔABC) = 17 sq. unit, then find the value of y.

42. If A (–3, 2) B (x, y) and C (1, 4) are the vertices of an isosceles triangle with $AB = BC$. Find the value of $(2x + y)$.

43. If the point P (3, 4) is equidistant from the points A ($a + b$, $b - a$) and B ($a - b$, $a + b$) then prove that $3b - 4a = 0$.

44. The vertices of quadrilateral $ABCD$ are A (–5, 7), B (–4, 5), C (–1, –6) and D (4, 5). Find the area of quadrilateral $ABCD$.

45. Find the ratio in which the line $3x + y = 12$ divides the line segment joining the points (1, 3) and (2, 7).

46. The line segment joining the points A (2, 1) and B (5, –8) is trisected at the points P and Q such that P is nearer to A. If P is also lies on line given by $2x - y + k = 0$, find the value of K.

47. The line segment joining the points (3, –4) and (1, 2) is trisected at the point P and Q. If the coordinantes of P and Q are (p –2) and $\left(\frac{5}{3}, q\right)$ respectively, find the values of p and q.

48. In ΔABC, the coordinates of A are (3, 2) and the coordinates of the mid point of AC and AB are (2, –1) and (1, 2) respectively. Find the coordinates of mid point of BC.

49. For the ΔABC with vertices A (5, 2), B (–5, –1) and C (3, –5). Show that the median AD divides the triangle into two triangles of equal area.

50. If P (x, y) is any point on the line joining the points A(a, 0) and B (0, b), then show that $\frac{x}{a} + \frac{y}{b} = 1$.

51. If the points (x, y), (–5, –2) and (3, –5) are collinear, prove that $3x + 8y + 31 = 0$.

ANSWERS

1. *c*	2. a
3. *b*	4. *b*
5. *c*	6. *d*
7. *c*	8. *a*
9. *a*	10. *d*

11. *b*

12. *c*

13. *d*

14. *b*

15. *c*

16. $a = \frac{1}{3}$

17. 2 units

18. $a = 2$

19. $p = -5, q = 2$

20. $(-3, 4)$

21. $b = -8$

22. $p = -1$

23. $x - y = 2$

24. $\left(-\frac{1}{3}, 0\right)$

25. $x = 1, 5$

26. 5 sq. unit

27. $(0, 1)$

28. $a = 0, b = 10$

29. $\left(\frac{10}{7}, \frac{33}{7}\right)$

30. $\left(\frac{11}{7}, \frac{18}{7}\right)$

31. 5 units

32. 5 units

33. $4\sqrt{2}$ units

34. $(-1, 3)$

35. $(0, -2)$

36. $K = 0, -6$

37. $\left(-\frac{3}{4}, 4\right)$

38. $K = \frac{21}{13}$

39. 2, 8

40. $\left(\frac{17}{8}, 0\right)$

41. -1

42. 1

44. 72 sq. unit

45. 6 : 1

46. $K = -8$

47. $p = \frac{7}{3}, q = 0$

48. $(0, -1)$.

CHAPTER 4

SOME APPLICATIONS OF TRIGONOMETRY

KEY POINTS

1. **Line of Sight :** The line of sight is the line drawn from the eye of an observer to the point in the object viewed by the observer.
2. **Angle of Elevation :** The angle of elevation is the angle formed by the line of sight with the horizontal, when it is above the horizontal level *i.e.* the case when we raise our head to look at the object.
3. **Angle of Depression :** The angle of depression is the angle formed by the line of sight with the horizontal when it is below the horizontal *i.e.* case when we lower our head to took at the object.

MULTIPLE CHOICE QUESTIONS

1. The length of the shadow of a man is equal to the height of man. The angle of elevation is

   (a) 90° (b) 60°

   (c) 45° (d) 30°

2. The length of the shadow of a pole 30*m* high at some instant is $10\sqrt{3}$ m. The angle of elevation of the sun is

   (a) 30° (b) 60°

   (c) 45° (d) 90°

3. In given fig. 1 *CE* || *AB*. The angle of elevation at points *A* and *D* respectively are

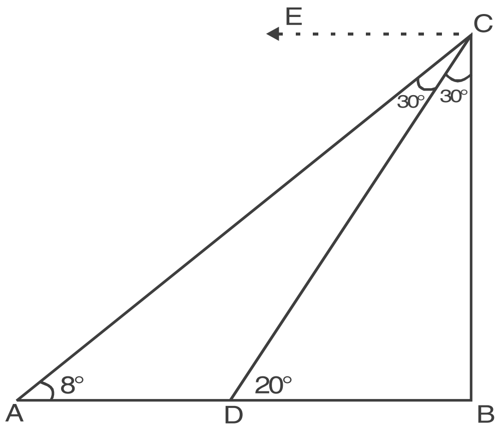


Fig. 1

(a) (30°, 60°) (b) (30°, 30°)

(c) (60°, 30°) (d) (45°, 45°)

4. The tops of two poles of height 10m and 18m are connected with wire. If wire makes an angle of 30° with horizontal, then length of wire is

(a) 10m (b) 18m

(c) 12m (d) 16m

5. From a point 20m away from the foot of the tower, the angle of elevation of the top of the tower is 30°. The height of the tower is

(a) $20\sqrt{3}\ m$ (b) $\dfrac{20}{\sqrt{3}}\ m$

(c) $40\sqrt{3}\ m$ (d) $\dfrac{40}{\sqrt{3}}\ m$

6. The ratio of the length of a tree and its shadow is $1 : \dfrac{1}{\sqrt{3}}$. The angle of elevation of the sun is

(a) 30° (b) 45°

(c) 60° (d) 90°

7. A kite is flying at a height of $50\sqrt{3}\ m$ above the level ground, attached to string inclined at 60° to the horizontal, the length of string is

(a) 100 m (b) 50 m

(c) 150 m (d) 75 m

8. In given fig. 2 the perimeter of rectangle *ABCD* is

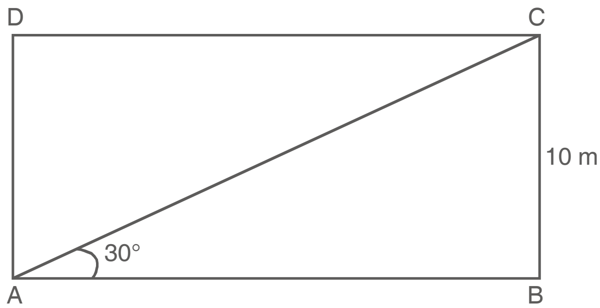


Fig.. 2

(a) 40 m (b) $20\left(\sqrt{3}+1\right)$ m

(c) 60 m (d) $10\left(\sqrt{3}+1\right)$ m

9. A tree is broken at a height of 10 m above the ground. The broken part touches the ground and makes an angle of 30° with the horizontal. The height of the tree is

(a) 30 m (b) 20 m

(c) 10 m (d) 15 m

10. In given fig. 3 $\tan\alpha = \frac{3}{4}$, if *AB* = 12m, then height *BC* is

(a) 8 m (b) 12 m

(c) 9 m (d) 10 m

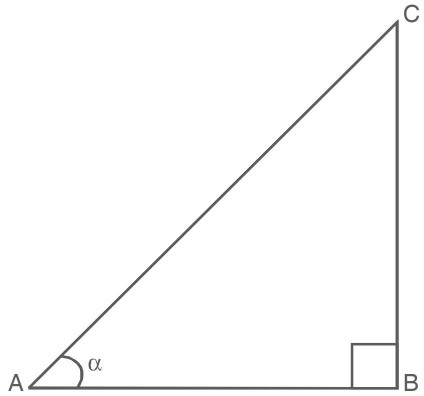


Fig.. 3

11. In given fig. 4 D is mid point of BC, $\angle CAB = \alpha_1$ and $\angle DAB = \beta_2$ then tan α_1 : tan β_2 is equal to

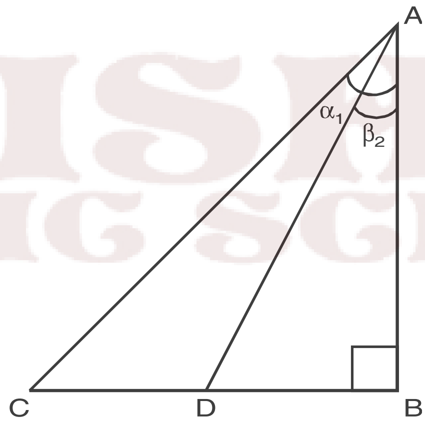


Fig. 4

(a) 2 : 1 (b) 1 : 2

(c) 1 : 1 (d) 1 : 3

12. In given fig. 5, $\tan\theta = \frac{8}{15}$ if PQ = 16 m, then the length of PR is

(a) 16 m (b) 34 m

(c) 32 m (d) 30 m

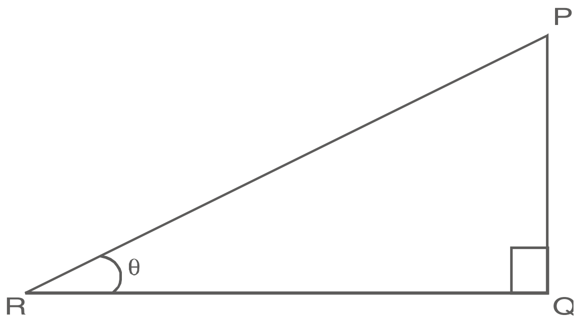


Fig. 5

13. The height of a tower is 50 m. When angle of elevation changes from 45° to 30°, the shadow of tower becomes x metres more, the value of x is

(a) 50 m (b) $50\left(\sqrt{3}-1\right)$ m

(c) $50\sqrt{3}$ m (d) $\frac{50}{\sqrt{3}}$ m

14. The angle of elevations of a building from two points on he ground 9m and 16m away from the foot of the building are complementary, the height of the building is

(a) 18 m (b) 16 m

(c) 10 m (d) 12 m

LONG ANSWER TYPE QUESTIONS

15. A pole of height 5m is fixed on the top of the tower. The angle of elevation of the top of the pole as observed from a point A on the ground is 60° and the angle of depression of the point A from the top of the tower is 45°. Find the height of tower. $\left(\text{Take } \sqrt{3} = 1.732\right)$

16. From a point on the ground the angle of elevations of the bottom and top of a water tank kept on the top of the 30m high building are 45° and 60° respectively. Find the height of the water tank.

17. The shadow of a tower standing on the level ground is found to be 60m shorter when the sun's altitude changes from 30° to 60°, find the height of tower.

18. A tree breaks due to storm and the broken part bends so that the top of the tree touches the ground making an angle 60° with the ground. The distance from the foot of the tree to the point where the top touches the ground is 5m. Find the total height of the tree.

19. The angle of elevation of a bird from a point on the ground is 60°, after 50 seconds flight the angle of elevation changes to 30°. If the bird is flying at the height of $500\sqrt{3}$ m. Find the speed of the bird.

20. The angle of elevation of a jet fighter plane from a point *A* on the ground is 60°. After a flight of 15 seconds, the angle of elevation changes to 30°. If the jet is flying at a speed of 720 km/h. find the constant height at which the jet is flying. $\left(\text{Take } \sqrt{3} = 1.732\right)$.

21. From a window 20m high above the ground in a street, the angle of elevation and depression of the top and the foot of another house opposite side of the street are 60° and 45° respectively. Find the height of opposite house.

22. An aeroplane flying at a height of 1800m observes angles of depressions of two points on the opposite bank of the river to be 60° and 45°, find the width of the river.

23. The angle of elevation of the top of the tower from two points *A* and *B* which are 15m apart, on the me side of the tower on the level ground are 30° and 60° respectively. Find the heigh of the tower and distance of point *B* from he base of the tower. $\left(\text{Take } \sqrt{3} = 1.732\right)$

24. The angle of elevation of the top of a 10m high building from a point *P* on the ground is 30°. A flag is hoisted at the top of the building and the angle of elevation of the top of the flag staff from *P* is 45°. Find the length of the flag staff and the distance of the building from point *P.*

25. The angle of elevation of a bird from a point 12 metres above a lake is 30° and the angle of depression of its reflection in the lake is 60°. Find the distance of the bird from the point of observation.

26. The angle of elevation of the cloud from a point 60m above a lake is 30° and angle of depression of the reflection of the cloud in the lake is 60°, find the height of the cloud.

27. A man on a cliff observes a boat at an angle of depression of 30°, which is approaching the shore to point '*A*' on the immediately beneath the observer

with a uniform speed, 12 minutes later, the angle of depression of the boat is found to be 60°. Find the time takes by the boat to reach the shore.

28. A man standing on the deck of a ship, 18m above the water level observes that the angle of elevation and depression of the top and the bottom of a cliff are 60° and 30° respectively. Find the distance of the cliff from the ship and height of the cliff.

29. A person standing on the bank of a river observes that the angle of elevation of the top of a tree standing on the opposite bank is 60°. When he moves 40m away from the bank he finds the angle of elevation to be 30°. Find the height of the tree and the width of the river.

30. An aeroplane, when 300 m high, passes vertically above another plane at an instant when the angle of elevation of two aeroplanes from the me point on the ground are 60° and 45° respectively. Find the vertical distance between the two planes.

31. The angle of depression of the top and bottom of a 10m tall building from the top of a tower are 30° and 45° respectively. Find the height of the tower and distance between building and tower.

32. A boy standing on a horizontal plane, finds a bird flying at a distance of 100m from him at an elevation of 30°. A girl, standing on the root of 20m high building, finds the angle of elevation of the me bird to be 45°. Both the boy and girl are on the opposite sides of the bird. Find the distance of bird from the girl.

33. At a point '*P*' on the level ground, the angle of elevation of a vertical tower is found to be such that its tangent is $\frac{3}{4}$. On walking 192 metres away from *P* the tangent of the angle is $\frac{5}{12}$. Find the height of the tower.

34. The angle of elevation of a building from two points *P* and *Q* on the level ground on the me side of the building are 36° and 54° respectively. If the distance of the points *P* and *Q* from the base of the building are 10m and 20m respectively, find the height of the building. $\left(\text{Take } \sqrt{2} = 1.414\right)$

35. A round balloon of radius '*r*' subtends an angle 'θ' at the eye of the observer while the angle of elevation of its centre is α. Prove that the height of the centre of the balloon is $r \sin\alpha \ \text{cosec}\frac{\theta}{2}$.

ANSWERS

1. *c*
2. *b*
3. *a*
4. *d*
5. *b*
6. *c*
7. *a*
8. *b*
9. *a*
10. *c*
11. *a*
12. *b*
13. *b*
14. *d*
15. 6.83 m
16. $30\left(\sqrt{3}-1\right)$ m
17. $30\sqrt{3}$ m
18. $5\left(2+\sqrt{3}\right)$ m
19. 20 m/sec.
20. 2598 m
21. $20\left(\sqrt{3}+1\right)$ m
22. $600\left(3+\sqrt{3}\right)$ m
23. Height = 12.97 m, distance = 7.5 m
24. Length of flag staff $= 10\left(\sqrt{2}-1\right)$ m, Distance of the building $= 10\sqrt{3}$ m.
25. $24\sqrt{3}$ m
26. 120 cm
27. 18 minutes
28. $18\sqrt{3}$ m, 72 m
29. Height = 34.64 m, Width of the river = 20 m.
30. $1000\left(3-\sqrt{3}\right)$ m
31. Height $= 5\left(3+\sqrt{3}\right)$ m, distance $= 5\left(3+\sqrt{3}\right)$ m
32. 30 m
33. 180 m
34. 14.14 m

CHAPTER 5

CIRCLE

KEY POINTS

1. **Tangent to a Circle :** It is a line that intersects the circle at only one point.
2. There is only one tangent at a point of the circle.
3. The proofs of the following theorems can be asked in the examination :
   (i) The tangent at any point of a circle is perpendicular to the radius through the point of contact.
   (ii) The lengths of tangents drawn from an external point to a circle are equal.

MULTIPLE CHOICE QUESTIONS

1. In the given fig. 1 PQ is tangent then $\angle POQ + \angle QPO$ is equal to

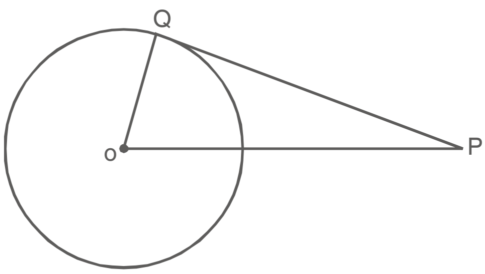


Fig. 1

(a) 120° (b) 90°

(c) 80° (d) 100°

2. If PQ is a tangent to a circle of radius 5cm and $PQ = 12$ cm, Q is point of contact, then OP is

(a) 13 cm (b) 17 cm

(c) 7 cm (d) $\sqrt{119}$ cm

3. In the given fig. 2 PQ and PR are tangents to the circle, $\angle QOP = 70°$, then $\angle QPR$ is equal to

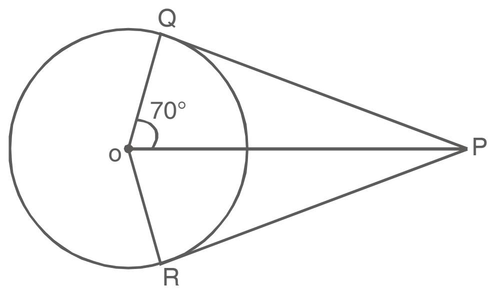


Fig. 2

(a) 35° (b) 70°

(c) 40° (d) 50°

4. In the given fig. 3 PQ is a tangent to the circle, $PQ = 8$ cm, $OQ = 6$ cm then the length of PS is

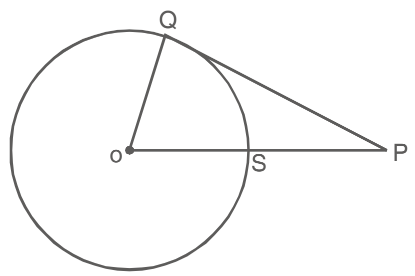


Fig. 3

(a) 10 cm (b) 2 cm

(c) 3 cm (d) 4 cm

5. In the given fig. 4 PQ is tangent to outer circle and PR is tangent to inner circle. If $PQ = 4$ cm, $OQ = 3$ cm and $OR = 2$ cm then the length of PR is

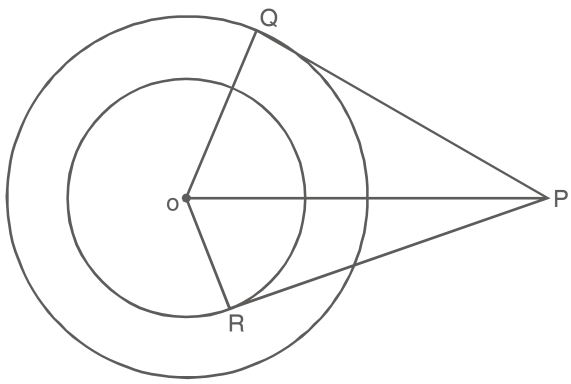


Fig. 4

(a) 5 cm (b) $\sqrt{21}$ cm

(c) 4 cm (d) 3 cm

6. In the given fig. 5 *P, Q* and *R* are the points of contact. If *AB* = 4 cm, *BP* = 2 cm then the perimeter of ΔABC is

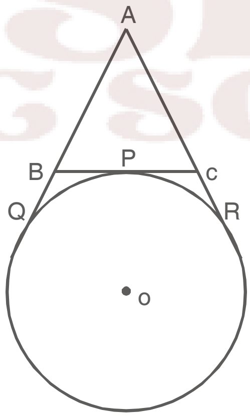


Fig. 5

(a) 12 cm (b) 8 cm

(c) 10 cm (d) 9 cm

7. In the given fig. 6 the perimeter of ΔABC is

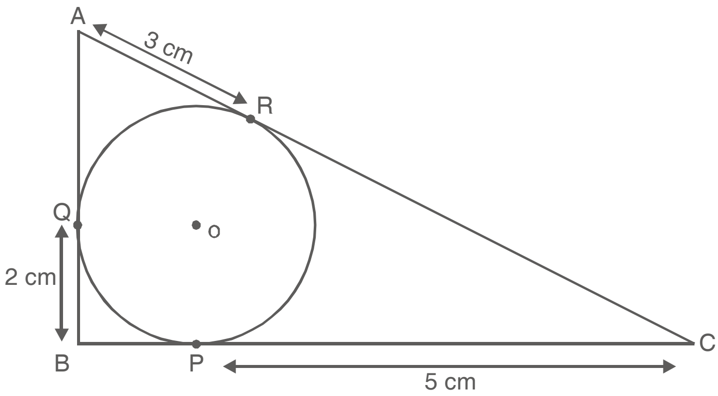


Fig. 6

(a) 10 cm (b) 15 cm

(c) 20 cm (d) 25 cm

8. The distance between two tangent parallel to each other to a circle is 12 cm. The radius of circle is

(a) 13 cm (b) 6 cm

(c) 10 cm (d) 8 cm

9. In the given fig. 7 a circle touches all sides of a quadrilateral. If $AB = 6$ cm, $BC = 5$ cm and $AD = 8$ cm. Then the length of side CD is

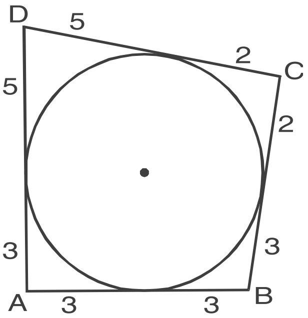


Fig. 7

(a) 6 cm (b) 8 cm

(c) 5 cm (d) 7 cm

10. In a circle of radius 17 cm, two parallel chords are drawn on opposite side of diameter. The distance between two chords is 23 cm and length of one chord is 16 cm, then the length of the other chord is

(a) 34 cm (b) 17 cm

(c) 15 cm (d) 30 cm

11. In the given fig. 8 P is point of contact then $\angle OPB$ is equal to

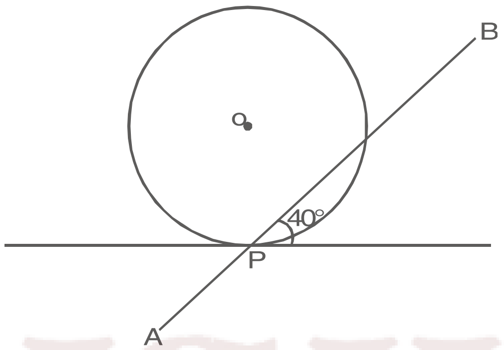


Fig. 8

(a) 50° (b) 40°

(c) 35° (d) 45°

12. In the given fig. 9 PQ and PR are tangents to the circle with centre O, if $\angle QPR = 45°$ then $\angle QOR$ is equal to

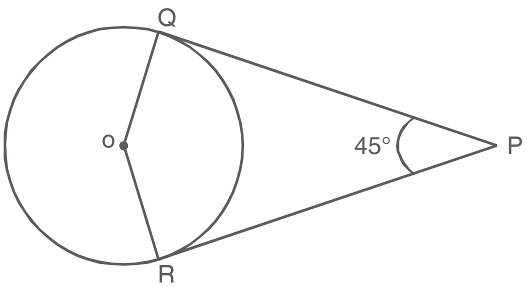


Fig. 9

(a) 90° (b) 110°

(c) 135° (d) 145°

13. In the given fig. 10 O is centre of the circle, PA and PB one tangents to the circle, then $\angle AQB$ is equal to

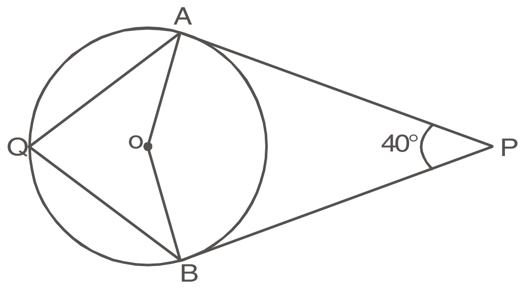


Fig. 10

(a) 70° (b) 80°

(c) 60° (d) 75°

14. In the given fig. 11 ΔABC is circumscribed touching the circle at *P, Q* and *R.* If *AP* = 4 cm, *BP* = 6 cm, *AC* = 12 cm, then value of *BC* is

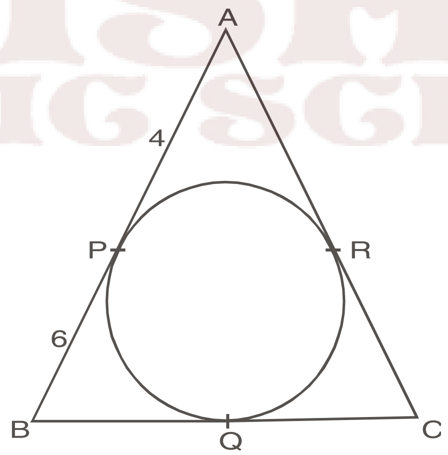


Fig. 11

(a) 6 cm (b) 14 cm

(c) 10 cm (d) 18 cm

15. In the given fig. 12 ΔABC is subscribing a circle and *P* is mid point of side *BC*. If *AR* = 4 cm, *AC* = 9 cm, then of *BC* is equal to

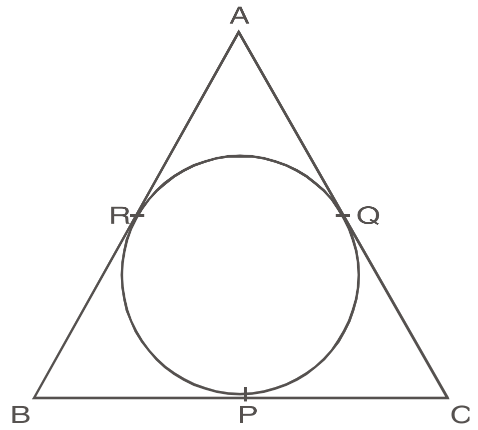


Fig. 12

(a) 10 cm (b) 11 cm

(c) 8 cm (d) 9 cm

SHORT ANSWER TYPE QUESTIONS

16. *AB* and *AC* are two tangents to a circle with centre *O*. If $\angle BOA = 2x$ and $\angle OAB = x$, then find the value of *x*.

17. An incircle is drawn touching the equal sides of an isosceles triangle at *E* and *F*. Show that the point *D*, where the circle touches the third side is the mid point of that side.

18. The length of tangent to a circle of radius 2.5 cm from an external point *P* is 6 cm. Find the distance of *P* from the nearest point of the circle.

19. *TP* and *TQ* are the tangents from the external point of a circle with centre *O*. If $\angle OPQ = 30°$, then find the measure of $\angle TQP$.

20. In the given fig. 13 $AP = 4$ cm, $BQ = 6$ cm and $AC = 9$ cm. Find the semi perimeter of ΔABC.

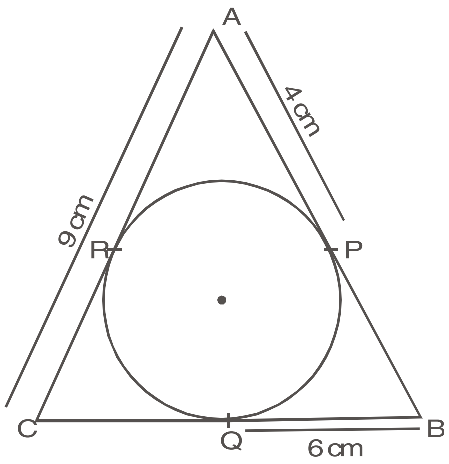


Fig. 13

21. In the given fig. 14 *OP* is equal to diameter of the circle with centre *O*. Prove that ΔABP is an equilateral triangle.

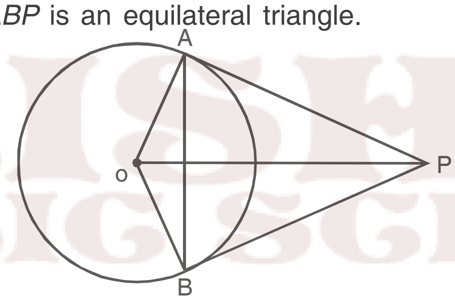


Fig. 14

22. In the given fig. (15) a semicircle is drawn outside the bigger semicircle. Diameter BE of smaller semicircle is half of the radius BF of the bigger semicircle. If radius of bigger semicircle is $4\sqrt{3}$ cm. Find the length of the tangent AC from A on a smaller semicircle.

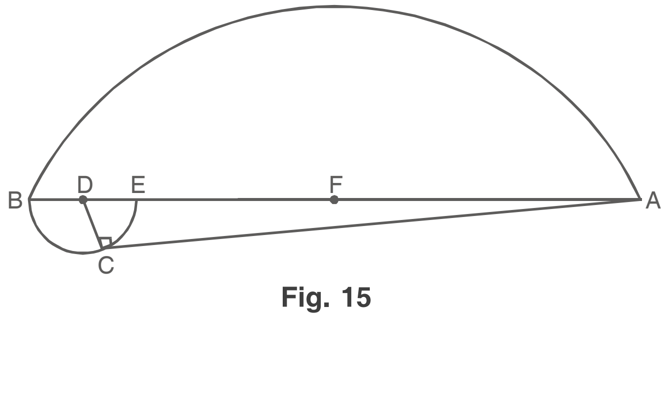


Fig. 15

23. In the fig. (16) PA and PB are tangent to circle with centre O. Find the value of X.

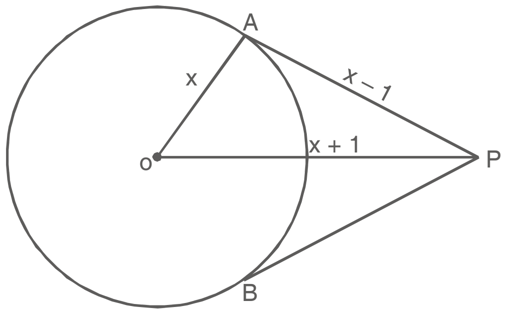


Fig. 16

24. On the side AB as diameter of a right angled triangle ABC a circle is drawn intersecting the hypotenuse AC in P. Prove that PB = PC.

25. In the given fig. (17) PQ is tangent to the circle with centre O. AP = 8 cm and length of tangent exceeds the radius by 1. Find the radius of the circle.

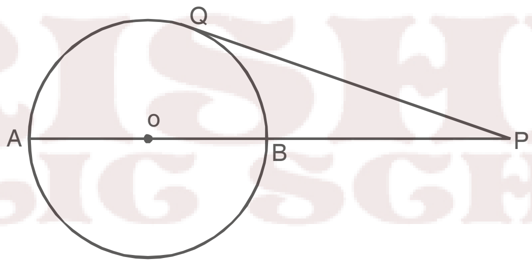


Fig. 17

26. A chord AB of 8 cm is drawn in a circle with centre O of radius 5 cm. Find the length of tangents from external point P to A and B.

27. In the given fig. (18) AB = AC, D is the mid point of AC, BD is the diameter of the circle, then prove that AE = 1/4 AC.

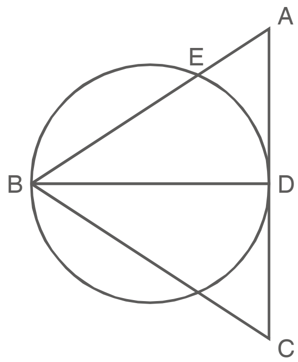


Fig. 18

28. In the given fig. (19) radii of two concentric circles are 5 cm and 8 cm. The length of tangent from P to bigger circle is 15 cm. Find the length of tangent to smaller circle.

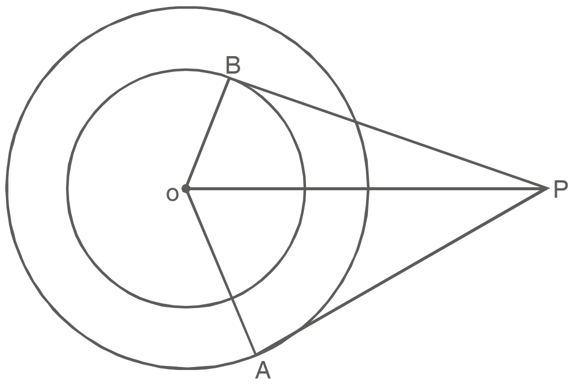


Fig. 19

29. An incircle is drawn touching the sides of a right angled triangle, the base and perpendicular of the triangle are 6 cm and 2.5 cm respectively. Find the radius of the circle.

30. In the given fig. (20) AB = 13 cm, BC = 7 cm. AD = 15 cm. Find PC.

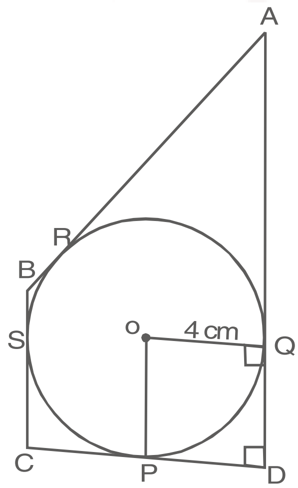


Fig. 20

31. In the given fig. (21) find the radius of the circle.

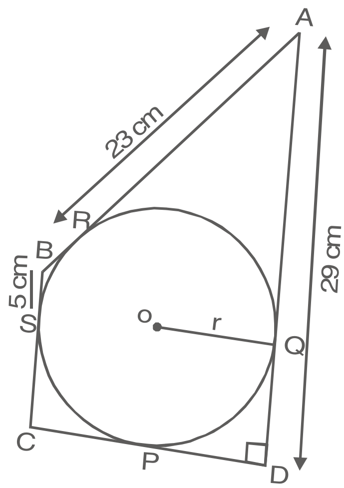


Fig. 21

32. In the given fig. (22) if radius of circle r = 3 cm. Find the perimeter of ΔABC.

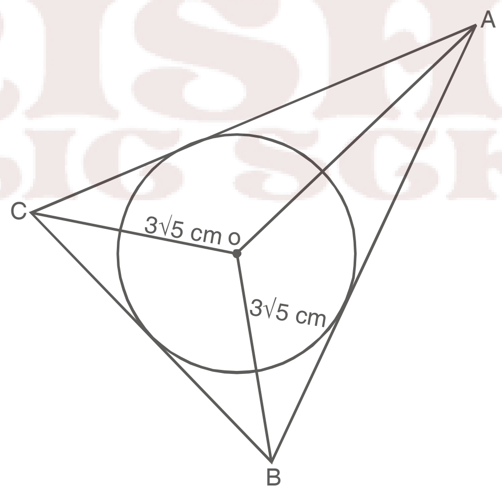


Fig. 22

33. PQ is a diameter of a circle and PR is the chord such that $\angle RPQ = 30°$. The tangent at R intersects PQ produced at S. Prove that RQ = QS.

34. In the given fig. (23) XP and XQ are tangents from X to the circle with centre O. R is a point on the circle. Prove that

XA + AR = XB + BR.

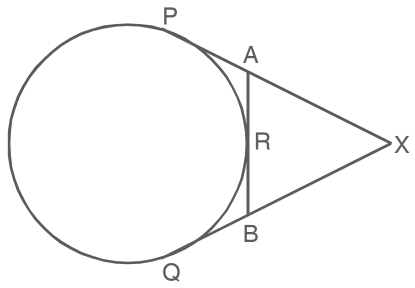


Fig. 23

LONG ANSWER TYPE QUESTIONS

35. Prove that the tangent at any point of a circle is perpendicular to the radius through the point of contact.

Rider :

1. Prove that in two concentric circles the chord of the larger circle which touches the smaller circle is bisected at the point of contact.
2. PT is a tangent to the circle with centre O and T is point of contact. It OT = 6 cm, OP = 10 cm find the length of tangent PT.
3. In the given fig. (24) PQ is tangent and PB is diameter. Find the value of x and y.

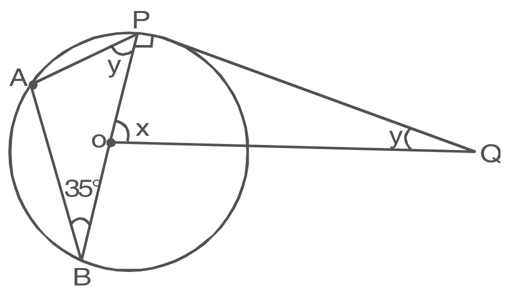


Fig. 24

4. In the given fig. (25) AC is diameter of the circle with centre O and A is point of contact, then find x.

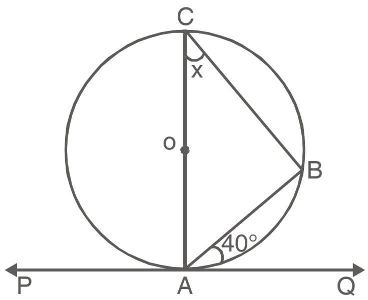


Fig. 25

36. Prove that the length of tangents, drawn from an external point to a circle are equal.

Rider :

1. In the given fig. (26) PA and PB are tangents from point P. Prove that KN = AK + BN.

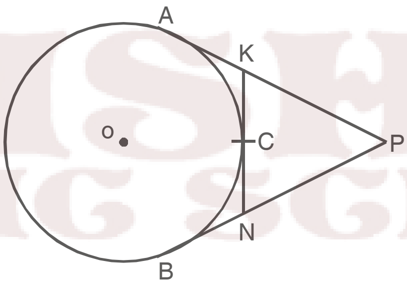


Fig. 26

2. Two concentric circles are of radii 5 cm and 3 cm. Find the length of the chord of the larger circle which is tangent to the smaller circle.

3. In the given fig. (27) PA and PB are tagents to the circle with centre O. Prove that OP is perpendicular bisector of AB.

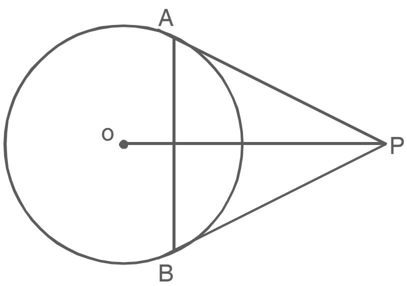


Fig. 27

4. In the given fig. (28) PQ is chord of length 6 cm of the circle of radius 6 cm. TP and TQ are tangents. Find ∠PTQ.

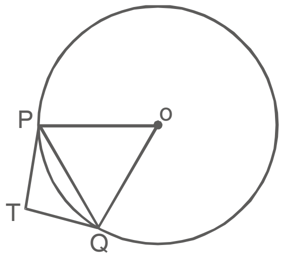


Fig. 28

ANSWERS

1.	*b*	2.	*d*
3.	*c*	4.	*d*
5.	*b*	6.	*a*
7.	*c*	8.	*b*
9.	*d*	10.	*a*
11.	*a*	12.	*c*
13.	*a*	14.	*b*
15.	*a*	16.	30°
18.	4 cm	19.	60°
20.	15 cm	22.	12 cm
23.	4 cm	25.	Radius = 3 cm.
26.	20/3 cm	28.	$2\sqrt{66}$ cm
29.	1 cm.	30.	5 cm
31.	11 cm.	32.	32 cm
35.(2)	8 cm	35.(3)	x = 35°, y = 55°
35.(4)	40°	36.(2)	8 cm 36.(4) 120°

CHAPTER 6

CONSTRUCTIONS

KEY POINTS

1. Construction should be neat and clean and as per scale given in question.
2. Steps of construction should be provided only to those questions where it is mentioned.

QUESTIONS

1. Draw a line segment AB = 7 cm. Take a point P on AB such that AP : PB = 3 : 4.
2. Draw a line segment PQ = 10 cm. Take a point A on PQ such that $\frac{PA}{PQ} = \frac{2}{5}$. Measure the length of PA and AQ.
3. Construct a ΔABC in which BC = 6.5 cm, AB = 4.5 cm and $\angle ACB = 60°$. Construct another triangle similar to ΔABC such that each side of new triangle is $\frac{4}{5}$ of the corresponding sides of ΔABC.
4. Draw a triangle XYZ such that XY = 5 cm, YZ = 7 cm and $\angle XYZ = 75°$. Now construct a $\Delta X'YZ' \sim \Delta XYZ$ with its sides $\frac{3}{2}$ times of the corresponding sides of ΔXYZ.
5. Construct an isoscales triangle whose base is 8 cm and altitude 5 cm and then construct anothertriangle whose sides are $\frac{3}{4}$ times the corresponding sides of the given triangle.

6. Draw an isosceles ΔABC with AB = AC and base BC = 7 cm and vertical angle is 120°. Construct $\Delta AB'C' \sim \Delta ABC$ with its sides $1\frac{1}{3}$ times of the corresponding sides of ΔABC.

7. Draw ΔPQR in which $\angle Q = 90°$, PQ = 6 cm, QR = 8 cm. Construct $\Delta P'QR' \sim \Delta PQR$ with its sides equal to 2/3rd of corresponding sides of ΔPQR.

8. Construct a right angled triangle in which base is 2 times of the perpendicular. Now construct a triangle similar to it with base 1.5 times of the original triangle.

9. Draw an equilateral triangle PQR with side 5cm. Now construct $\Delta PQ'R'$ such that $\frac{PQ}{PQ'} = \frac{1}{2}$. Measure PQ′.

10. Draw a circle of radius 4 cm with centre O. Take a point circle from P such that OP = 6cm. Draw tangents PA and PB to circle P. Measure the length of PA and PB.

11. Draw a line segment AB = 8 cm. Taking AB as diameter a circle with centre O. Now draw $OP \perp AB$. Through P draw a tangent to the circle.

12. Draw a circle of radius OP = 3 cm. Draw $\angle POQ = 45°$ such that OQ = 5 cm. Now draw two tangents from Q to given circle.

13. Draw a circle with centre O and radius 3.5 cm. Now draw two tangents PQ and PB from an external point draw two tangents PA and PB from an external point P such that $\angle APB = 45°$. What is the value of $\angle AOB + \angle APB$.

14. Draw a circle of radius 4 cm. Now draw a set of tangents from an external point P such that the angle between the two tangents is half of the central angle made by joining the point of contact to the centre.

15. Draw a line segment AB = 9 cm. Taking A and B as centres draw two circles of radius 5 cm and 3 cm respectively. Now draw tangents to each circle from the centre of the other.

16. Draw a circle of radius 3.5 cm with centre O. Take point P such that OP = 6 cm. OP cuts the circle at T. Draw two tangents PQ and PR. Join Q to R. Through T draw AB parallel to QR such that A and B are point on PQ and PR.

17. Draw a circle of diameter 7 cm. Draw a pair of tangents to the circle, which are inclined to each other at an angle of 60°.

18. Draw a circle with centre O and radius 3.5 cm. Take a horizontal diamater. Extend it to both sides to point P and Q such that OP = OQ = 7 cm. Draw tangents PA and QB one above the diameter and the other below the diameter. Is PA || BQ.

CHAPTER 7

MENSURATION (CONTINUED) SURCE AREAS AND VOLUMES

KEY POINTS

1. $c = 2\pi r$ where $c \rightarrow$ circumference of the circle π be taken as 22/7 or 3.14 (app.) and 'r' be the radius of the circle.

2. Area of circle $= \pi r^2$ where 'r' is the radius of the circle.

3. Area of Semi circle $= \dfrac{\pi r^2}{2}$.

4. Area enclosed by two concentric circles

$$= \pi \, (R^2 - r^2)$$

$$= \pi \, (R + r) \, (R - r); \; R > r$$

where 'R' and 'r' are radii of two concentric circles.

5. The are length 'l' of a sector of angle 'θ' in a circle of radius 'r' in given by

$$l = \frac{\theta}{360^\circ} \times (\text{circumference of the circle})$$

$$= \frac{q}{\cancel{360^\circ}_{180}} \times \cancel{2}^{1} \pi r$$

$$l = \frac{\theta}{180^\circ} \times \pi r$$

6. If the arc subtends an angle θ, then area of the corresponding sector is $\dfrac{\theta}{360^\circ} \times \pi r^2$.

7. Angle described by minute hand in 60 minutes = 360°. Angle described by minute hand in 1 minute $= \left(\dfrac{360°}{60}\right) = 6°$.

8. Total Surce area of cube of side a units = $6a^2$ units.

9. Volume of cube of side a units = a^3 cubic units.

10. Total surce area of cuboid of dimensions l, b and $h = 2(l \times b + b \times h + h \times l)$ square units.

11. Volume of cuboid of cylinder of dimensions l, b and $h = l \times b \times h$ cubic units.

12. Curved surce area of cylinder of radius r and height $h = 2\pi rh$ square units.

13. Total surce area of cylinder of radius r and height $h = 2\pi r(r + h)$ square units.

14. Volume of cylinder of radius r and height $h = \pi r^2 h$ cubic units.

15. Curved surce area of cone of radius r height h and slant height $l = \pi rl$ square units where $l = \sqrt{r^2 + h^2}$.

16. Total surce area of cone = $\pi r(l + r)$ sq. units.

17. Volume of cone $= \dfrac{1}{3}\pi r^2 h$ units.

18. Total curved surce area of sphere of radius r units = $4\pi r^2$ sq. units.

19. Curved surce area of hemisphere of radius r units $2\pi r^2$ sq. units.

20. Total surce area of hemisphere of radius r units = $3\pi r^2$ sq. units.

21. Volume of sphere of radius r units $= \dfrac{4}{3}\pi r^3$ cubic units.

22. Volume of hemisphere of radius r units $= \dfrac{2}{3}\pi r^3$ cubic units.

23. Curved surce of frustum = $\pi l(r + R)$ sq. units, where l slant height of frustum and radii of circular ends are r and R.

24. Total surce area of frustum = $l\pi(r + R) + \pi(r^2 + R^2)$ sq. units.

25. Volume of Frustum $= \frac{1}{3}\pi h\left(r^2 + R^2 + rR\right)$ cubic units.

MULTIPLE CHOICE QUESTIONS

1. Find the area of circle whose diameter is 'd'

   (a) 2pd (b) $\frac{\pi d^2}{4}$

   (c) p.d (d) pd^2

2. If the circumeference and area of a circle are numerically equal then what is the radius of the circle equal to

   (a) r = 1 (b) r = 7

   (c) r = 2 (d) r = c

3. The radius of a circle is 7 cm. What is the perimeter of the semi circle?

   (a) 36 cm (b) 14 cm

   (c) 7p (d) 14p

4. The radius of two circles are 13 cm and 6 cm respectively. What is the radius of the circle which has circumference equal to the sum of the circumference of two circles?

   (a) 19p (b) 19 cm

   (c) 25 cm (d) 32 cm

5. The circumference of two circles are in the ratio 4 : 5 what is the ratio of the areas of these circles.

   (a) 4 : 5 (b) 16 : 25

   (c) 64 : 125 (d) 8 : 10

6. The area of an equilateral triangle is $\sqrt{3}$ m^2 its one side is

   (a) 4 m (b) $3\sqrt{3}$ m

   (c) $\frac{3\sqrt{3}}{4}$ m (d) 2 m

7. The volume of a cuboid is 440 cm^3. The area of its base is 66 cm^2. What is its height?

(a) $\frac{40}{3}$ cm (b) $\frac{20}{3}$ cm

(c) 440 cm (d) 66 cm

8. Volume of the cubes is in the ratio of 8 : 125. The ratio of their surce areas is

(a) 8 : 125 (b) 2 : 5

(c) 4 : 25 (d) 16 : 25

9. If the perimeter of a sector is 'l' and radius is 'r' then the area of the sector is

(a) $l \cdot r$ (b) $l \cdot r^2$

(c) $\frac{lr^2}{2}$ (d) $l^2 \cdot r$

10. An arc of a circle is of length 5π cm and the section it bounds has an area of 10π cm^2. Then the radius of circle is :

(a) 2 cm (b) 4 cm

(c) $2\sqrt{2}$ cm (d) 8 cm

11. Three cubes each of side 'a' are joined from end to end to form a cuboid. The volume of the new cuboids :

(a) a^2 (b) $3a^3$

(c) a^3 (d) $6a^3$

12. A wire is in the form of a circle of radius 7 cm. It is bent into a square the area of the square is :

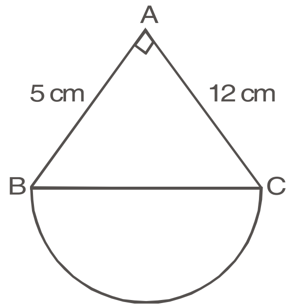
(a) 11 cm^2 (b) 121 cm^2

(c) 154 cm^2 (d) 44 cm^2

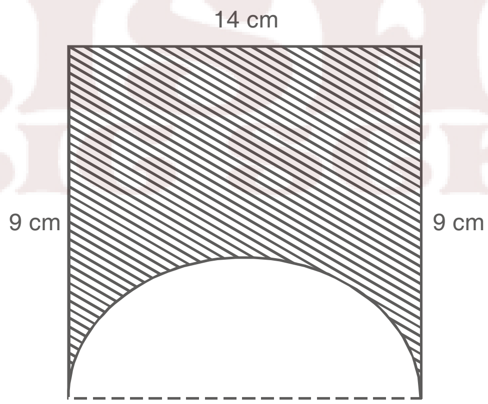
SHORT ANSWER TYPE QUESTIONS

13. The volume and surce area of a sphere are numerically equal. Find the radius of the sphere.

14. Find the perimeter of the figure in which a semicircle is drawn on BC as diameter. $\angle BAC = 90°$.



15. Find the area of shaded region in the figure.



16. The numerical difference between circumference and diameter is 30 cm. What is the radius of the circle?

17. What is the perimeter of a sector of angle 45° of a circle with radius 7 cm.

18. From each vertex of trapezium a sector of radius 7 cm has been cut off. Write the total area cut off.

19. Write the ratio of the areas of two sectors having angles 120° and 90°.

20. How many cubes of side 4 cm can be cut from a cuboid measuring $(16 \times 12 \times 8)$ cm^3.

21. The diameter and height of a cylinder and a cone are equal. What is the ratio of their volume.

22. A cylinder, a cone and a hemisphere are of equal base and have the me height. What is the ratio in their volumes?

23. A bicycle wheel makes 5000 revolutions in moving 10 km. Write the perimeter of wheel.

24. The sum of the radius of the base and the height of a solid cyliner is 15 cm. If total surce area is 660 cm^2. Write the radius of the base of cylinder.

25. Find the height of largest right circular cone that can be cut out of a cube whose volume is 729 cm^3.

26. What is the ratio of the areas of a circle and an equilateral triangle whose diameter and a side of triangle are equal.

27. If the circumference of the circle exceeds its diameter by 30 cm. What is the diameter of the circle

28. The length of an arc of a circle of radius 12 cm is 10π cm. Write the angle measure of this arc.

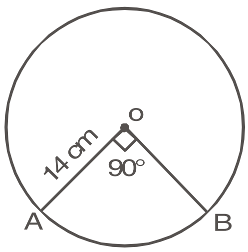
29. The cost of fencing a circular field of the rate of Rs. 10 per meter is Rs. 440. What is the radius of the circular field?

30. Find the perimeter of the protactor if its diameter is 14 cm.

31. A path of 5 m is build round the circular park of radius 15m. Find the area of the path.

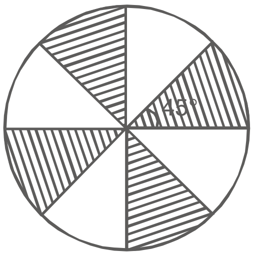
32. The radii of two circles are 4 cm and 3 cm respectively. Find the radius of a circle having area equal ot the sum of the areas of the circles.

33. In the figure find length of arc AB if 'O' is thecentre of the circle and radius is 14 cm. $\left(\pi = \frac{22}{7}\right)$

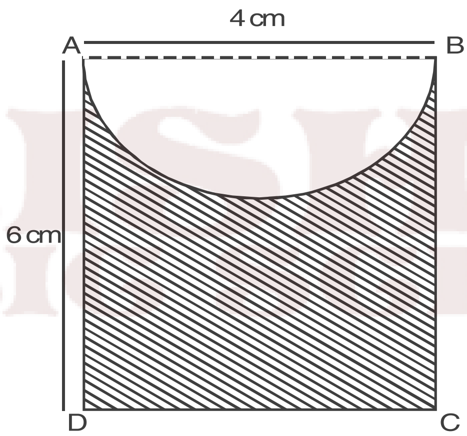


34. ABC is an equilateral triangle of side 30m. A Cow is tied at vertex A by mean of the 10m long rope. What is the area the Cow can graze in?

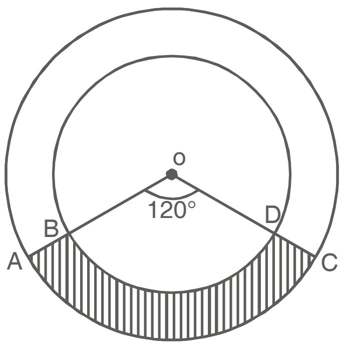
35. Find the area of the four blades of me size of radius 20 cm and central angle 45° of a circular n.



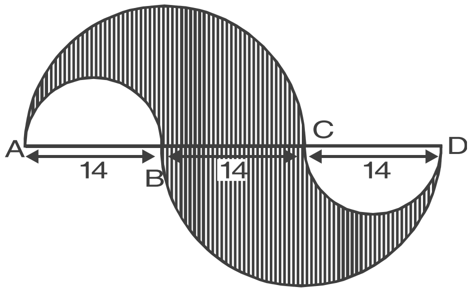
36. Find the perimeter of the shaded region.



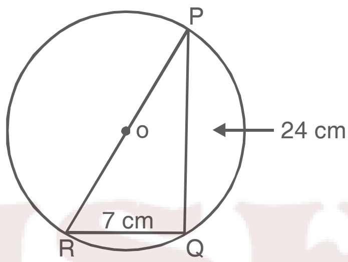
37. Two concentric circle with centre 'O' and radius 7 cm and 14 cm. If $\angle AOC = 120°$ what is the area of shaded region?



38. Find the perimeter of the shaded portion.



39. Find the circumference of the circle with centre 'O'.



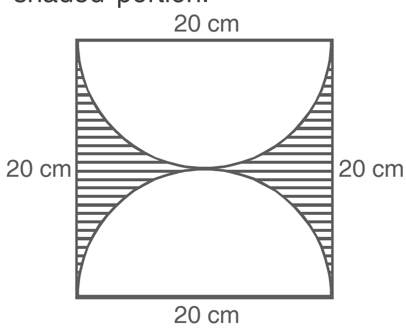
40. The radius of two circles are in the ratio 3 : 4 and sum of the areas of two circles is equal to the area of third circle. What is the radius of third circle. If the radius of first is 6 cm.

41. What is the area of the largest triangle that can be inscribed in a semicircle of radius r cm.

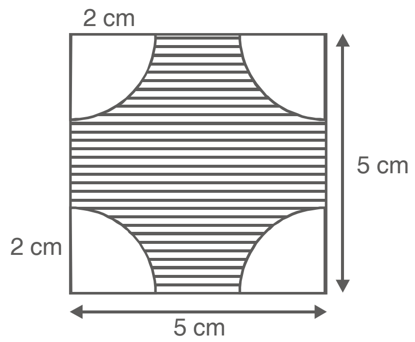
42. A piece of wire 20 cm long is bent into an arc of a circle subtending an angle of 60° at he centre then what is the radius of the Circle?

43. The minute hand of a clock is $\sqrt{12}$ cm long. What is the area described by the minute hand between 8.00 a.m to 8.05 a.m.?

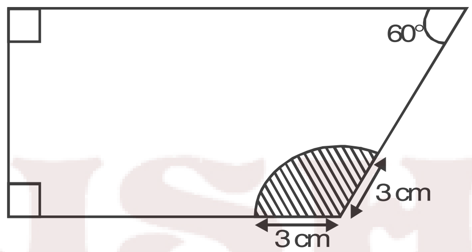
44. Find the area of shaded portion.



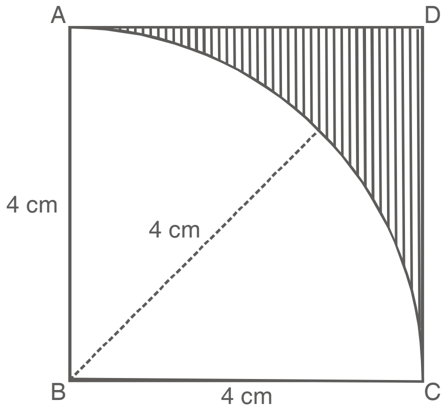
45. Find the area of shaded portion.



46. In the figure find the area of sector.



47. ABCD is a square kite of side 4 cm. What is the are of the shaded portion.



48. The volume of cube is $8a^3$. Find its surce area.

49. The length of a diagonal of a cube is 17.32 cm. Find the volume of cube (use $\sqrt{3} = 1.732$).

50. Three cubes of the me metal, whose edges are 6, 8, 10 cm are melted and formed into a Single cube. Find the diagonal of the single cube.

LONG ANSWER TYPE QUESTIONS

51. The height of frustum is 4 cm and the radii of two basere 3 cm and 6 cm respectively. Find the slant height of the frustum.

52. Volume of right circular cylinder is 448π cm^3 height of cylinder is 7cm. Find the radius.

53. If lateral surce area of a cube is 64 cm^2. What is its edge?

54. The area of a rhombus is 24 cm^2 and one of its diagonal is 8 cm. What is other diagonal of the rhombus?

55. What is the length of the largest rod that can be put in a box of inner dimensions 30cm, 24 cm and 18 cm?

56. Curved surce area of a cylinder is 16 πcm^2, radius is 4cm, then find its height.

57. 50 circular plates each of equal radius of 7 cm are placed one over the other to form a cylinder. Find the height and volume of the cylinder if thickness of plate is $\frac{1}{2}$ cm.

58. A well of diameter 2m is dug 14 m deep. Find the volume of the earth dug out.

59. A largest sphere is carved out of a cube of side 7 cm. Find the radius.

60. If the semi vertical angle of a cone of height 3 cm is 60°. Find its volume.

61. Find the edge of cube if volume of the cube is equal to the volume of cuboid of dimensions $(8 \times 4 \times 2)$ cm.

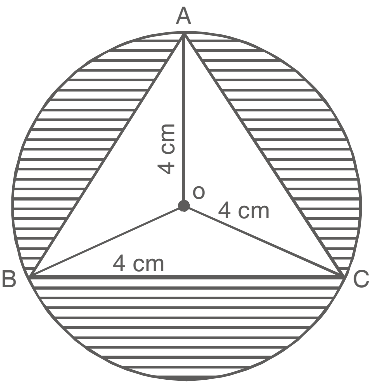
62. Find the volume of cone of height 2h and radius r.

63. Is it possible to have a right circular cylinder closed at both ends, whose flat area is equal to its total curve surce.

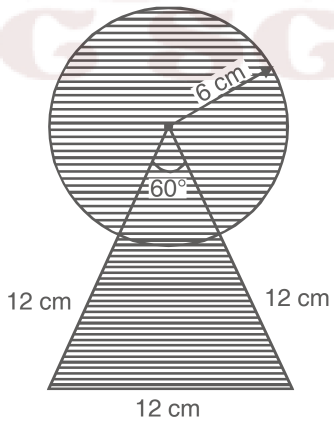
64. In a shower, there is 5 cm rain lls. Find in cubic meter the volume of water that lls on 2 hectares of ground. (1 hectare = 10000 m^2).

65. What is the total surce area of a solid hemisphereof radius R.

66. In figure, ΔABC is equilateral triangle. The radius of the circle is 4 cm. Find the Area of shaded portion.



67. Find the area of Shaded portion.



68. Four Cows are tied with a rope of 7 cm at four corners of a quadrilateral field of unequal sides. Find the total area grazed.

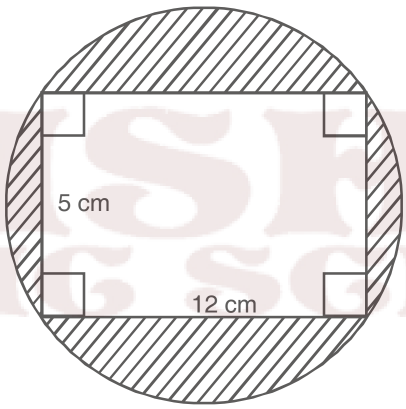
69. A solid consists of a right circular cylinder with a right circular cone at the top. The height of cone is 'h' cm. The total volume of the solid is 3 times the volume of the cone. Find the height of the cylinder.

70. A cylindrical vessel of 36 cm height and 18 cm radius of the base is filled with nd. The nd is emptied on the ground and a conical heap of nd is formed. The height of conical heap is 27 cm. Find the radius of base of nd.

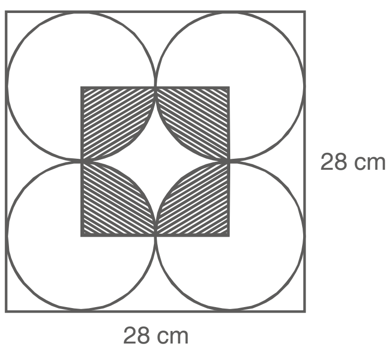
71. The radii of circular ends of bucket are 5.5 cm and 15.5 cm and its height is 24 cm. Find the surce area of bucket.

72. Water flow out through a circular pipe whose internal diameter is 2 cm at the rate of 6m/sec. into a cylinderical tank. If radius of base of the tank is 60 cm. How much will the level of the water rise in half an hour?

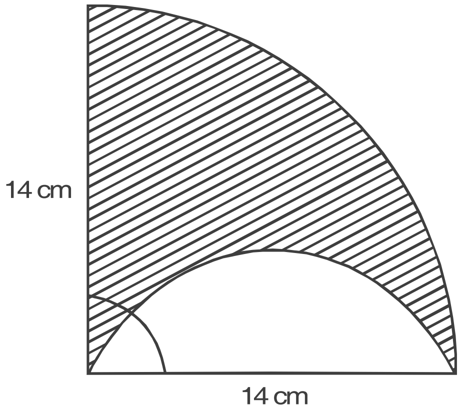
73. In the figure along side. Find the area of the Shaded portion.



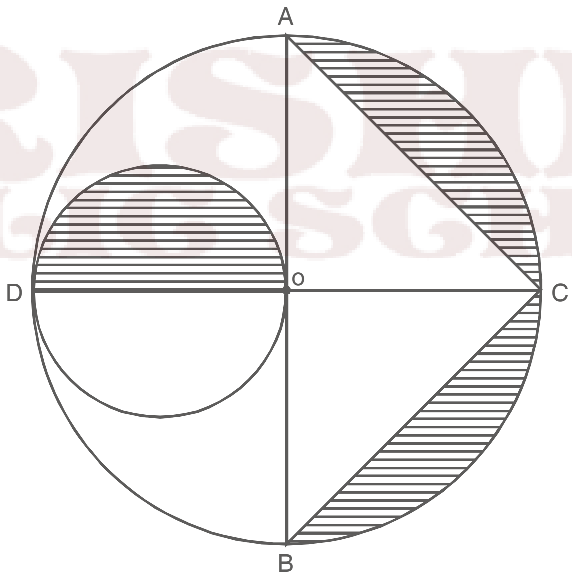
74. Find the shaded area.



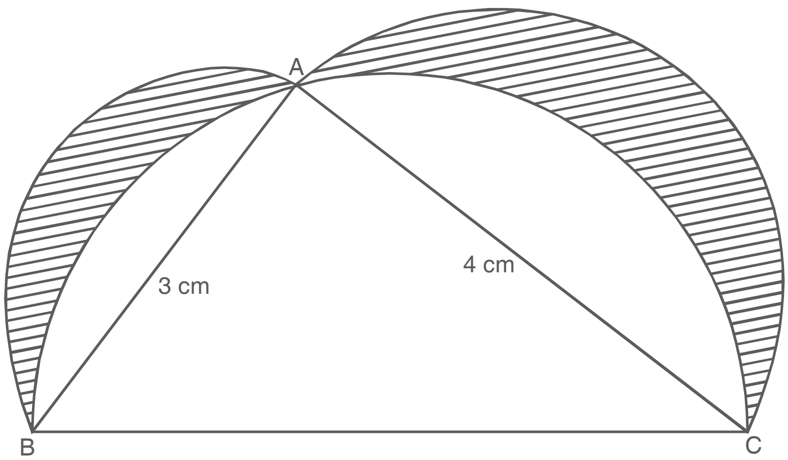
75. Find the shaded area.



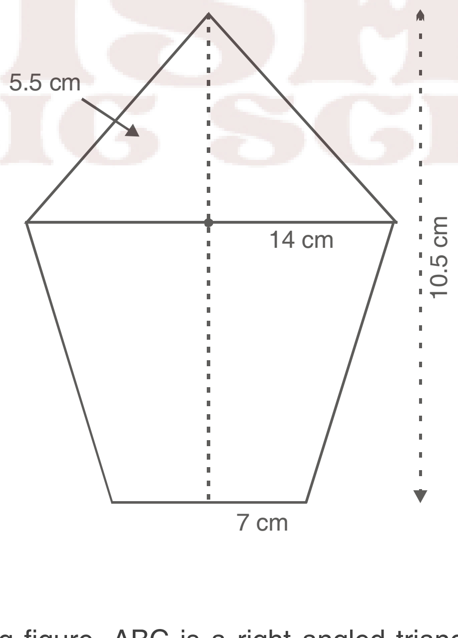
76. AB and CD ae two perpendicular diameters and CD = 8 cm find the area of Shaded portion.



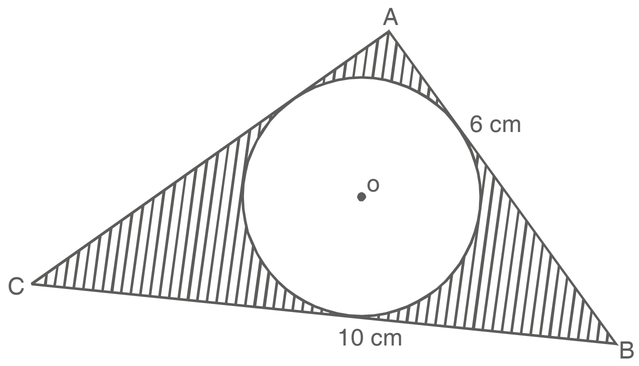
77. In the adjoining figure ABC is a right angled triangle, right angled at A. Semi circle are drawn on AB, AC and BC as diamaters. Find the area of shaded portion.



78. A toy is in the form of a conemounted on a cone frustum. If the radius of the top and bottom are 14 cm and 7 cm and the height of cone and toy are 5.5 cm and 10.5 cm respectively. Find the volume of toy.



79. In the adjoining figure, ABC is a right angled triangle at A. Find the area of Shaded region if AB = 6 cm, BC = 10 cm and 0 is the centre of the incircle of ΔABC (take $\pi = 3.14$).



ANSWERS

1.	b	2.	c
3.	a	4.	b
5.	b	6.	d
7.	b	8.	c
9.	c	10.	b
11.	b	12.	b
13.	3 units	14.	$37\frac{3}{7}$ cm.
15.	49 cm^2	16.	14 cm
17.	19.5 cm	18.	154 cm^2
19.	4 : 3	20.	24
21.	3 : 1	22.	3 : 1 : 2
23.	2 m	24.	7 cm
25.	27 cm	26.	$\pi : \sqrt{3}$
27.	14 cm	28.	150°

29. 7 m

30. 36 cm

31. 550 m^2

32. 5 cm

33. 22 cm

34. $\frac{50}{3}\pi m^2$

35. 200π

36. $(16 + \pi)$ cm

37. 154 cm^2

38. 42π

39. 25π

40. 10 cm

41. r^2

42. $\frac{60}{\pi}$ cm

43. π cm

44. 86 cm^2

45. $(25 - 4\pi)$ cm^2

46. 3π cm^2

47. $(16 - 4\pi)$ cm^2

48. 24 a^2

49. 1000 cm^3

50. $12\sqrt{3}$ cm

51. 5 cm

52. 8 cm

53. 4 cm

54. 6 cm

55. $30\sqrt{2}$ cm

56. 2 cm

57. 25 cm; 3850 cm^3

58. 44 m^3

59. 3.5 cm

60. 27π

61. 4 cm

62. $\frac{2}{3}\pi . r^2 . h$

63. Yes, when r = h

64. 1000 m^3

65. $3\pi R^2$

66. 29.46 cm^3

67. $\left(\frac{660}{7} + 36\sqrt{3}\right) cm^2$

68. 154 cm^2

69. $\frac{2}{3}$ h.

70. 36 cm

71. 1716 cm^2

72. 3 m

73. $\frac{1019}{14}$ cm^2

74. 154 m^2

75. 77 cm^2

76. $\frac{108}{7}$ cm^2

77. 6 cm^2

78. 2926 cm^3

79. 11.44 cm^2.

[**Hint :** Join 0 to A, B and C.

area of ΔABC = area of ΔOAB + area of ΔOBC + area of ΔOAC

$$= \frac{1}{2} AB \times r + \frac{1}{2} \times BC \times r + \frac{1}{2} AC \times r$$

$\Rightarrow$ (r = 2 cm)]

CHAPTER 8

PROBABLITY

1. The Theoretical probablity of an event E written as (E) is

$$P(E) = \frac{\text{Number of outcomes vourable to E}}{\text{Number of all possible outcomes of the experiment.}}$$

2. The sum of the probability of all the elementary events of an experiment is 1.

3. The probability of a sure event is 1 and probabaility of an impossible event is 0.

4. If E is an event, in general, it is true that P(E) + P ($\overline{E}$) = 1.

5. From the definition of the probability, the numerator is always less than or equal to the denominator therefore $O \leq P(E) \leq 1$.

MULTIPLE CHOICE QUESTIONS

1. If E is an event then P(E) + P$(\overline{E})$ = ... ?

   (a) 0 (b) 1

   (c) 2 (d) –1

2. The probability of an event that is cerain to happen is :

   (a) 0 (b) 2

   (c) 1 (d) –1

3. Which of the following can not be the probability of an event :

   (a) $\frac{2}{3}$ (b) $\frac{-3}{2}$

   (c) 15% (d) 0.7

4. If P(E) is .65 what is P (Not E)?

(a) .35 (b) .25

(c) 1 (d) 0

5. If P(E) is 38% of an event what is the probability of ilure of this event?

(a) 12% (b) 62%

(c) 1 (d) 0

6. A bag contains 9 Red and 7 blue marbles. A marble is taken out randomly, what is the P (red marble)?

(a) $\frac{7}{16}$ (b) $\frac{9}{16}$

(c) $\frac{18}{16}$ (d) $\frac{14}{16}$

7. In a Survey it is found that every fifth person possess a vehicle what is the probability of a person not possessing the vehicle?

(a) $\frac{1}{5}$ (b) $\frac{4}{5}$

(c) $\frac{3}{5}$ (d) 1

8. Anand and Sumit are friends what is the probability that they both have birthday on 11th Nov. (ignoring leap year).

(a) $\frac{1}{12}$ (b) $\frac{1}{7}$

(c) $\frac{1}{365}$ (d) $\frac{1}{366}$

9. The number of ce cards in a well shuffled pack of cards are :

(a) 12 (b) 16

(c) 4 (d) 52

10. A die is thrown once. What is the probability of getting an even prime number?

(a) $\frac{3}{6}$ (b) $\frac{1}{6}$

(c) $\frac{1}{2}$ (d) $\frac{1}{3}$

11. The probability of an impossible event is :

(a) 0 (b) 1

(c) –1 (d) ∞

12. Cards marked with numbers 1 to 20 are placed in a bag and mixed. One card is drawn what is the probability that card drawn is between 8 and 15.

(a) $\frac{8}{20}$ (b) $\frac{6}{20}$

(c) $\frac{15}{20}$ (d) 0

SHORT ANSWER TYPE QUESTIONS

13. A game of chance of a spnning wheel has number 1 to 10. What is the probability of getting a number less than equal to 5 when wheel comes to rest?

14. Two dice are rolled once what is the probability of getting a doublet?

15. A die is rolled once. What is the probability of getting a prime number?

16. A bank A.T.M. has notes of denomination 100, 500 and 1000 in equal numbers. What is the probability of getting a note of Rs. 1000.

17. What is the probability of getting a number greater than 6 in a single throw of a die.

18. A selection committee interviewed 50 people for the post of les manager. Out of which 35 are males and 15 are females. What is the probability of a female candidate being Selected.

19. A bag contains cards numbering from 5 to 25. One card is drawn from the bag. Find the probability that the card has numbers from 10 to 15.

20. In 1000 lottery tickets thre are 5 prize winning tickets. Find the probability of winning a prize. if a person buys one tickets.

21. It is known that in a box of 600 screws, 42 screws are defective. One screw is taken out at random from this box. Find the probability that it is not defective.

22. Write all the possible outcomes when a coin is tossed twice.

23. Two dice are rolled simultaneously. Find the probability that the sum is more than and equal to 10.

24. From the well shuffled pack of 52 cards. Two Black king and Two Red Aces are removed. What is the probability of getting a ce card.

25. In a leap year what is the probability of 53 Sundays.

26. A box contains card numbered from 2 to 101. One card is drawn at random. What is the probability of getting a numer which is a perfect square.

27. A box contains orange, mango and lemon flavoured candies. A candy is drawn randomly. (If p (not lemon) = $\frac{11}{15}$ and (P (mango) = $\frac{1}{3}$ then what is P(orange)?

28. From the well shuffled pack of 52 cards. Few cards of me colour are missing. If P (Red card) = $\frac{1}{3}$ and P (Black card) = $\frac{2}{3}$ then which colour of cards are missing and how many?

29. A bag contains 5 red balls and 'n' green balls. If the P(green ball) = 3 × P (red ball) then what is the value of n.

30. If from the well shuffled pack of cards all the aces are removed, find the probability of getting red card.

31. What is the probability of getting a total of less than 12 in the throws of two dice?

32. From the data (1, 4, 9, 16, 25, 29). If 29 is removed what is the probability of getting a prime number.

33. A card is drawn from an ordinary pack of pluging cards and a person bets that it is a spade or an ace. What are the odds against his winning the bet.

LONG ANSWER TYPE

34. A coin is tossed thrice then find the probability of

    (i) 2 heads (ii) 2 tails (iii) 3 heads.

35. The king, queen and jack of clubs are removed from a deck of 52 playing cards and the remaining cards are shuffled. A card is drawn from the remaining cards. Find the probability of getting a card of (i) heart; (ii) queen; (iii) Clubs.

36. A box contains 5 Red balls, 8 white balls and 4 Green balls. One ball is taken out of the box at random. What is the probability that ball is (i) red; (ii) white; (iii) Not green.

37. 12 defective pens are mixed with 120 good ones. One pen is taken out at random from this lot. Determine the probability that the pen taken out is not defective.

38. (i) A lot of 20 bulbs contain 5 defective bulbs. One bulb is drawn at random from the lot. What is the probability that this bulb is defective.

    (ii) Suppose the bulb drawn in (i) is not defective and is not replaced. Now one bulb is drawn at random from the rest. What is the probability that this bulb is not defective.

39. A box contains 90 discs which are numbered from 1 to 90. If one disc is drawn at random from the box, find the probability that it bears (i) a two digit number (ii) a perfect square number (ii) a number divisible by 5.

40. A game consists of tossing a one rupee coin 3 times and noting its outcome each time. Anand wins if all the tosses give the me result *i.e.,* three heads or three tails and loses otherwise. Calculate the probability that Anand will lose the game.

41. A die is thrown twice. What is the probability of getting : (i) The Sum of 7; (ii) The sum of greater than 10; (iii) 5 will not come up either time.

42. A box contains 12 balls out of which x are black. If one ball is drawn at random from the box, what is the probability that it will be black ball?

If 6 more black balls are put in the box, the probability of drawing a black ball is now double of what it was. Find x.

43. A jar contains 24 balls, some are green and other are blue. If a ball is drawn at random from the jar, the probability that it is green is $\frac{2}{3}$. Find the number of blue balls in the jar.

ANSWERS

1. b	2. c
3. b	4. a
5. b	6. b
7. b	8. c
9. a	10. b
11. a	12. b
13. $\frac{1}{2}$	14. $\frac{1}{6}$
15. $\frac{1}{2}$	16. $\frac{1}{3}$
17. 0	18. $\frac{3}{10}$
19. $\frac{2}{7}$	20. $\frac{1}{200}$
21. $\frac{93}{100}$	22. S = [HH, TT, HT, TH]
23. $\frac{1}{6}$	24. $\frac{5}{24}$

25. $\frac{2}{7}$

26. $\frac{9}{100}$

27. $\frac{2}{5}$

28. Red, 13

29. 15

30. $\frac{1}{2}$

31. $\frac{35}{36}$

32. zero

33. $\frac{9}{13}$

34. (i) $\frac{3}{8}$; (ii) $\frac{3}{8}$; (iii) $\frac{1}{8}$

35. (i) $\frac{13}{49}$; (ii) $\frac{3}{49}$; (iii) $\frac{10}{49}$

36. (i) $\frac{5}{17}$; (ii) $\frac{8}{17}$; (iii) $\frac{13}{17}$

37. $\frac{9}{10}$

38. (i) $\frac{1}{4}$; (ii) $\frac{14}{19}$

39. (i) $\frac{9}{10}$; (ii) $\frac{1}{10}$; (iii) $\frac{1}{5}$

40. $\frac{3}{4}$

41. (i) $\frac{1}{6}$; (ii) $\frac{1}{12}$; (iii) $\frac{25}{36}$

42. n = 3

43. 8.

BLUE PRINT
SAMPLE QUESTION PAPER

Topic/Type	MCQ 1 Mark	(I) 2 Marks	(II) 3 Maks	LA 4 Marks	Total
Algebra 10 (20)		4 (4)	3 (6)	2 (6)	1 (4)
Geometry	1 (1)	1 (2)	3 (9)	1 (4)	6 (16)
Mensuration	1 (1)	2 (4)	1 (3)	3 (13)	7 (20)
Some Application Trigonometry	1 (1)	—	1 (3)	1 (4)	3 (8)
Coordinate Geometry	2 (2)	1 (2)	2 (6)	—	5 (10)
Probability	1 (1)	1 (2)	1 (3)	—	3 (6)
Total	10 (10)	8 (16)	10 (30)	6 (24)	34 (80)

QUESTION PAPER (SOLVED)

CLASS X

Time allowed : 3 to 3½ hours
Maximum marks : 80

General Instructions

1. All question are compulsory.
2. The question paper consists of 34 questions divided into four sections A, B, C and D. Section A comprises of 10 questions of 1 mark each. Section B comprises of 8 questions of 2 marks each. Section C comprises of 10 questions of 3 marks each and Section D comprises of 6 questions of 4 marks each.
3. Question numbers 1 to 10 in Section A are multiple choice questions where you are to select one correct option out of the given four.
4. There is no overall choice. How ever, internal choice has been provided in 1 question of 2 marks 3 questions of three marks each and 2 questions of 4 marks each. You have to attempt only one of the alternatives in all such questions.
5. Use of calculators is not permitted.

SECTION A

Question number 1 to 10 are of 1 mark each

1. If a number 'x' is Selected at random from the number –3, –2, –1, 0, 1, 2, 3. The probability of $|n| < 2$ is –

   (a) $\frac{5}{7}$ (b) $\frac{2}{7}$

(c) $\frac{3}{7}$ (d) $\frac{1}{7}$

2. If one root of $3x^2 - 5x + k = 0$ is 1, then the value of 'k' is

(a) –2 (b) –8

(c) 8 (d) 2

3. The sum of first n terms of A.P. is n^2, then common difference is

(a) 1 (b) 2

(c) 3 (d) 4

4. The distance between the points (3, 0) and (0, x) is 5 units, the value of x will be

(a) 3 (b) 4

(c) 5 (d) 6

5. The length of the shadow of a pole 30m high at some instant is $10\sqrt{3}$ m. The angle of elevation of the sun is

(a) 60° (b) 30°

(c) 45° (d) 90°

6. If n^{th} term of an A.P. 4, 9, 14, ... is 124 then n is

(a) 25 (b) 26

(c) 27 (d) 24

7. The coordinates of the point where the line $\frac{x}{2} + \frac{y}{3} = 1$ intersect x axis is :

(a) (2, 0) (b) (0, 2)

(c) (3, 0) (d) (0, 3)

8. If the roots of the quadratic equation $ax^2 + bx + c = 0$, $a \neq 0$ are equal, then C is

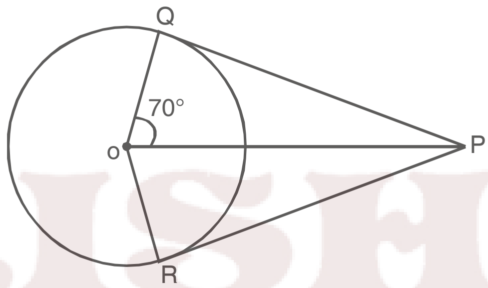
(a) $-\frac{b}{2a}$ (b) $\frac{b}{2a}$

(c) $\frac{-b^2}{4a}$ (d) $\frac{b^2}{4a}$

9. In the given fig. PQ and PR are tangent to the circle, $\angle QOP = 70°$, then $\angle QPR$ is equal to

   (a) 35° (b) 70°

   (c) 40° (d) 50°



10. A wire is in the form of a circle of radius 14 cm. If it is bent in the form of square. The side of square is :

   (a) 44 cm (b) 22 cm

   (c) 88 cm (d) 14 cm

SECTION B

11. In 8 times the 8th term is equal to 12 times the 12th term of an A.P. then find its 20th term.

12. An in-circle is drawm touching the equal sides of an isosceles triangle at E and F. Show that the point D. Where circle touches the third side is the mid point of that side.

13. The wheel of a bicycle makes 5000 rounds o cover the distance of 11 km. Find the diameter of the wheel.

14. Find the area of triangle whose vertices are (1, –1), (–3, 5) and (2, –7).

15. If the product of two consecutive natural numbers is 30, then find the numbers.

16. the sum of n terms of two A.P.'s are in the ratio 3n + 8 : 7n + 15. Find the ratio of their 9th terms.

17. The length of an area of a Sector is 5π cm and the area of Sector is 20π cm^2. Find the radius of the circle.

18. Cards with numbers 2, 3, 4, ... 101 are placed in a beg and mined throughly. One card is drawn at random. Find the probability that the number on the card is

    (i) an odd number

    (ii) A composite number less than 26.

OR

Two dice are rolled simultaneously. Find the probability that the sum is more than and equal to 10.

SECTION C

19. Find the roots of the quadratic equation $2x^2 + 5x - 7 = 0$ by the method of completing the square.

20. The sum of first 9 terms of an A.P. is 171 and sum of first 24 terms is 996, find A.P.

OR

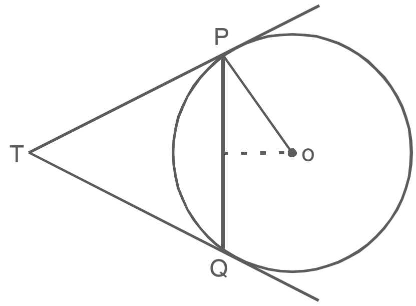
The sum of first 16 terms of an A.P. is 528 and sum of next 16 terms is 1552. Find its 19th term.

21. Constant a ΔABC in which BC = 6.5 cm. AB = 4.5 cm and $\angle ACB = 60°$. Construct another triangle similar to ΔABC such that each side of new triangle is $\frac{4}{5}$ of the corresponding sides of ΔABC.

22. PQ is a diameter of circle and PR is a chord such that $\angle RPQ = 30°$ and $\angle QSR = 30°$. The tangent at R intersects PQ produced at S. Prove that RQ = QS.

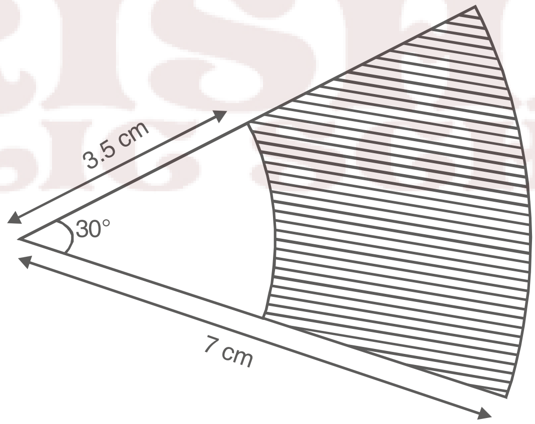
OR

In given Fig. PQ is a chord of length 8 cm of a ircle of radius 5 cm. The tangent at P and Q intersect at a point T. Find the length of TP.



23. Draw a circle of radius 3.5 cm with centre O. Now draw two tangents PA and PB from an external point P such that $\angle APB = 45°$. Measure the length of PA and PB.

24. Find the area of Shaded portion.



25. An observer 1.5 cm tall is 28.5 m away from a chimney. The angle of elevation of the top of the chimney from his eyes is 30°. Find the height of the chimney. (Take $\sqrt{3} = 1.73$).

26. Find the ratio in which the line $3x + y = 12$ divides the line segment joining the points (1, 3) and (2, 7).

OR

Show that the points (–2, 3), (8, 3) and (6, 7) are the vertices of a right angled triangle.

27. A point P on x-axis divides the line segment joining the points (4, 5) and (1, –3) in certain ratio. Find the coordinates of point P.

28. There are 54 marbles of blue, green and white coloured in a jar. The probability to select a blue marble is $\frac{1}{3}$ and the probability to select a green marble is $\frac{4}{9}$. How many white marbles are there?

SECTION D

29. Prove that the length of tangents drawn from an external point to a circle are equal.

OR

Prove that the opposite sides of a quadrilateral circumscribing a circle substend supplementary angles at the centre of the circle.

30. A pole of height 5 m is fixed on the top of the tower. The angle of elevation of the top of the pole as observed from a point 'A' on the ground is 60° and the angle of depression of the point A from the top of the tower is 45°. Find the height of the tower. (Take $\sqrt{3} = 1.732$).

31. The side of a square is 4 cm more than the other square. If the sum of areas of these two squares is 400 square cm. Find the sides of each square.

32. A cylinder whose height is two third of its diameter has the me volume as a sphere of radius solved 4cm. Calculate the radius of the base of the cylinder.

33. Marbles of diameter 1.4 cm are dropped in to a cylindrical beaker of diameter 7 cm containing some water find the number of Marble dropped so that water level rises by 536 cm.

34. Three cubes of metal whose edges are in the ratio 3 : 4 : 5 are melted into a single cube whose diagonal is $12\sqrt{3}$ cm find the edges of the three cubes.

OR

A cylinrical pipe has inner diameter of 4cm and water flows through it at the rate of 20m. per minute. How long would it take to fill a conical tank. Whose diameter of base is 80 cm and depth 72cm.

ANSWERS

1.	*c*	2.	*d*
3.	*b*	4.	*b*
5.	*a*	6.	*a*
7.	*a*	8.	*d*
9.	*c*	10.	*b*

11. Let first term and common difference of the A.P. are a and d respectively.

$$8\ (a + 7d) = 12\ (a + 11d)$$

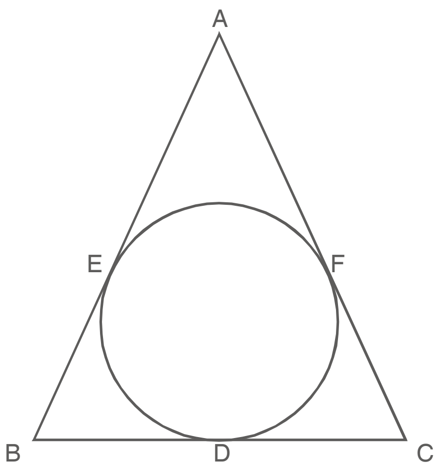
$$4a + 76d = 0$$

$$a + 19d = 0$$

$$a_{20} = 0.$$

12. AB = AC (given)

AE = AF (length of tangents from an external point)



$$AB - AE = AC - AF$$

$$BE = CF$$

but $BE = BD$ and $CF = CD$

$\therefore$ $BD = CD$

or D is mid point of BC

13. 5000 rounds = 11 km

= 11000 meter

$$1 \text{ round} = \frac{11000}{5000} \text{ m} = \frac{11}{5} \text{ m} = \frac{1100}{5} \text{ cm}$$

$$2\pi r = \frac{1100}{5} \text{ cm} \Rightarrow 2r = 70 \text{ cm}.$$

14. $\Delta = \frac{1}{2}\left[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right]$

$$= \frac{1}{2}\left[1(5 + 7) - 3(-7 + 1) + 2(-1 - 5)\right]$$

$$= \frac{1}{2}[12 + 18 - 12]$$

= 9 sq. units

15. Let two consecutive natural number are x and x + 1 :

$$x(x + 1) = 30$$

$$x^2 + x - 30 = 0$$

$$(x + 6)(x - 5) = 0$$

$x = 5, -6$ (Not natural), $\therefore$ Natural numbers are 5 and 6.

16. Let first terms and common differences of two A.P.'s are a_1, d_1 and a_2, d_2 respectively.

$$\frac{\frac{n}{2}\left\{2a_1 + (n-1)\,d_1\right\}}{\frac{n}{2}\left\{2a_2 + (n-1)\,d_2\right\}} = \frac{3n+8}{7n+15}$$

$$\frac{2\left\{a_1 + \frac{n-1}{2}d_1\right\}}{2\left\{a_2 + \frac{n-1}{2}d_2\right\}} = \frac{3n+8}{7n+15}$$

$$\frac{a_1 + 8d_1}{a_2 + 8d_2} = \frac{3 \times 17 + 8}{7 \times 17 + 15} = \frac{59}{134}.$$

{for 9th term

$$\frac{n-1}{2} = 8$$

$$n = 17\}.$$

$\therefore$ ratio of 9th terms is 59 : 134.

17. Area of a Sector $= \dfrac{l \, . \, r}{2}$

$$\Rightarrow \qquad 20\pi = \frac{5 \, . \, \pi \, . \, r}{2}$$

$$\Rightarrow \qquad r = 8\text{cm}$$

18. (i) Probability of an odd number $= \dfrac{50}{100} = \dfrac{1}{2}$

(ii) Probability of a composite number $= \dfrac{15}{100} = \dfrac{3}{20}$

OR

Probability $= \dfrac{1}{9}$

19. $$2x^2 + 5x - 7 = 0$$

$$x^2 + \frac{5}{2}x - \frac{7}{2} = 0 \quad \text{(Dividing by 2)}$$

$$x^2 + \frac{5}{2}x + \left(\frac{5}{4}\right)^2 = \frac{7}{2} + \left(\frac{5}{4}\right)^2$$

(Adding $\left(\frac{5}{4}\right)^2$ in both sides)

$$\left(x + \frac{5}{4}\right)^2 = \frac{7}{2} + \frac{25}{16} = \frac{81}{16}$$

$$x + \frac{5}{4} = \pm\frac{9}{4}$$

$$x = 1, -\frac{7}{2}$$

20. Let first term and common difference of A.P. and a and d respectively

$$\frac{9}{2}[2a + 8d] = 171$$

$$a + 4d = 19$$
...(i)

$$\frac{24}{2}\{2a + 3d\} = 996$$

$$2a + 23d = 83$$
...(ii)

Solving (i) and (ii) $a = 7$, $d = 3$

$\therefore$ A.P. is 7, 10, 13, ...

OR

Let first term and common difference of the A.P. are a and d respectively.

$$\frac{16}{2}\{2a + 15d\} = 528 \qquad \{S_{16} = 528\}$$

$$2a + 15d = 66 \qquad \text{...(i)}$$

$$\frac{32}{2}\{2a + 31d\} = 528 = 1552 \qquad \{S_{22} - S_{16} = 1552\}$$

$$2a + 31d = 130 \qquad \text{...(ii)}$$

From (i) and (ii) $a = 3$, $d = 4$

$\therefore$ 19th term $= 3 + 18 \times 4 = 75$.

21. Neat and correct construction.

22. $\angle RPQ = 30°$

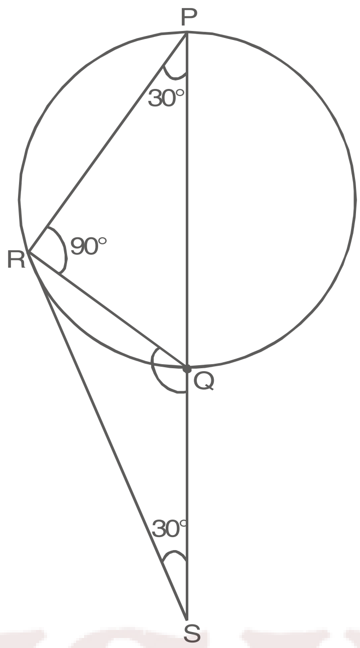
$\Rightarrow \quad \angle RQP = 60°$

$\Rightarrow \quad \angle RQS = 120°$

$\Rightarrow \quad \angle SRP = 30°$

Now $\quad \angle RSQ = \angle SRQ = 30°$

$\Rightarrow \quad QR = QS$



OR

Joint OT

Now $\Delta OPT \sim \Delta OMP$

$$\Rightarrow \quad \frac{OP}{OM} = \frac{PT}{MP}$$

$$\Rightarrow \quad \frac{5}{3} = \frac{n}{4}$$

$$\Rightarrow \quad n = \frac{20}{3} \text{ cm}$$

23. Neat and correct construction.

24. Area of Shaded Portion $= \frac{Q}{360}\pi R^2 - \frac{Q}{360}\pi r^2$

$$= \frac{30}{360}\pi \times 7^2 - \frac{30}{360}\pi \times (3.5)^2$$

$$= \frac{\pi}{12}\left[7^2 - \left(\frac{7}{2}\right)^2\right]$$

$$= \frac{22}{7 \times 12} \times 7^2 \left[1 - \frac{1}{4}\right]$$

$$= \frac{22 \times 7 \times 7}{7 \times 12} \times \frac{3}{4}$$

$$= \frac{77}{8}\ \text{cm}^2$$

25. In right ΔABC,

$$\frac{AB}{BC} = \tan 30^\circ$$

$$\frac{AB}{28.5} = \frac{1}{\sqrt{3}}$$

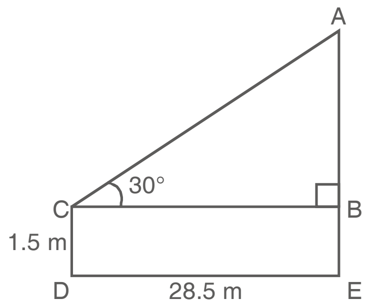
$$AB = 28.5 \times \frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$= 9.5 \times 1.73\ \text{m}$$

$$= 16.46\ \text{m}$$

Height of the Chimney $= 16.46 + 1.5$ m

$= 17.96$ m



26. Let the required ratio be K : 1

The coordinates of $P\left(\frac{2k+1}{k+1}, \frac{7k+3}{k+1}\right)$

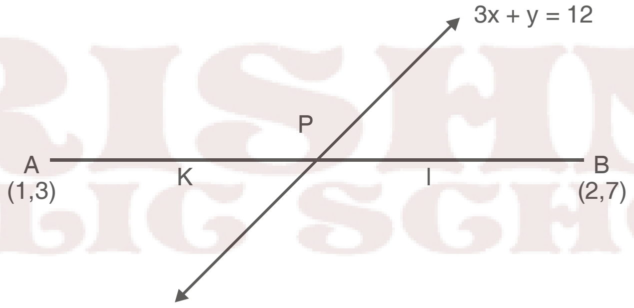
P lies on line $3x + y = 12$

$$\therefore \qquad 3\left(\frac{2k+1}{k+1}\right) + \frac{7k+3}{k+1} = 12$$

$$6k + 3 + 7k + 3 = 12k + 12$$

$$k = 6$$

$\therefore$ The required ratio is 6 : 1.



OR

A(–2, 3), B(8, 3)

C(6, 7)

$$AB^2 = (8+2)^2 + (3-3)^2$$

$$= (10)^2 + (0)^2$$

$$= 100$$

$$BC^2 = (6-8)^2 + (7-3)^2$$

$$= (-2)^2 + (4)^2$$

$$= 20$$

$$AC^2 = (6+2)^2 + (7-3)^2$$

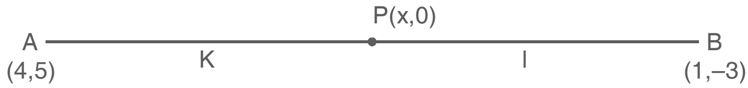
$$= (8)^2 + (4)^2$$

$= 80$

$\because \quad AB^2 = BC^2 + AC^2$

$\because$ By converse of Pythagoras theorem, ΔABC is a right angled triangle.

27. The coordinates of any point on x-axis be (x, 0) Let the required ratio be K : 1



$$0 = \frac{-3k + 5}{k + 1}$$

$$-3k + 5 = 0$$

$$k = \frac{5}{3}$$

$$\text{Ratio} = 5 : 3$$

$$x = \frac{k + 4}{k + 1}$$

$$= \frac{\frac{5}{3} + 4}{\frac{5}{3} + 1}$$

$$= \frac{\frac{17}{3}}{\frac{8}{3}} = \frac{17}{8}$$

The coordinates of P are $\left(\frac{17}{8}, 0\right)$.

28. Let there are x blue marbles

y green marbles

and z white marbles

According to question $x + y + z = 54$

Probabilit of Selecting blue marbles $= \frac{x}{54} = \frac{1}{3}$

$\Rightarrow \quad x = \frac{54}{3}$

$\Rightarrow \quad x = 18.$

Similarly Prob. of selecting green marbles $= \frac{y}{54}$

But $\frac{y}{54} = \frac{4}{9}$

$\Rightarrow \quad y = \frac{4 \times \cancel{54}\,6}{\cancel{9}}$

$y = 24$

But $n + y + z = 54$

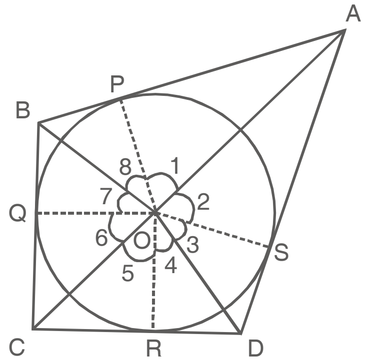
$\Rightarrow \quad 18 + 24 + z = 54$

$\Rightarrow \quad z = 12$

white marbles are 12. **Ans.**

29. Fig. + given + To prove + const.

Proof.



OR

Join OP, OQ, OR and OS.

$$\Delta AOP \cong \Delta AOS \text{ (by SSS)}$$

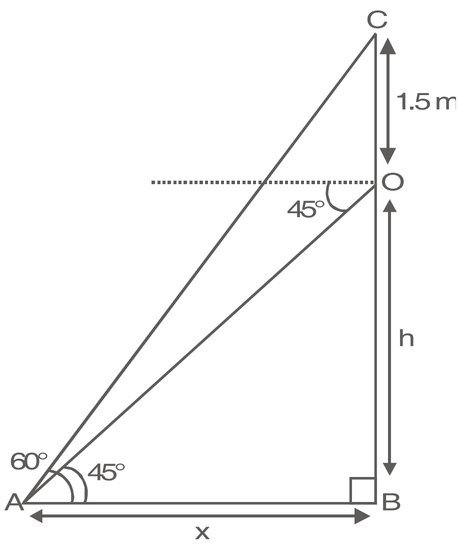
$$\angle 1 = \angle 2,\ \angle 3 = \angle 4,\ \angle 5 = \angle 6,\ \angle 7 = \angle 8$$

$$\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8 = 360$$

$$2(\angle 2 + \angle 3 + \angle 6 + \angle 7) = 360$$

$$\angle AOD + \angle BOC = 180°$$

30. Correct Fig.



In right ΔABD,

$$\frac{BD}{AB} = \tan 45^\circ$$

$$\frac{h}{x} = 1$$

$$h = x \qquad \text{...(1)}$$

In right ΔABC, $\dfrac{BC}{AB} = \tan 60^\circ$

$$\frac{h+5}{x} = \sqrt{3}$$

$$h + 5 = x\sqrt{3} \qquad \text{...(ii)}$$

$$h(\sqrt{3} - 1) = 5$$

$$h = \frac{5\left(\sqrt{3}+1\right)}{\left(\sqrt{3}-1\right)\left(\sqrt{3}+1\right)}$$

$h = x = 6.83$ m or height of tomer = 6.83 m.

31. Let the side of one square = x cm

and side of other square = (x – 4) cm

$$x^2 + (x - 4)^2 = 400$$

$$x^2 - 4x - 192 = 0$$

$$x = 16, -12$$

$\therefore$ Sides of squares are 16 cm and 12 cm.

32. Let diameter of cylender = 2r and radius is 'r'

$\Rightarrow$ heigh of cylender = $\dfrac{2}{3} \times 2r = \dfrac{4r}{3}$

Now volume of cylender = y of Sphere of radius 4 cm.

$\Rightarrow \qquad \pi \cdot r^2 \times \frac{\cancel{4}r}{\cancel{3}} = \frac{\cancel{4}}{\cancel{3}} \cdot \cancel{\pi} \times 4 \times 4 \times 4$

$r^3 = 4^3$

$\Rightarrow \qquad r = 4$ cm.

33. Let 'n' marbles are dropped

Now V of 'n' marbles = 4 of raised water in cylender

$$\Rightarrow \qquad n \times \frac{4}{3}\pi\, r^3 = \pi r^2 . H$$

$$\Rightarrow \qquad n \times \frac{4}{3}\pi \times \frac{14}{20} \times \frac{14}{20} \times \frac{14}{20} = \pi \times \frac{7}{2} \times \frac{7}{2} \times 5.6$$

$\Rightarrow \qquad n = 150$ marbles.

34. Let the sides of the cubes are 3x, 4x and $5x^3$

therefore volumes of three cubes $= (3x)^3 + (4x)^3 + (5x)^3$

$= 27x^3 + 64x^3 + 125x^3$

$= 216x^3$

But v of three cubes = v of one big cube

$216x^3 = a^3$

(let side of big cube is 'a').

$\Rightarrow \qquad a = 6x$

Now the length of diagonal of big cube is

$$\sqrt{(6x)^2 + (6x)^2 + (6x)^2} = \sqrt{108x^2}$$

$$\text{ii } \sqrt{108x^2} = 12\sqrt{3}$$

$\Rightarrow \qquad 108x^2 = 144 \times 3 \Rightarrow n = 2$

therefore the sides are 6.8 and 10 cm.

OR

Let pipe can fill in 't' minutes

therefore V. of water flowed in pipe in 't' minutes

= V. of water in cone.

$$\Rightarrow \quad \pi \times r^2 \times h \times t = \frac{1}{3}\pi r^2 . h$$

$$\Rightarrow \quad \cancel{\pi} \times 2 \times 2 \times 2000 \times t = \frac{1}{3}\cancel{\pi} \times 40 \times 40 \times 72$$

$$\Rightarrow \quad t = \frac{24}{5}\text{ min}.$$

or 4 min 48 sec. **Ans.**

Time allowed : 3 to 3½ hours
Maximum Marks : 80

General Instructions

1. All question are compulsory.
2. The question paper consists of 34 questions divided into four sections A, B, C and D. Section A comprises of 10 questions of 1 mark each. Section B comprises of 8 questions of 2 marks each. Section C comprises of 10 questions of 3 marks each and Section D comprises of 6 questions of 4 marks each.
3. Question numbers 1 to 10 in Section A are multiple choice questions where you are to select one correct option out of the given four.
4. There is no overall choice. How ever, internal choice has been provided in 1 question of 2 marks 3 questions of three marks each and 2 questions of 4 marks each. You have to attempt only one of the alternatives in all such questions.
5. Use of calculators is not permitted.

SECTION A

Question number 1 to 10 are of 1 mark each

1. The sum of roots of the quadratic equation $2x^2 + 13x + 11 = 0$ is

   (a) -13 (b) $-\frac{13}{2}$

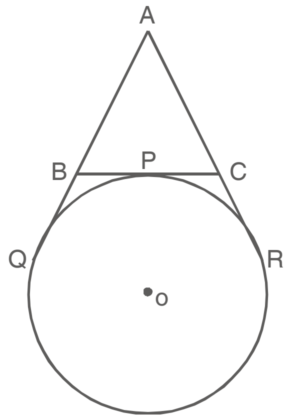
   (c) $\frac{11}{2}$ (d) -11

2. n^{th} term of the A.P. $-5, -3, -1, \ldots\ldots\ldots\ldots$ is

(a) $2n - 7$ (b) $7 - 2n$

(c) $2n + 7$ (d) $2n + 1$

3. In the given fig. P, Q and R are the points of contact. If $AB = 6$ cm, $BP = 3$ cm, then the perimeter of ΔABC is



(a) 12 cm (b) 18 cm

(c) 9 cm (d) 15 cm

4. The tops of the two poles of height 8 m and 12 m are connected with wire. If wire makes an angle 30° with the horizontal, then the length of wire is–

(a) 10 m (b) 12 m

(c) 8 m (d) 4 m

5. The distance between the line $y + 3 = 0$ and $2y - 5 = 0$ is–

(a) 8 unit (b) $\frac{11}{2}$ unit

(c) 6 unit (d) 5 unit

6. The probability of a question calculated to be right is $\frac{x}{12}$. If the probability of the question colculated not to be right is $\frac{2}{3}$ then x = ...

(a) 2 (b) 3

(c) 4 (d) 6

7. If –9, –14, –19, ... is an A.P. then the value of $a_{30} - a_{20}$ is

(a) –50 (b) 50

(c) 10 (d) None of these

8. A right circular cylinder of height 45 cm and radius 4 cm is made by melting of spheres of radius 6 cm each. Find the number of spheres.

(a) 3 (b) 4

(c) 5 (d) 6

9. At any instant, the shadow of a pole is equal to its height, the angle of elevation of the sum is

(a) 30° (b) 45°

(c) 60° (d) 90°

10. The perimeter of triangle formed by the points (0, 0), (3, 0) and (0, 3) is

(a) 6 unit (b) 9 unit

(c) $2\left(1+\sqrt{3}\right)$ unit (d) $3\left(2+\sqrt{2}\right)$ unit

SECTION B

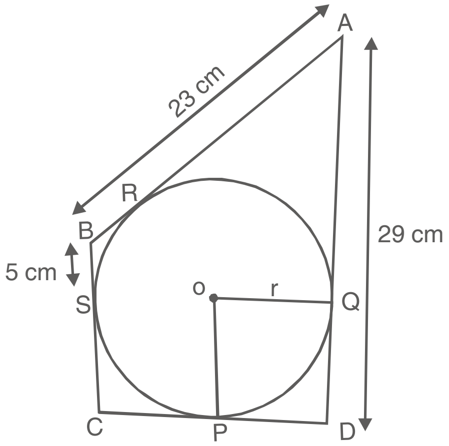
11. If the first term of an A.P. is 3 and 6^{th} term is 23 then find its 17^{th} term.

12. For what value of 'm' the roots of the quadratic equation : $4x^2 + mx + 1 = 0$ are real?

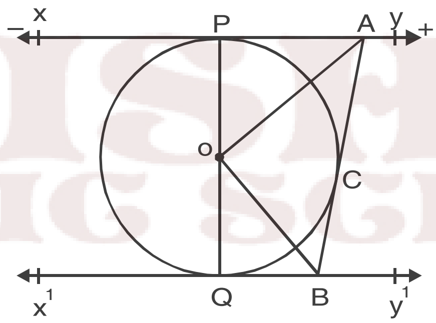
13. Two concentric circles are of radii 5 cm and 3 cm. Find the length of chord of the larger circle which touches the smaller circle.

OR

In given fig. find the radius of the circle.



14. In given fig. *XY* and *X´Y´* are two parallel tangents to a circle with centre *O* and another tangent *AB* with point of contact *C* intersecting *XY* at *A* and *X´ Y´* at *B*. Prove that < *AOB* = 90°.



15. Three balls are made by melting a ball of radius 3 cm out of these three the radius of two balls are 1.5 cm and 2 cm respectively. Find the radius of third ball.

16. The angle of elevation of the top of a tower from two points at a distance of 4 m and 9 m from the base and on the me straight line with it are complementary. Find the height of the tower.

17. Find a point on *y*-axis which is equidistant from the points (–2, 5) and (2, –3).

18. All kings, queens and jacks have been removed from a pack of cards and remaining cards are well shuffled. A card is drawn at random. Find the probability that it is–

    (a) A ce card. (b) A black card.

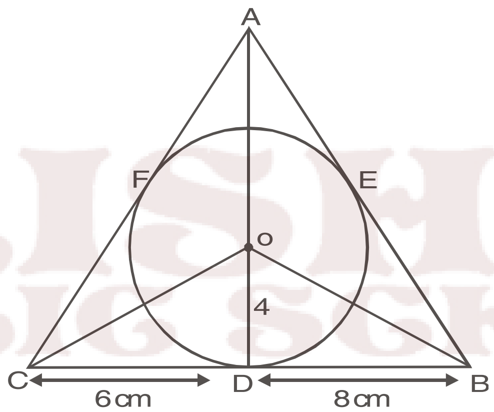
SECTION C

19. Construct an isosceles triangle whose base is 8 cm and altitude 5 cm and then construct another triangle whose sides are $\frac{3}{4}$ times the corresponding sides of the given triangle.

20. Solve the equation

$$\frac{x-1}{x-2} + \frac{x-3}{x-4} = \frac{10}{3}, \; x \neq 2, \; x \neq 4.$$

21. In given fig. a ΔABC is drawn to circum scribe a circle of radius 4 cm. D, E and F are points of contact. Find the sides AB and AC.



22. A copper of 2.2 dcm^3 is melted and recast into a wire of diameter .50 cm. Find the length of wire.

OR

Find the area swept by a minute hand of length 14 cm in one minute.

23. Find the sum of the A.P. 6 + 12 + 18 + ... + 120.

24. The sum of 4^{th} and 8^{th} term of on A.P. is 24 and then sum of 5^{th} and 10^{th} term is 39, find the A.P.

OR

If n^{th} term of an A.P. is $3–2n$, then find hte sum of its 40 terms.

25. The slant height of right circular cone is 10cm and its height is 8 cm. It is cut by a plane parallel to its base passing thorugh the mid point of the height find ratio of the volume of two parts.

26. In right angled ΔABC, $\angle B = 90°$ and $AB = \sqrt{34}$ unit. The coordinates of points B and C are (4, 2) and (–1, y) respectively. If $ar\ (\Delta ABC) = 17$ sq. unit, then find the value of y.

27. A number 'x' is selected from the numbers 1, 2, 3 and the another number 'y' is selected from the numbers 1, 4, 9 what is the probability that the promet of (x, y) is less than 9.

OR

A bag contains 12 balls out of which x are black. If 6 more black balls are put in the box, the probability of drawing a black ball is double of what it was before. Find x.

28. If the points (x, y), (–5, –2) and (3, –5) are collinear prove that $3x + 8y + 31 = 0$.

SECTION D

29. Two water taps together can fill a tank in $9\frac{3}{8}$ hours. The tap of larger diameter takes 10 hours less than the smaller one to fill the tank seperatly. Find the time in which each tap can separately fill the bank.

OR

A motor boat, whose speed is 9 km/h in still water goes 12 km. down stream and comes back in a total time 3 hours. Find the speed of the stream.

30. Prove that the tangent at any point of a circle is perpendicular to the radius through the point of contact.

31. From solid cylinder of height 28cm and radius 12cm, a conical cavity of height 16cm, and radius 12cm, is drilled out. Find (a) the volume (b) total surce area of remaining solid.

32. A container, shaped like a right circular cylinder, having diameter 12cm and height 15 cm is full of ice-cream. This ice-cream is to be filled in to

cones of height 12cm and diameter 6cm, having a hemi spherical shap on the top, find the number of such cones which can be filled with ice-cream.

33. From a point 100 m above a lake, the angle of elevation of astationary helicopter is 30° and the angle of depression of its reflection in the lake is 60°. Find the distance of the helicopter from the point of observation.

34. A hemispherical bowl of internal diameter 36 cm is full of liquid. Thus liquid is to be filled in cylindrical bottles of radius 3 cm and height 65 cm. How many bottles are required to empty the bowl?

OR

The inner circumference of a circular track is 440 cm. The track is 14 cm wide. Find the cast of leveling it at 20 paise/m. Also find the cast of putting up a fencing along outer circle at Rs. 2 metre.

ANSWERS

1. *b*	2. *a*
3. *b*	4. *c*
5. *b*	6. *c*
7. *a*	8. *c*
9. *b*	10. *d*
11. 67	12. $m \geq 4$ or $m \leq -4$
13. 8 cm or $\frac{11}{2}$ cm	15. 5 cm
16. 6 *m*	17. (0, 1)
18. (a) 0, (b) $\frac{1}{2}$	20. 5, $\frac{5}{2}$
21. AB = 15 cm, AC = 13 cm	22. 112 m or 10.26 cm^2
23. 1260	24. –13, –8, –3, ... OR –1520

25. 8 : 7

26. $y = -1, 5$

27. $\frac{5}{9}$ OR 3.

29. 15 hours, 25 hours OR 3 km/h

31. $10258\frac{2}{7}$ cm^3, $3318\frac{6}{7}$ cm^2

32. 10

33. 200 m

34. 72 OR Rs. 1355.20, Rs. 1056

KEY POINTS AND PRACTICE PAPERS

SUBJECT : MATHEMATICS

CLASS X

How to use this study material?

Dear Children,

This study material contains gist of the topics/units along with the assignments for self assessment. Here are some tips to use this study material while revision during pre-boards and finally in board examination.

- Go through the syllabus given in the beginning. Identify the units carrying more weightage.
- Suggestive blue print and design of question paper is a guideline for you to have clear picture about the form of the question paper.
- Revise each of the topics/ units and attempt the questions given for self assessment.
- After attempting the self assessment part, consult the question bank where questions carrying one, two , three, four marks are given. Revise them.
- After revision of all the units, solve the mple paper, and do self assessment with the value points.
- Must study the marking scheme/solution for CBSE previous year paper which will enable you to know the coverage of content under different questions.
- Underline or highlight key ideas to have birds eye view of all the units at the time of examination.
- Write down your own notes and make summaries with the help of this study material.
- Turn the theoretical information into outline mind maps.
- Make a separate revision note book for diagrams and numericals as well.
- Discuss your 'DOUBTS' with your teacher/other students.
- Use part2 for -2 and -4

Important:

- Slow learners may revise the knowledge part first.
- Bright students may emphasize the application part of the question paper.

INDEX

COURSE STRUCTURE
CLASS X

- As per CCE guidelines, the syllabus of Mathematics for class X has been divided term wise.
- The units specified for each term shall be assessed through both formative and summative assessment.
- In each term, there shall be two formative assessments each carrying 10% weightage.
- The summative assessment in I term will carry 30% weightage and the summative assessment in the II term will carry 30% weightage.
- Listed laboratory activities and projects will necesrily be assessed through formative assessments.

FIRST TERM (I)	
UNITS	
I NUMBER SYSTEM Real Numbers	
II ALGEBRA Polynomials, pair of linear equations in two variables.	
III GEOMETRY Triangles	
V TRIGONOMETRY Introduction to trigonometry, trigonometric identity.	22
VII STATISTICS	

SECOND TERM (II)	
UNITS	
II ALGEBRA(contd) Quadratic equations, arithmetic progressions	
III GEOMETRY(contd) Circles, constructions	
IV MENSURATION Areas related to Circles, Surce Area & Volumes	
V TRIGONOMETRY(Contd) Heights and Distances.	
VI COORDINATE GEOMETRY	
VII PROBABILITY	

DETAILS OF THE CONCEPTS TO BE MASTERED BY EVERY CHILD OF CLASS X WITH EXERCISE AND EXAMPLES OF NCERT TEXT BOOK

-I

SYMBOLS USED

*:-Important Questions, **:- Very important Questions, ***:- Very very important Questions

S.No	TOPIC	CONCEPTS	DEGREE OF IMPORTANCE	References(NCERT BOOK)
01	Real Number	Euclid's division Lemma & Algorithm	***	Example -1,2,3,4 Ex:1.1 Q:1,2,4
		Fundamental Theorem of Arithmetic	***	Example -5,7,8 Ex:1.2 Q:4,5
		Revisiting Irrational Numbers	***	Example -9,10,11 Ex: 1.3 Q:1.2 Th:1.4
		Revisiting Rational Number and their decimal Expansion	**	Ex -1.4 Q:1
02	Polynomials	Meaning of the zero of Polynomial	*	Ex -2.1 Q:1
		Relationship between zeroes and coefficients of a polynomial	**	Example -2,3 Ex-2.2 Q:1
		Forming a quadratic polynomial	**	Ex -2.2 Q:2
		Division algorithm for a polynomial	*	Ex -2.3 Q:1,2
		Finding the zeroes of a polynomial	***	Example: 9 Ex -2.3 Q:1,2,3,4,5 Ex-2.4,3,4,5
03	Pair of Linear Equations in two variables	Graphical algebraic representation	*	Example:2,3 Ex -3.4 Q:1,3
		Consistency of pair of liner equations	**	Ex -3.2 Q:2,4
		Graphical method of solution	***	Example: 4,5 Ex -3.2 Q:7
		Algebraic methods of solution a. Substitution method b. Elimination method c. Cross multiplication method d. Equation reducible to pair of liner equation in two variables	**	 Ex -3.3 Q:1,3 Example-13 Ex:3.4 Q:1,2 Example-15,16 Ex:3.5 Q:1,2,4 Example-19 Ex-3.6 Q :1(ii),(viii),2 (ii),(iii)
04	TRIANGLES	1) Similarity of Triangles	***	Theo:6.1 Example:1,2,3 Ex:6.2 Q:2,4,6,9,10

		2) Criteria for Similarity of Triangles	**	Example:6,7 Ex:6.3 Q:4,5,6,10,13,16
		3) Area of Similar Triangles	***	Example:9 The:6.6 Ex:6.4 Q:3,5,6,7
		4) Pythagoras Theorem	***	Theo:6.8 & 6.9 Example:10,12,14, Ex:6.5 Q:4,5,6,7,13,14,15,16
05	Introduction to Trigonometry	1) Trigonometric Ratios	*	Ex:8.1 Q:1,2,3,6,8,10
		2) Trigonometric ratios of some specific angles	**	Example:10,11 Ex:8.2 Q:1,3
		3) Trigonometric ratios of complementary angles	**	Example:14,15 Ex:8.3 Q:2,3,4,6
		4) Trigonometric Identities	***	Ex:8.4 Q:5 (iii,v,viii)
06	STATISTICS	CONCEPT 1 Mean of grouped data		
		1. Direct Method	***	Example:2 Ex:14.1 Q:1&3
		2. Assumed Mean Method	*	Ex:14.1 Q:6
		3. Step Deviation Method	*	Ex:14.1 Q:9
		CONCEPT 2		
		Mode of grouped data	***	Example:5 Ex:14.2 Q:1,5
		CONCEPT 3		
		Median of grouped data	***	Example:7,8 Ex:14.3 Q1,3,5
		CONCEPT 4		
		Graphical representation of c.f.(ogive)	**	Example:9 Ex:14.4 Q:1,2,3

1.Real numbers

(Key Points)

1. **Euclid's Division lemma:- G**iven Positive integers a and b there exist unique integers q and r tisfying
   a=bq +r, where 0≤r<b, where a, b, q and r are respectively called as dividend, divisor, quotient and remainder.

2. **Euclid's division Algorithm:-** To obtain the HCF of two positive integers y c and d, with c>0, follow the steps below:

   Step I: Apply Euclid's division lemma, to c and d, so we find whole numbers, q and r such that c =dq +r, 0$\leq r < d$.
   Step II: If r=0, d is the HCF of c and d. If r $\neq 0, apply\ the$ division lemma to d and r.
   Step III: Continue the process till the remainder is zero. The divisor at this stage will be the required HCF

3. **The Fundamental theorem of Arithmetic:-**
   Every composite number can be expressed (ctorised) as a product of primes, and this ctorization is unique, apart from the order in which the prime ctors occur.
   Ex.: $24 = 2\ X\ 2\ X\ 2\ X\ 3 = 3\ X\ 2\ X\ 2\ X\ 2$

   Theorem: LET x be a rational number whose decimal expansion terminates. Then x can be expressed in the form
   Of $\frac{p}{q}$ where $p\ and\ q$ are co-prime and the prime ctorition of q is the form of $2^n.5^m$, where n, m are non negative integers.
   Ex. $\frac{7}{10} = \frac{7}{2\ X\ 5} = 0.7$

   Theorem: LET x=$\frac{p}{q}$ be a rational number such that the prime ctorition of q is not of the form of $2^n.5^m$, where n, m are non negative integers. Then x has a decimal expansion which is non terminating repeating (recurring).
   Ex. $\frac{7}{6} = \frac{7}{2\ X\ 3} = 1.1666 \ldots \ldots ..$

   Theorem: For any two positive integers a and b,
   HCF (a,b) X LCM (a,b)=a X b
   Ex.: 4 & 6; HCF (4,6) = 2, LCM (4,6) = 12; HCF X LCM = 2 X 12 =24
   Ans. : a X b = 24

(Level- 1)

1. If $\frac{p}{q}$ is a rational number ($q \neq 0$). What is the condition on q so that the decimal representation of $\frac{p}{q}$ is terminating?

   Ans. q is form of $2^n.5^m$ $where\ n, m\ are\ non\ negative\ integers.$

2. Write a rational number between $\sqrt{2}\ and\ \sqrt{3}$.

   Ans. 1.5

3. The decimal expansion of the rational no. $\frac{43}{2^4.5^3}$ $will\ terminate\ after\ how\ many\ of\ decimals?$

   Ans. After 4 places of decimal.

4. Find the $(\ HCF\ X\ LCM\ ) for\ the\ numbers$ $100\ and\ 190.$

Ans. 19000

5. State whether the number $(\sqrt{2}-\sqrt{3})(\sqrt{2}+\sqrt{3})$ *is* rational or irrational justify.

Ans. Rational

6. Write one rational and one irrational number lying between 0.25 and 0.32.

Ans. One rational no. =0.26, one irrational no. = 0.27010010001………

7. Express 107 in the form of 4q + 3 for some positive integer.

Ans. 4 X 26 + 3

8. Write whether the rational number $\frac{51}{1500}$ will have a terminating decimal expansion or a non terminating repeating decimal expansion.

Ans. Terminating.

(level - 2)

1. Use Euclid's division algorithm to find the HCF of 1288 and 575.

Ans. 23.

2. Check whether $5\ X\ 3\ X\ 11 + 11\ and\ 5\ X\ 7 + 7\ X\ 3 + 3$ are composite number and justify.

Ans. Composite number.

3. Check whether 6^ncan end with the digit 0, where n is any natural number.

Ans. No, 6^ncan not end with the digit 0.

4. Given that LCM (26, 169) = 338, write HCF (26, 169).]

Ans. 13

5. Find the HCF and LCM of 6, 72 and 120 using the prime ctorization method.

Ans. HCF = 6
LCM = 360

(level - 3)

1. Show that $\sqrt{3}$ is an irrational number.
2. Show that $5+3\sqrt{2}$ is an irrational number.
3. Show that square of an odd positive integer is of the form 8m + 1, for some integer m.
4. Find the LCM & HCF of 26 and 91 and verify that $LCM\ X\ HCF = product\ of\ the\ two\ numbers$.

Ans. LCM=182, HCF=13

(PROBLEMS FOR SELF EVALUATION/HOTS)

1. State the fundamental theorem of Arithmetic.
2. Express 2658 as a product of its prime ctors.
3. Show that the square of an odd positive integers is of the form 8m + 1 for some whole number m.
4. Find the LCM and HCF of 17, 23 and 29.

5. Prove that $\sqrt{2}$ is not a rational number.
6. Find the largest positive integer that will divide 122, 150 and 115 leaving remainder 5, 7 and 11 respectively.
7. Show that there is no positive integer n for which $\sqrt{n-1} + \sqrt{n+1}$ *is rational*.
8. Using prime ctorization method, find the HCF and LCM of 72, 126 and 168. Also show that $HCF\ X\ LCM\ \neq product\ of\ the\ three\ numbers$.

2. Polynomials
(Key Points)

Polynomial:

An expression of the form $p(x) = a_0 + a_1x + a_2x^2 + \text{-----} + a_nx^n$ where $a_n \neq 0$ is called a polynomial in variable x of degree n. where; a_0, a_1, ----- a_n are real numbers and each power of x is a non negative integer.
Ex.:- $2x^2 - 5x + 1$ is a polynomial of degree 2.
Note: $\sqrt{x} +\ 3$ *is not a polynomial*.

- A polynomial $p(x) = ax + b$ *of degree* 1 *is called a linear polynomial*. Ex. 5x -3, 2x etc
- A polynomial $p(x) = ax^2 + bx + c$ *of degree* 2 *is called a quadratic polynomial*. Ex. $2x^2 + x - 1$, $1 - 5x + x^2$ etc.
- A polynomial $p(x) = ax^3 +\ bx^2 + cx + d$ *of degree* 3 *is called a cubic polynomial*.
  Ex. $\sqrt{3}x^3 -\ x +\ \sqrt{5},\ x^3 -\ 1$ etc.

Zeroes of a polynomial: A real number k is called a zero of polynomial $p(x) if\ p(x) = 0$. The graph of $y = p(x)$ intersects the X- axis.
- A linear polynomial has only one zero.
- A Quadratic polynomial has two zeroes.
- A Cubic polynomial has three zeroes.

For a quadratic polynomial: If α, β are zeroes of $P\ (x) = ax^2 +\ bx + c$ then :

1. Sum of zeroes = α +β = $\frac{-b}{a}$ = $\frac{-Coefficient\ of\ x}{coefficient\ of\ x^2}$
2. Product of zeroes = α . β = $\frac{c}{a}$ = $\frac{Constant\ term}{coefficient\ of\ x^2}$

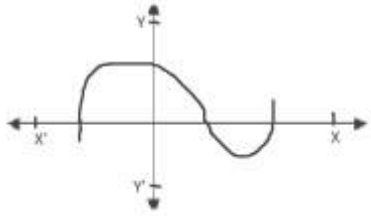
- A quadratic polynomial whose zeroes are α and β, is given by:
  $p\ (x) =\ x^2 - (\alpha + \beta)x +\ \alpha\beta$
  $= x^2 -\ (sum\ of\ zeroes)x + product\ of\ zeroes.$
- If $\alpha, \beta\ and\ \gamma$ are zeroes of the cubic polynomial $ax^3 + bx^2 + cx + d$ then:
  * $\alpha + \beta + \gamma\ = \frac{-b}{a}$
  * $\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a}$
  * $\alpha\beta\gamma = \frac{-d}{a}$

Division algorithm for polynomials: If $p(x) and\ g(x) are\ any\ two\ polynomials\ with\ g(x) \neq 0$, then we can find polynomials $q(x) and\ r(x) such\ that$:

$p(x) = q(x)X\ g(x) + r(x), where\ r(x) = 0\ or\ degree\ of\ r(x) < degree\ of\ g(x).$

(Level - 1)

1. In a graph of $y = p(x), find\ the\ number\ of\ zeroes\ of\ p(x).$



Ans. 3.

2. If α, β are the zeroes of $f\ (x) = x^2 + x + 1$, then find $\frac{1}{\alpha} + \frac{1}{\beta}$.

Ans. (-1)

3. Find a quadratic polynomial whose zeroes are $\frac{-2}{\sqrt{3}}\ and\ \frac{\sqrt{3}}{4}$.

Ans. $x^2 - \left(\frac{-2}{\sqrt{3}} + \frac{\sqrt{3}}{4}\right)x + (-\frac{1}{2})$

4. If $p\ (x) = \frac{1}{3}x^2 - 5x + \frac{3}{2}$ then find its sum and product of zeroes.

Ans. Sum=15, Product = $\frac{9}{2}$

5. If the sum of zeroes of a given polynomial $f\ (x) = x^3 - 3kx^2 - x + 30$ is 6. Find the value of K.

Ans. $\alpha + \beta + \gamma = \frac{-b}{a} = \frac{3k}{1} = 6$

$\therefore$ k = 2

6. Find the zero of polynomial $3x + 4$.

Ans. -4/3

7. Write the degree of zero polynomial.

Ans. Not defined.

(Level - 2)

1. Form a cubic polynomial with zeroes 3, 2 and -1.

Hints/Ans. $p(x) = x^3 - (\alpha + \beta + \gamma)x^2 + (\alpha\beta + \beta\gamma + \gamma\alpha)x - \alpha\beta\gamma$

2. Find the zeroes of the quadratic polynomial $6x^2 - 3 - 7x$ and verify the relationship between the zeroes and the coefficients.

Ans. Zeroes are 3/2 & -1/3.

3. For what value of k, (-4) is a zero of polynomial $x^2 - x - (2k + 2)$?

Ans. k=9

4. Give an example of polynomials $p(x), g(x), q(x) and\ r(x)\ which\ satisfy\ division\ algorithm\ and\ deg.p(x) = deg.g(x).$

Ans. $3x^2 + 2x + 1,\ x^2,\ 3,\ 2x + 1$

5. Find the zeroes of $4u^2 + 8u$.

Ans. 0, -2

6.Find a quadratic polynomial, whose the sum and product of its zeroes are $\frac{1}{4}, -1$.

Ans. $x^2 - \frac{1}{4}x - 1$

(Level - 3)

1. Find the zeroes of polynomial $x^3 - 2x^2 - x + 2$

Ans. -1, 1, 2

2. If the zeroes of the polynomial $x^3 - 3x^2 + x + 1$ are $\alpha - \beta,\ \alpha,\ \alpha + \beta$. Find $\alpha\ and\ \beta$

Ans. $\alpha = 1,\ \beta = \pm\sqrt{2}$

3. Divide $f(x) = 6x^3 + 11x^2 - 39x - 65$ by $g(x) = x^2 - 1 + x$

Ans. Quotient=$6x + 5$; Remainder = $-38x - 60$

4. Check whether the polynomial $t^2 - 3$ is a ctor of polynomial $2t^4 + 3t^3 - 2t^2 - 9t - 12$ $by\ applying\ the\ division\ algorithm.$

Ans. Remainder=0, Quotient=$2t^2$ + 3t + 4, Given Polynomial is a ctor.

(Level - 4)

1. Obtain all zeroes of $f(x) = x^3 + 13x^2 + 32x + 20$

Ans. -1, -2, -10

2. Obtain all other zeroes of $3x^4 + 6x^3 - 2x^2 - 10x - 5$, if two of its zeroes are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$

Ans. -1 & -1

3. On dividing $x^3 - 3x^2 + x + 2\ by$ a polynomial $g(x)$,the quotient and remainder were $x - 2$ and $-2x + 4$ respectively, find $g(x)$.

Ans. $x^2 - x$+1

(PROBLEMS FOR SELF-EVALUATION)

1. Check whether $g(x) = 3x - 2$ is a ctor of $p(x) = 3x^3 + x^2 - 20x + 12$.
2. Find quotient and remainder applying the division algorithm on dividing $p(x) = x^3 - 6x^2$+2x -4 by $g(x) = x - 1$.
3. Find zeros of the polynomial $2x^2 - 8x + 6$
4. Find the quadratic polynomial whose sum and product of its zeros are $\frac{2}{3}$, $\frac{-1}{3}$respectively.
5. Find the zeroes of polynomial $x^3 - 2x^2 - x + 2$
6. If one of the zeroes of the polynomial $2x^2 + px + 4 = 0\ is$ 2, find the other root, also find the value of p.
7. If $\alpha\ and\ \beta$ are the zeroes of the polynomial $kx^2 + 4x$+4 show that $\alpha^2 + \beta^2 = 24$, find the value of k.
8. If $\alpha\ and\ \beta$ are the zeroes of the equation $6x^2 + x - 2 = 0, find\ \frac{\alpha}{\beta} + \frac{\beta}{\alpha}$

---xxx---

3. Pair of linear equations in two variables
(Key Points)

- An equation of the form ax + by + c = 0, where a, b, c are real nos ($a \neq 0$, $b \neq 0$) is called a linear equation in two variables x and y.
  Ex : (i) x – 5y + 2 =0
  (ii) $\frac{3}{2}x - y = 1$

- The general form for a pair of linear equations in two variables x and y is
  $a_1x + b_1y + c_1 = 0$
  $a_2x + b_2y + c_2 = 0$
  where a_1, b_1, c_1, a_2, b_2, c_2 are all real nos and $a_1 \neq 0$, $b_1 \neq 0$, $a_2 \neq 0$, $b_2 \neq 0$.
  Examples: $x + 3y - 6 = 0$
  $2x - 3y - 12 = 0$

- Graphical representation of a pair of linear equations in two variables:
  $a_1x + b_1y + c_1 = 0$
  $a_2x + b_2y + c_2 = 0$

  (i) will represent intersecting lines if $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$
  i.e. unique solution. And this type of equations are called consistent pair of linear equations.
  Ex: x – 2y = 0
  3x + 4y – 20 = 0

  (ii) will represent overlapping or coincident lines if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$
  i.e. Infinitely many solutions, consistent or dependent pair of linear equations
  Ex: 2x + 3y – 9 = 0
  4x + 6y – 18 = 0

  (iii) will represent parallel lines if $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$
  i.e. no solution and called inconsistent pair of linear equations
  Ex: x + 2y – 4 = 0
  2x + 4y – 12 = 0

  (iv) Algebraic methods of solving a pair of linear equations:

(i) Substitution method
(ii) Elimination Method
(iii) Cross multiplication method

(Level - 1)

1. Find the value of 'a' so that the point(3,9) lies on the line represented by 2x-3y=5

   Ans: a= $\frac{1}{3}$

2. Find the value of k so that the lines 2x – 3y = 9 and kx-9y =18 will be parallel.

   Ans: k= 6

3. Find the value of k for which x + 2y =5, 3x+ky+15=0 is inconsistent

Ans: k= 6

4. Check whether given pair of lines is consistent or not 5x – 1 = 2y, y = $\frac{-1}{2} + \frac{5}{2}x$

Ans: consistent

5. Determine the value of 'a' if the system of linear equations 3x+2y -4 =0 and 9x – y – 3 = 0 will represent intersecting lines.

Ans: a $\neq \frac{-3}{2}$

6. Write any one equation of the line which is parallel to √2x – √3y =5

Ans: $5\sqrt{2}x - 5\sqrt{3}y = 5\sqrt{5}$

7. Find the point of intersection of line -3x + 7y =3 with x-axis

Ans: (-1, 0)

8. For what value of k the following pair has infinite number of solutions.

   (k-3)x + 3y = k

   k(x+y)=12

Ans: k= 6

9. Write condition so that $a_1x + b_1y = c_1$ and $a_2x + b_2y = c_2$ have unique solution.

Ans: $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

(Level - 2)

1. 5 pencils and 7pens together cost Rs. 50 whereas 7 pencils and 5 pens together cost Rs. 46. Find the cost of one pencil and that of one pen.

Ans: Cost of one pencil = Rs. 3

Cost of one pen = Rs. 5

2. Solve the equations:

   3x – y = 3

   7x + 2y = 20

Ans: x=2, y=3

3. Find the fraction which becomes to 2/3 when the numerator is increased by 2 and equal to 4/7 when the denominator is increased by 4

Ans: 28/45

4. Solve the equation:

   px + qy = p – q

   qx – py = p + q

Ans: x = 1, y = -1

(Level - 3)

1. Solve the equation using the method of substitution:

$$3x - 5y = -1$$
$$x - y = -1$$

Ans. $x = -2, \quad y = -1$

2. Solve the equations:

$$\frac{1}{2x} - \frac{1}{y} = -1$$
$$\frac{1}{x} + \frac{1}{2y} = 8 \qquad \text{Where, } x \neq 0, y \neq 0$$

Ans. $x = \frac{1}{6}, y = \frac{1}{4}$

3. Solve the equations by using the method of cross multiplication:

$$x + y = 7$$

5x + 12y =7

Ans. $x = 11, y = -4$

4. A man has only 20 pai coins and 25 pai coins in his purse, If he has 50 coins in all totaling Rs. 11.25, how many coins of each kind does he have.

Ans. 25 coins of each kind

5. For what value of k, will the system of equations

$x + 2y = 5$

$3x + ky - 15 = 0$ has a unique solution.

Ans. $k \neq 6$

(level - 4)

1. Draw the graphs of the equations

4x – y = 4

4x + y = 12

Determine the vertices of the triangle formed by the lines representing these equations and the x-axis. Shade the triangular region so formed

Ans: (2,4)(1,0)(3,0)

2. Solve Graphically

x – y = -1 and

3x + 2y = 12

Calculate the area bounded by these lines and the x- axis ,

Ans: x = 2, y = 3 and area = 7.5 unit ^{2}

3. Solve :- $\frac{10}{x+y} + \frac{2}{x-y} = 4$

$\frac{15}{x+y} + \frac{5}{x-y}$ = -2

Ans: x = 3 , y = 2

4. Ritu can row downstream 20 km in 2 hr , and upstream 4 km in 2 hr . find her speed of rowing in still water and the speed of the current. (HOTS)

Ans: Speed of the rowing is still water = 6 km/hr

Speed of the current = 4 km/hr .

5. In a $\Delta\, ABC$,$\angle C$ = 3 ,∠B = 2 (∠A +∠B) find the these angle. (HOTS)

Ans: ∠a = 20^0 , ∠b = 40^0 , ∠c = 120^0 .

6. 8 men and 12 boys can finish a piece of work in 10 days while 6 men and 8 boys can finish it in 14 days. Find the time taken by 1 man alone and that by one boy alone to finish the work . (HOTS)

Ans: One man can finish work in 140 days

One boys can finish work in 280 days

7. Find the value of K for which the system of linear equations 2x+5y = 3 , (k +1)x + 2(k + 2) y = 2K Will have infinite number of solutions.

(HOTS)

Ans: K = 3

(SELF EVALUTION/HOTS)

1. Solve for x and y:

   x + y = a + b

   ax – by= $a^2 - b^2$

2. For what value of k will the equation x +5y-7=0 and 4x +20y +k=0 represent coincident lines?

3. Solve graphically: 3x +y +1=0

   2x -3y +8=0

4. The sum of digits of a two digit number is 9. If 27is subtracted from the number, the digits are reversed. Find the number.

5. Draw the graph of x + 2y – 7 =0 and 2x – y -4 = 0. Shade the area bounded by these lines and Y-axis.

6. Students of a class are made to stand in rows. If one student is extra in a row, there would be 2 rows less. If one student is less in a row there would be 3 rows more. Find the number of the students in the class.

7. A man travels 370 km partly by train and partly by car. If he covers 250 km by train and the rest by the car it takes him 4 hours, but if he travels 130 km by train and the rest by car, he takes 18 minutes longer. Find the speed of the train and that of the car

8. Given linear equation 2x +3y-8=0, write another linear equation such that the geometrical representation of the pair so formed is (i) intersecting lines, (ii) Parallel Lines.

TRIANGLES

KEY POINTS

1. **Similar Triangles:-** Two triangles are id to be similar, if (a) their corresponding angles are equal and (b) their corresponding sides are in proportion (or are in the me ration).

2. Basic proportionality Theorem [or Thales theorem].

3. Converse of Basic proportionality Theorem.

4. Criteria for similarity of Triangles.

   (a) AA or AAA similarity criterion.
   (b) S similarity criterion.
   (c) SSS similarity criterion.

5. Areas of similar triangles.

6. Pythagoras theorem.

7. Converse of Pythagoras theorem.

(Level -1)

1. If in two triangles, corresponding angles are equal, then the two triangles are…………..

Ans. Equiangular then similar

2. ΔABC is a right angled at B. BD is perpendicular upon AC. If AD=a, CD=b, then AB^2=

Ans. a(a+b)

3. The area of two similar triangles are $32cm^2$ and $48cm^2$.If the square of a side of the first Δ is $24cm^2$,then the square of the corresponding side of 2^{nd} $triangle$ will be

Ans. $36cm^2$

4. ABC is a triangle with DE|| BC. If AD=2cm, BD=4cm then find the value DE:BC

Ans. 1:3

5. In ΔABC,DE ||BC, if AD=4x-3,DB=3x-1,AE=8x-7and BC=5x-3,then find the values of x are:

Ans. $1, -\frac{1}{2}$

6. The perimeters of two similar triangles are 40cm and 50 cm respectively, find the ratio of the area of the first triangle to the area of the 2^{nd} triangle:

Ans. 16:25

7. A man goes 150m due east and then 200m due north. How r is he from the starting point?

Ans. 250 m

8. A ladder reaches a window which is 12m above the ground on one side of the street. Keeping its foot at the me point, the ladder is turned to the other side of the street to reach a window 9m high. If the length of the ladder is 15m, find the width of the street.

Ans. 21m

9. BO and CO are respectively the bisector of ∠B and ∠C of ΔABC.AO produced meets BC at P,then find AB/AC

Ans.$\frac{BP}{PC}$

10.In ▲ABC,the bisectors of ∠B intersects the side AC at D.A line parallel to side AC intersects line segments AB,DB and CB at points P,R,Q respectively.Then, Find AB XCQ

Ans. BC X AP

11.If ΔABC is an equilateral triangle such that AD⊥BC,then AD^2=……….

Ans. $3CD^2$

12.If ΔABC and ΔDEF are similar triangles such that ∠A=47^0,and∠E=83^0 ,then find ∠C

Ans. 50^0

13.Two isosceles triangles have equal angles and their areas are in the ratio 16:25,then find the ratio of their corresponding heights

Ans. 4:5

14.Two poles of heights 6m and 11m stand vertically upright on a plane ground.If the distance between their feet is 12m,then find the distance between their tops.

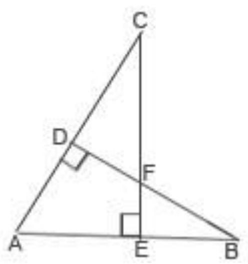
Ans.13m

15.The lengths of the diagonals of a rhombus are 16cm and 12cm.Then, find the length of the side of the rhombus .

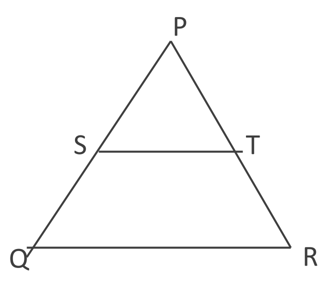
Ans. 10cm

(Level - 2)

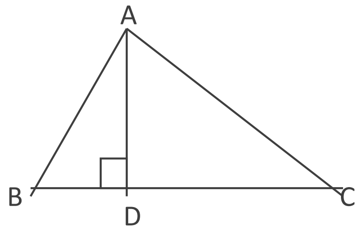
1.In given fig. BD⊥AC and CE⊥AB then prove that
(a)ΔAEC~ΔADB
(b)CA/AB=CE/DB



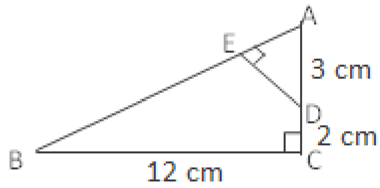
2. In the given figure fig .$\frac{PS}{SQ}=\frac{PT}{TR}$, and ∠PST=∠PQR. Prove that ΔPQR is an isosceles triangle.



3.In given fig AD⊥BC and $\angle B<90^0$,prove that $AC^2=AB^2 + BC^2 - 2BC \times BD$



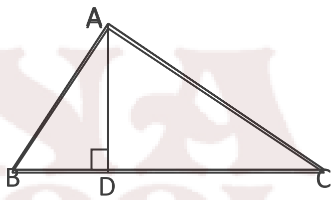
4.In given fig. ΔABC is right angled at C and DE⊥AB. Prove that ΔABC~ΔADE and hence find length of AE and DE.



Ans.$\frac{15}{17},\frac{36}{17}$

5. In a ΔABC ,if DE||AC and DF||AE, prove that$\frac{EF}{BF}=\frac{EC}{BE}$

6.In given fig.AD⊥BC,if $\frac{BD}{AD}=\frac{DA}{DC}$, prove that ABC is a right angled triangle.

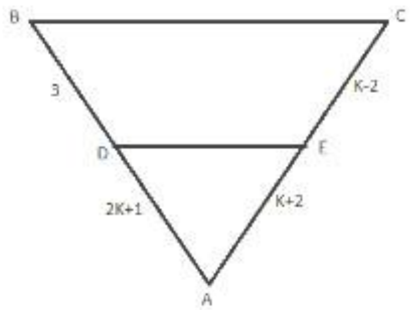


7.Two Δs ABC and DEF are similar. If ar(ΔDEF)=243cm²,ar(ΔABC)=108cm² and BC=6cm,find EF.

Ans. 9 cm

8.What is the value of K in given figure if DE||BC.

Ans. K=4, -1



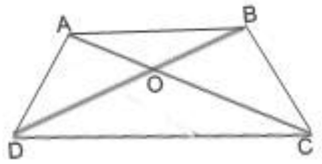
9. A pole of length 10m casts a shadow 2m long on the ground. At the me time a tower casts a shadow of length 60m on the ground then find the height of the tower.

Ans. 300m

<u>Level - 3</u>

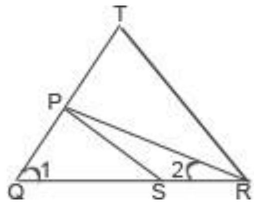
1.In given figure,AB||DCand $\frac{AO}{OC}=\frac{BO}{OD}$ then find the value of x, if .
$OA = 2x + 7, OB = 4x,OD = 4x - 4 \; and \; OC = 2x + 4$

Ans. 7



2.PQR is a right angled triangle with $\angle$P=90^0. If PM $\perp$QR, then show that $PM^2 = QM\ X\ \ MR$

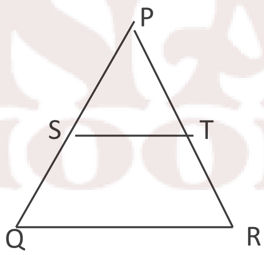
3. In given fig. $\frac{QR}{QS}$=$\frac{QT}{PR}$ and $\angle$1=$\angle$2.Show that ΔPQS~ΔTQR.



4. Find the length of altitude of an equilateral triangle of side 2cm.
Ans. $\sqrt{3}$ cm

5. In a trapezium ABCD,O is the point of intersection of AC and BD,AB||CD and AB=2CD.If the area of ΔAOB=84cm² then find area of ΔCOD. Ans. 21 cm^2

6.In given fig. $\frac{PS}{SQ}$=$\frac{PT}{TR}$=3.If area of ΔPQR is 32cm², then find the area of the quad.STQR Ans.14 cm^2



7. M is the mid-point of the side CD of a ||gm ABCD. The line BM is drawn intersecting AC at L and AD produced at E. Prove that EL=2BL.

8.Prove that the ratio of the area of two similar Δs is equal to the square of the ratio of their corresponding medians.

9. D and E are points on the sides CA and CB respectively of ΔABC, right angled at C.Prove that AE²+BD²=AB²+DE².

10 .ABC and DBC are two Δs on the me base BC and on the me side of BC with $\angle$A=$\angle$D=90^0.If CA and BD meet each other at E, show that AE x EC=BE x ED.

Level - 4

1.Prove that in a right angled triangle the square of hypotenuse is equal to the sum of the squares of the other two sides.

2.If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, prove that the other two sides are divided into the me ratio.

3. Δ ABC is right angled at B and D is midpoint of side BC. Prove that $AC^2 = 4AD^2 - 3AB^2$

4. Prove that the ratio of the areas of two similar triangles is equal to the ratio of square of their corresponding sides.

5. In a Δ , if the square of one side is equal to sum of the squares of the other two sides, prove that the angle opposite to the first side is a right angle.

6. In an equilateral Δ ABC, D is a point on the side BC, such that BD = $\frac{1}{3}$BC. Prove that $9AD^2 = 7AB^2$

7. P and Q are the mid points of side CA and CB respectively of Δ ABC right angled at C. Prove that $4(AQ^2 + BP^2) = 5AB^2$.

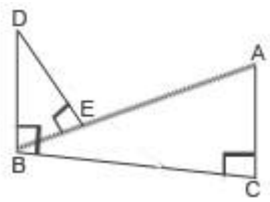
8. CM and RN are respectively the medians of ΔABC and ΔPQR. If ΔABC~ΔPQR, prove that

(i) ΔAMC~ΔPNR (ii)CM/RN=AB/PQ (iii)ΔCMB~ΔRNQ

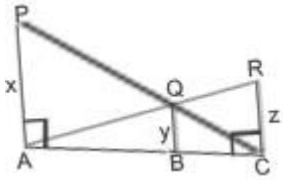
SELF EVALUATION

1.The diagonal BD of a ||gm ABCD intersects the line segment AE at the point F, where E is any point on the side BC. Prove that DF x EF=FB x .

2. In fig.DB⊥BC,DE⊥AB and AC⊥BC. Prove that BE/DE=AC/BC.



3. In given fig. PA, QB, RC are each perpendicular to AC. Prove that $\frac{1}{x}+\frac{1}{z}=\frac{1}{y}$



4. Prove that three times the sum of the squares of the sides of a triangle is equal to four times the sum of the squares of the medians of the triangle.

5. ABC is a right triangle with $\angle A = 90^0$,A circle is inscribed in it. The lengths of the two sides containing the right angle are 6 cm and 8 cm. find the radius of the incircle. Ans. 4cm

6. ABC is a right triangle, right angled at C. If p is the length of the perpendicular from C to AB and a, b, c have the usual meaning, then prove that

(i) cp=ab (ii) $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$

7. In a trapezium ABCD, AB||DC and DC=2AB.EF||AB, where E and F lie on the side BC and AD respectively such that BE/EC=4/3.Diagonal DB intersects EF at G. Prove that EF=11AB.

8. Sides AB, AC and median AD of a triangle ABC are respectively proportional to sides PQ, PR and median PM of another triangle PQR. Show that ΔABC~ΔPQR.

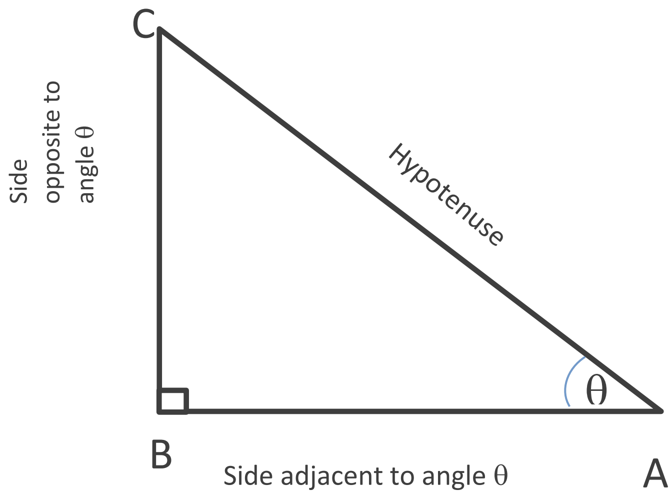
INTRODUCTION TO TRIGONOMETRY

IMPORTANT CONCEPTS

TAKE A LOOK:

1. Trigonometric ratios of an acute angle of a right angled triangle.

$$\sin\theta = \frac{\text{Side opposite to}\angle\theta}{\text{Hypotenuse}} = \frac{BC}{AC}$$



$$\cos\theta = \frac{\text{Side adjacent to}\angle\theta}{\text{Hypotenuse}} = \frac{AB}{AC}$$

$$\tan\theta = \frac{\text{Side opposite to}\angle\theta}{\text{Side adjacent to }\angle\theta} = \frac{BC}{AB}$$

$$cot = \frac{1}{Tan\ \theta} = \frac{\text{Side Adjacent to}\angle\theta}{\text{Side Opposite to }\angle\theta} = \frac{AB}{BC}$$

$$\sec\theta = \frac{1}{\cos\theta} = \frac{\text{Hypotenuse}}{\text{Side adjacent to }\angle\theta} = \frac{AC}{AB}$$

$$\operatorname{cosec}\theta = \frac{1}{\sin\theta} = \frac{\text{Hypotenuse}}{\text{side Opposite to }\angle\theta} = \frac{AC}{BC}$$

2. Relationship between different trigonometric ratios

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$

$$\tan\theta = \frac{1}{\cot\theta}$$

$$\cos\theta = \frac{1}{\sec\theta}$$

$$\sin\theta = \frac{1}{\operatorname{cosec}\theta}$$

3. Trigonometric Identities.
   (i) $\sin^2\theta + \cos^2\theta = 1$
   (ii) $1 + \tan^2\theta = \sec^2\theta$
   (iii) $1 + \cot^2\theta = \operatorname{cosec}^2\theta$

4. Trigonometric Ratios of some specific angles.

θ	0°	30°	45°	60°	90°
sin θ	0	½	1/√2	√3/2	1
cos θ	1	√3/2	1/√2	1/2	0
tan θ	0	1/√3	1	√3	Not defined
cot θ	Not defined	√3	1	1/√3	0
sec θ	1	2/√3	√2	2	Not defined
cosec θ	Not defined	2	√2	2/√3	1

5. Trigonometric ratios of complementary angles.
   (i) $\sin(90^o - \theta) = \cos\theta$
   (ii) $\cos(90^o - \theta) = \sin\theta$
   (iii) $\tan(90^o - \theta) = \cot\theta$
   (iv) $\cot(90^o - \theta) = \tan\theta$
   (v) $\sec(90^o - \theta) = \text{cosec}\,\theta$
   (vi) $\text{cosec}(90^o - \theta) = \sec\theta$

(Level – 1)

1. If θ and 3θ-30° are acute angles such that sinθ=cos(3θ-30°),then find the value of tanθ.

Ans. $\frac{1}{\sqrt{3}}$

2. Find the value of $\frac{(\cos 30°+\sin 60°)}{(1+\cos 60°+\sin 30°)}$

Ans. 2√3

3. Find the value of $(\sin\theta+\cos\theta)^2+(\cos\theta-\sin\theta)^2$

Ans. 2

4. If tanθ= $\frac{3}{4}$ then find the value of $\cos^2\theta-\sin^2\theta$

Ans. $\frac{7}{25}$

5. If secθ+tanθ=p, then find the value of secθ-tanθ

Ans. $\frac{1}{p}$

6. change $\sec^4\theta-\sec^2\theta$ in terms of tanθ.

Ans. $\tan^4\theta+\tan^2\theta$

7. If cotθ=1/√3 then find the value of $(1-\cos^2\theta)/(1+\cos^2\theta)$

Ans. $\frac{3}{5}$

8. If $\cot\theta + \frac{1}{\cot\theta}$ =2 then find the value of $\cot^2\theta + \frac{1}{\cot^2\theta}$.

Ans. 2

9. Ifsinθ=a/b,then find the value ofsecθ+tanθ

Ans. $\sqrt{\frac{b+a}{b-a}}$

10. Ifcos x=cos60° cos30°+sin60° sin30°,then find the value of x

Ans. 30°

11. If 0°≤ x ≤90° and $2\sin^2 x=1/2$,then find the value of x

Ans. 30°

12. Find the value of $\text{cosec}^2 30°-\sin^2 45°-\sec^2 60°$

Ans. -2

13. Simplify (secθ+tanθ)(1-sinθ)

Ans. $\cos\vartheta$

Level - 2

1. If secα=5/4 then evaluate tanα/(1+tan²α). Ans: $\frac{12}{25}$

2. If A+B =90°,then prove that $\sqrt{\frac{tanA\ tanB+tanA\ cotB}{sinA\ secB} - \frac{sin^2B}{cos^2B}}$ = tanA

3. Prove that co/(1-sinA)+co/(1+sinA) =2secA.

4. Prove that $\sqrt{\frac{secA-1}{secA+1}}$. + $\sqrt{\frac{secA+1}{secA-1}}$ = 2cosecA

5. Prove that $(\sin\theta+\operatorname{cosec}\theta)^2 + (\cos\theta+\sec\theta)^2 = 7+\tan^2\theta+\cot^2\theta$.

6. Evalute $\frac{11\sin 70°}{7\cos 20°}$-$\frac{4\cos 53°\ \operatorname{cosec} 37°}{7\tan 15°\tan 35°\tan 55°\tan 75°}$ Ans:1

7. Prove that $\sqrt{\frac{cosecA-1}{cosecA+1}}$ + $\sqrt{\frac{cosecA+1}{cosecA-1}}$ = 2secA.

8. In a right angle triangle ABC,right angled at B, if tanA=1, then verify that 2sinA co = 1.

9. If tan (A-B)=√3, and sinA =1, then find A and B. Ans:90°& 30°

10. If θ is an acute angle and sinθ=cosθ, find the value of $3\tan^2\theta + 2\sin^2\theta - 1$. Ans:3

11. If $\frac{x}{a}$cosθ + $\frac{y}{b}$ sin θ = 1 and $\frac{x}{a}$sinθ – $\frac{y}{b}$ cosθ = 1,prove that $x^2/a^2 + y^2/b^2 = 2$.

Level - 3

1. Evaluate the following :- $\sin^2 25° + \sin^2 65° + \sqrt{3}(\tan 5°\ \tan 15°\ \tan 30°\ \tan 75°\ \tan 85°)$.

   Ans:2

2. If $\frac{\cos\alpha}{\cos\beta}$ = m, and $\frac{\cos\alpha}{\sin\beta}$= n , show that $(m^2+n^2)\cos^2\beta = n^2$.

3. Prove that $\tan^2\theta + \cot^2\theta + 2 = \operatorname{cosec}^2\theta\ \sec^2\theta$.

4. Prove that $(\tan A - \tan B)^2 + (1+\tan A\tan B)^2 = \sec^2 A\sec^2 B$.

5. If (cosθ-sinθ) = √2 sinθ, then show that cosθ + sinθ = √2 cos θ.

6. Prove that $(\sin\theta+\sec\theta)^2 + (\cos\theta + \operatorname{cosec}\theta)^2 = (1+\sec\theta\operatorname{cosec}\theta)^2$.

7. Prove that $\sin\theta/(1-\cos\theta) + \tan\theta/(1+\cos\theta) = \sec\theta\operatorname{cosec}\theta + \cot\theta$.

8. Prove that $(\sin\theta - \operatorname{cosec}\theta)(\cos\theta - \sec\theta) = \frac{1}{\tan\theta+\cot\theta}$.

9. Ifcotθ $\frac{15}{8}$, evaluate $(2 + 2\sin\theta)(1 - \sin\theta)/(1+\cos\theta)(2 - 2\sin\theta)$.

Level - 4

1. Prove that $(\sec\theta+\tan\theta - 1)/(\tan\theta - \sec\theta+1) = \cos\theta/(1 - \sin\theta)$.

2. If x = r sinAcosC, y=rsinAsinC, z=rco, Prove that $r^2=x^2+y^2+z^2$.

3. Prove that $\frac{1}{\sec\theta-\tan\theta} - \frac{1}{\cos\theta} = \frac{1}{\cos\theta} - \frac{1}{\sec\theta+\tan\theta}$.

4. If $x= a\sin\theta$, $y= b\tan\theta$, prove that $\frac{a^2}{x^2} - \frac{b^2}{y^2} = 1$.

5. Prove that: $\frac{\cos\theta}{1-\tan\theta} - \frac{\sin^2\theta}{\sin\theta-\cos\theta} = \sin\theta + \cos\theta$

6. Evaluate $\frac{\sin^2\theta+\sin^2(90°-\theta)}{3(\sec^2 61°-\cot^2 29°)} - \frac{3\cot^2 30°\sin^2 54°\sec^2 36°}{2(\operatorname{cosec}^2 65°-\tan^2 25°)}$. Ans. $-\frac{25}{6}$

7. Prove that $\frac{1+\cos A+\sin A}{1+\cos A - \sin A} = \frac{1+\sin A}{\cos A}$.

8. Prove that $\frac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta} = \tan\theta$.

9. Prove that $\frac{1+\tan^2 A}{1+\cot^2 A} = \left(\frac{1-\tan A}{1 - \cot A}\right)^2 = \tan^2 A$.

10. Ifcotθ= $\frac{7}{8}$, evaluate (i)$\cos^2\theta + \sin^2\theta$ (ii)$\cos^2\theta - \sin^2\theta$. Ans. 1, $-\frac{15}{113}$

Self Evaluation

1. If $a\cos\theta + b\sin\theta = c$, then prove that $a\sin\theta - b\cos\theta = \mp\sqrt{\mathbf{a}^2 + \mathbf{b}^2 - \mathbf{c}^2}$.

2. If A,B,C are interior angles of **triangle ABC, show that cosec²**$\left(\frac{B+C}{2}\right)$ **- tan²** $\frac{A}{2}$ = 1.

3. If$\sin\theta + \sin^2\theta + \sin^3\boldsymbol{\theta} = 1$, prove that $\cos^6\theta - 4\cos^4\theta + 8\cos^2\boldsymbol{\theta} = 4$.

4. IftanA = ntanB, sinA = msinB, prove that $\cos^2 A = (m^2 - 1)/(n^2-1)$.

5. Evaluate [secθcosec(90°- θ) – tanθ cot(90°θ) + $\sin^2 55°\ \sin^2 35°$] **/**

(tan10°tan20°tan60°tan70°tan80°). Ans: $\frac{2}{\sqrt{3}}$

6. If secθ + tanθ=p, prove that sinθ = **(**p^2-1**)**/**(**p^2+1**)**.

STATISTICS
KEY POINTS

The three measures of central tendency are :

i. Mean
ii. Median
iii. Mode

- Mean Of grouped frequency distribution can be calculated by the following methods.

(i) <u>Direct Method</u>

Mean = $\bar{X} = \frac{\sum_{i=1}^{n} fixi}{\sum_{i=1}^{n} fi}$

Where X_i is the class mark of the i^{th} class interval and f_i frequency of that class

(ii) <u>Assumed Mean method or Shortcut method</u>

Mean = $\bar{X}$ = a +$\frac{\sum_{i=1}^{n} fidi}{\sum_{i=1}^{n} fi}$

Where a = assumed mean

And $d_i = X_i - a$

(iii) **<u>Step deviation method</u>**.

Mean = $\bar{X}$ = a +$\frac{\sum_{i=1}^{n} fiui}{\sum_{i=1}^{n} fi}\ x\ h$

Where a = assumed mean

h = class size

And $u_i = (X_i - a)/h$

- Median of a grouped frequency distribution can be calculated by

Median = l + $\left(\frac{\frac{n}{2}-cf}{f}\right) x\ h$

Where

l = lower limit of median class

n = number of observations

cf = cumulative frequency of class preceding the median class

f = frequency of median class

h = class size of the median class.

- Mode of grouped data can be calculated by the following formula.

Mode = l + $\left(\frac{f1-fo}{2f1-fo-f2}\right) x\ h$

Where

l = lower limit of modal class

h = size of class interval

f1 = Frequency of the modal class

fo = frequency of class preceding the modal class

f2= frequency of class succeeding the modal class

- Empirical relationship between the three measures of central tendency.
  3 Median = Mode + 2 Mean
  Or, Mode = 3 Median – 2 Mean
- Ogive
  Ogive is the graphical representation of the cumulative frequency distribution. It is of two types:
  (i) Less than type ogive.
  (ii) More than type ogive

- Median by graphical method
  The x-coordinated of the point of intersection of ‘less than ogive’ and ‘more than ogive’ gives the median.

LEVEL – 1

<table>
<tr><th>Slno</th><th>Question</th><th>Ans</th></tr>
<tr><td>1</td><td>What is the mean of 1st ten prime numbers ?</td><td>12.9</td></tr>
<tr><td>2</td><td>What measure of central tendency is represented by the abscis of the point where less than ogive and more than ogive intersect?</td><td>Median</td></tr>
<tr><td>3</td><td>If the mode of a data is 45 and mean is 27, then median is ___________.</td><td>33</td></tr>
<tr><td>4</td><td>Find the mode of the following
<table>
<tr><td>X_i</td><td>35</td><td>38</td><td>40</td><td>42</td><td>44</td><td></td></tr>
<tr><td>f_i</td><td>5</td><td>9</td><td>10</td><td>7</td><td>2</td><td></td></tr>
</table></td><td>Mode =40</td></tr>
<tr><td>5</td><td>Write the median class of the following distribution.
<table>
<tr><td>Class</td><td>0-10</td><td>10-20</td><td>20-30</td><td>30-40</td><td>40-50</td><td>50-60</td><td>60-70</td></tr>
<tr><td>Frequency</td><td>4</td><td>4</td><td>8</td><td>10</td><td>12</td><td>8</td><td>4</td></tr>
</table></td><td>30-40</td></tr>
</table>

LEVEL – 2

<table>
<tr><th>Slno</th><th>Question</th><th>Ans</th></tr>
<tr><td>1</td><td>Calculate the mean of the following distribution
<table>
<tr><td>Class interval</td><td>50-60</td><td>60-70</td><td>70-80</td><td>80-90</td><td>90-100</td></tr>
<tr><td>Frequency</td><td>8</td><td>6</td><td>12</td><td>11</td><td>13</td></tr>
</table></td><td>78</td></tr>
<tr><td>2</td><td>Find the mode of the following frequency distribution
<table>
<tr><td>Marks</td><td>10-20</td><td>20-30</td><td>30-40</td><td>40-50</td><td>50-60</td></tr>
<tr><td>No. of students</td><td>12</td><td>35</td><td>45</td><td>25</td><td>13</td></tr>
</table></td><td>33.33</td></tr>
<tr><td>3</td><td>Find the median of the following distribution
<table>
<tr><td>Class interval</td><td>0-10</td><td>10-20</td><td>20-30</td><td>30-40</td><td>40-50</td><td>50-60</td></tr>
<tr><td>Frequency</td><td>5</td><td>8</td><td>20</td><td>15</td><td>7</td><td>5</td></tr>
</table></td><td>28.5</td></tr>
<tr><td>4</td><td>A class teacher has the following absentee record of 40 students of a class for the whole term.
<table>
<tr><td>No. of days</td><td>0-6</td><td>6-10</td><td>10-14</td><td>14-20</td><td>20-28</td><td>28-38</td><td>38-40</td></tr>
<tr><td>No. of students</td><td>11</td><td>10</td><td>7</td><td>4</td><td>4</td><td>3</td><td>1</td></tr>
</table>
Write the above distribution as less than type cumulative frequency distribution.</td><td></td></tr>
<tr><td></td><td>Answer :
<table>
<tr><td>No. of days</td><td>Less Than 6</td><td>Less Than 10</td><td>Less Than 14</td><td>Less Than 20</td><td>Less Than 28</td><td>Less Than 38</td><td>Less Than 40</td></tr>
<tr><td>No. of students</td><td>11</td><td>21</td><td>28</td><td>32</td><td>36</td><td>39</td><td>40</td></tr>
</table></td><td></td></tr>
</table>

LEVEL – 3

Slno | Question | Ans

1 If the mean distribution is 25

Class	0-10	10-20	20-30	30-40	40-50
Frequency	5	18	15	P	6

Then find p.

Ans: P=16

2 Find the mean of the following frequency distribution using step deviation method

Class	0-10	10-20	20-30	30-40	40-50
Frequency	7	12	13	10	8

Ans: 25

3 Find the value of p if the median of the following frequency distribution is 50

Class	20-30	30-40	40-50	50-60	60-70	70-80	80-90
Frequency	25	15	P	6	24	12	8

Ans: P=10

4 Find the median of the following data

Marks	Less Than 10	Less Than 30	Less Than 50	Less Than 70	Less Than 90	Less Than 110	Less Than 130	Less than 150
Frequency	0	10	25	43	65	87	96	100

Ans: 76.36

LEVEL – 4

Slno | Question | Ans

1 The mean of the following frequency distribution is 57.6 and the sum of the observations is 50. Find the missing frequencies f_1 and f_2.

Class	0-20	20-40	40-60	60-80	80-100	100-120	Total
Frequency	7	f_1	12	f_2	8	5	50

Ans: f_1 =8 and f_2 =10

2 The following distribution give the daily income of 65 workers of a ctory

Daily income (in Rs)	100-120	120-140	140-160	160-180	180-200
No. of workers	14	16	10	16	9

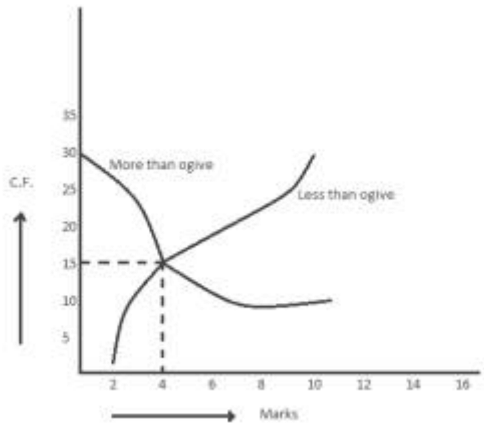
Convert the above to a more than type cumulative frequency distribution and draw its ogive.

3 Draw a less than type and more than type ogives for the following distribution on the me graph. Also find the median from the graph.

Marks	30-39	40-49	50-59	60-69	70-79	80-89	90-99
No. of students	14	6	10	20	30	8	12

SELF – EVALUATION

1. What is the value of the median of the data using the graph in figure of less than ogive and more than ogive?



2. If mean =60 and median =50, then find mode using empirical relationship.
3. Find the value of p, if the mean of the following distribution is 18.

Variate (x_i)	13	15	17	19	20+p	23
Frequency (f_i)	8	2	3	4	5p	6

4. Find the mean, mode and median for the following data.

Classes	0-10	10-20	20-30	30-40	40-50	50-60	60-70
frequency	5	8	15	20	14	8	5

5. The median of the following data is 52.5. find the value of x and y, if the total frequency is 100.

Class Interval	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80	80-90	90-100
frequency	2	5	X	12	17	20	Y	9	7	4

6. Draw ‘less than ogive’ and ‘more than ogive’ for the following distribution and hence find its median.

Classes	20-30	30-40	40-50	50-60	60-70	70-80	80-90
frequency	10	8	12	24	6	25	15

7. Find the mean marks for the following data.

Marks	Below 10	Below 20	Below 30	Below 40	Below 50	Below 60	Below 70	Below 80	Below 90	Below 100
No. of students	5	9	17	29	45	60	70	78	83	85

8. The following table shows age distribution of persons in a particular region. Calculate the median age.

Age in	Below	Below	Below	Below	Below	Below	Below	Below

years	10	20	30	40	50	60	70	80
No. of persons	200	500	900	1200	1400	1500	1550	1560

9. If the median of the following data is 32.5. Find the value of x and y.

Class Interval	0-10	10-20	20-30	30-40	40-50	50-60	60-70	Total
frequency	x	5	9	12	y	3	2	40

CLASS – X

MATHEMATICS

Time : 3 hours Maximum Marks :

General Instructions:

1. *All questions are compulsory.*
2. *The question paper consists of 34 questions divided into four sections A, B, C and D. Section A comprises of 8 questions of 1 mark each, Section B comprises of 6 questions of 2 marks each. Section C comprises of 10 questions of 3 marks each and Section D comprises of 10 questions of 4 marks each.*
3. *Question numbers 1 to 8 in Section A are multiple choice questions where you are to select on correct option out of the given four.*
4. *There is no overall choice. However, internal choice has been provided in 1 question of two marks, 3 questions of three marks each and 2 questions of four marks each. You have to attempt only one of the alternatives in all such questions.*
5. *Use of calculator is not permitted.*
6. *An additional 15 minutes time has been allotted to read this question paper only.*

SECTION A

[1] If the system of liner equations x - ky =2 and 3x + 2y =-5 has a unique solution, then the value of k is:-

[a] k = $\frac{2}{3}$ [b] k ≠- $\frac{2}{3}$ [c]k =$\frac{3}{2}$ [d] k ≠-$\frac{3}{2}$

[2] If tanθ = $\frac{3}{4}$ then the value of $\frac{1-cos\theta}{1+cos\theta}$is :-

[a]$-\frac{1}{9}$ [b]$\frac{2}{9}$ [c] 1 [d]$\frac{1}{9}$

[3] If sin3x = cos[x-26^0] and 3x is an actual angle, then the value of x is :-

[a] 29^0 [b]26^0 [c]29^0 [d]13^0

[4] If x=2^{3}x 3 x 5^2 , y=2^{2}x3^3 ,then HCF [x , y] is :-

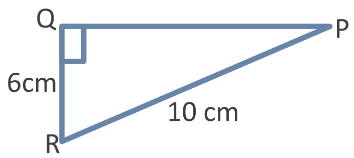
[a] 12 [b]108 [c]6 [d]36

[5] If a positive integer p is divided by 3, then the remainder can be:-

[a]1 or 3 [b]1,2 or3 [c]0,1 or 2 [d]2 or 3

[6] If the given figure, the value of tanP – cotRis:-

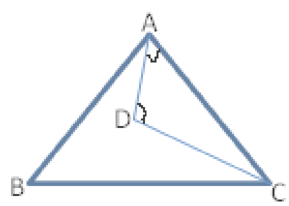
[a]1 [b]0 [c] -1 [d] 2



[7] Construction of a cumulative frequency table is useful in determining the :-

[a]Mean [b] Median [c] Mode [d]All the above

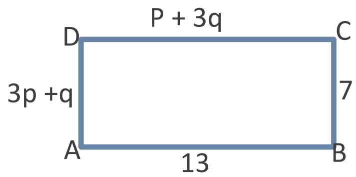
[8]In the given figure, if ∠A= ∠D = 90^0, AD=6cm , CD = 8cm and BC =26cm then ar(ΔABC) is :-



[a]240cm^2 [b]48cm^2 [c]120^2 [d]260cm^2

SECTION – B

[9]Find the value of p and q in the given figure, if ABCD is a rectangle



[10]If α and $\frac{1}{\alpha}$ are the zeroes of the polynomial $p(x) = 4x^2 - 2x + k - 4$, then find the value of k

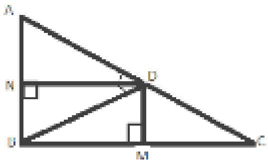
Or

Divide the polynomial p(x) =$5 - 3x + 3x^2 - x^3$ by g(x) = $x^2 - x + 1$ and find the quotient and remainder

[11] Without actually performing the long division, state whether $\frac{39}{343}$ will have a terminating or non-terminating, repeating decimal expansion

[12]Find the value of k , if $\frac{\cos 35^0}{\sin 55^0} + \frac{2\sin\theta}{\cos(90^0 - \theta)} = \frac{k}{2}$

[13] ABC is right angle triangle with $\angle ABC = 90^0$, BD⊥AC , DM ⊥BC , and DN⊥AB. prove that $DM^2 = DN\ X\ BC$.



[14] The following table gives production yield per hectare of wheat of 100 rms of village:-

Production (in kg/hec)	25-35	35-45	45-55	55-65	65-75	75-85
No.of rms	4	6	10	26	35	19

Write the above distribution to a more than type distribution.

SECTION - C

[15] Prove that $\frac{7\sqrt{7}}{4}$is irrational.

Or

Prove that $(16-5\sqrt{7})$is irrational.

[16] If one diagonal of a trapezium divides the other diagonal in the ratio 1:2 . Prove that one of the parallel sides is double the other?

[17] Prove that:$\frac{\sin\theta}{\cot\theta+\mathrm{cosec}\theta} = 2 + \frac{\sin\theta}{\cot\theta-\mathrm{cosec}\theta}$?

[18] The sum of the numerator and denominator, of a fraction is 8. If 3 is added to both the numerator and the denominator, the fraction become $\frac{3}{4}$. Find the fraction.

Or

Seven times a two digit number is equal to 4 times the number obtained by reversing the order of its digits. If the difference of digit is 3, find the number.

[19] If one zero of the polynomial p(x) = $3x^2-8x+2k+1$ is seven times of other, then find the zeores and the value of k.

[20] If $\sin\theta + \sin^2\theta + \sin^3\theta = 1$, prove that $\cos^6\theta - 4\cos^4\theta + 8\cos^2\theta = 4$.

[21]Find the mean of the following data, using step-deviation method:-

Class Interval	0-20	20-40	40-60	60-80	80-100	100-120
Frequency	7	8	12	10	8	5

Or

Class Interval	0-20	20-40	40-60	60-80	80-100
Frequency	17	28	32	p	19

If the mean of the above data is 50, then find the value of p?

[22] Prove that tanθ-cotθ=$\frac{2\,sin^2\theta-1}{sin\theta\ cos\theta}$

[23] InΔ ABC, if AD is the median, then show that AB^2+AC^2=2[$AD^2 + BD^2$].

[24] Find the median of the following data:

Class-interval	10-20	20-30	30-40	40-50	50-60	60-70	70-80
Frequency	12	30	34	65	46	25	18

SECTION-D

[25] Prove that, if a line is drawn parallel to one side of a triangle, to intersect the other two sides in distinct points the other two sides are divided into the me ratio.

Or

Prove that in a right triangle the square of the hypotenus is equal to the sum of the square of the other two sides?

[26] If x=a sinθ , y=b tanθ .prove that $\frac{a^2}{x^2}$ -$\frac{b^2}{y^2}$ =1.

[27] On dividing $3x^3+4x^2+5x-13$ by a polynomial g(x), the quotient and remainder are 3x+10 and 16x+43 respectively, Find the polynomial g(x).

[

28] The fraction become $\frac{9}{11}$,if 2 is added to both the numerator and the denominator. If 3 is added to both the numerator and the denominator, it becomes $\frac{5}{6}$. Find the fraction.

[29] The following table shows the ages of the patients admitted in a hospital during a year:

AGE(in years)	5-15	15-25	25-35	35-45	45-55	55-65
No. of patients	6	11	21	23	14	5

Find the mode and mean of the data given above.

[30] The perpendicular from A on the side BC of the ΔABC intersects BC at D such that DB=3CD Prove that $2AB^2 = 2AC^2 + BC^2$.

[31] Draw the graph of following eq^n:-
2x+3y = 12 and x-y =1
Shade the region between the two lines and x – axis. Also, determine the vertices of the triangle so formed.

[32]Prove that: -$\frac{cos\theta}{1-tan\theta}$ +$\frac{sin^2\theta}{sin\theta-cos\theta}$= $\sin\theta + cos\theta$

Or

Evaluate: -$\frac{sin^2\theta+sin^2(90^0-\theta)}{3(sec^2 61^0-cot^2 29^{0)}}$ --$\frac{3\ cot^2 30^0 sin^2 54^0 sec^2 36^0}{2(cosec^2\ 65^0-tan^2 25^0)}$.

[33] In a sports meet, the number of players in Football, Hockey and Athletics are 48,60,132, respectively. Find the minimum number of room required, if in each room the me number of player are to be seated and all of them being in the me sports ?

[34] The following distribution gives the daily income of 65 workers of a ctory :-

Daily income (in Rs)	100-120	120 – 140	140 -160	160- 180	180- 200
No. of worker	14	16	10	16	9

Convert the distribution above to a more than type cumulative frequency distribution and draw its ogive.

Sl.no marks

1. (b) 1

2. (d) 1

3. (a) 1

4. (a) 1

5. (c) 1

6. (b) 1

7. (b) 1

8. (c) 1

SECTION-B

9. Since opposite sides of the rectangle are equal 1

So. P+3q=13, 3p+q=7

Solving p=1, q=4 1

10. Since α and $\frac{1}{\alpha}$ are the zeros of the polynomial

P(x)= $4x^2$-2x+k-4

So, $\alpha \times \frac{1}{\alpha} = \frac{k-4}{4}$ 1

⇒ $1 = \frac{k-4}{4}$ 1

⇒k= 8

or

x^2-x+1)-x³+3x^2-3x+5(-x+2

-x³+x^2-x 1$\frac{1}{2}$

+ - +

2x^2-2x+5

2x^2-2x+2

- + -

3

---------------- 1/2

So, quotient = -x+2 , remainder = 3

11. Here, $\frac{39}{343} = \frac{3 \times 13}{7 \times 7 \times 7}$ $\frac{1}{2}$

Since denominator contains prime ctor 7 other than 2 or 5 $\frac{1}{2}$

So, $\frac{39}{343}$ will have a non-terminating repeating decimal expansion. 1

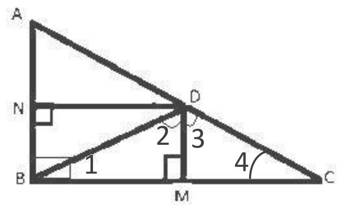
12. we have , $\frac{\cos 35°}{\sin 55°} + \frac{2sin\theta}{\cos(90° - \theta} = \frac{k}{2}$

$\frac{\cos 35°}{\cos(90° - 35°} + \frac{2sin\theta}{sin\theta} = \frac{k}{2}$ 1

$1 + 2 = \frac{k}{2}$ 1

K = 6

13. 1



∵ $DN \perp$ AB and $\angle B = 90^0$, DM ⊥ BC

So, DN ||BC and DM || AB

so, DNBM is a ||grm.

⇨ DN =BM

In ΔBDM and ΔDCM, 1

∠1=∠3, ∠2=∠4,

by AA-Similarity ΔBDM~ΔDCM

⇨ $\frac{DM}{CM} = \frac{BM}{DM}$ 1

⇨ DM^2=CM X BM

⇨ DM^2=CM X DN Proved.

14. 1 +1

More than type	Commutative frequency
more than 25	100
more than 35	96
more than 45	90
more than 55	80
more than 65	54
more than 75	19

SECTION-C

15\. Let us assume that, on contrary $\frac{7\sqrt{7}}{4}$ is rational

$\therefore \frac{7\sqrt{7}}{4} = \frac{a}{b}, where\ a, b\ are\ integers\ with\ b \neq 0$ 1

⇨ $\sqrt{7} = \frac{4a}{7b}$ 1

$\because \frac{4a}{7b}$ is a rational number. So, $\sqrt{7}$is rational .

but this contradicts the ct that $\sqrt{7}\ \overline{\text{is}}$ irrational.

so our assumption is wrong. 1

$\therefore \sqrt{7}$is irrational.

OR

on contrary, Let 16-5$\sqrt{7}$ is rational 1

so, 16-5$\sqrt{7}$=$\frac{a}{b}$, where a, b are integers with b$\neq 0$

⇨ $\sqrt{7}$= $\frac{16b-a}{5b} = \frac{integer}{integer}$ 1

⇨ $\sqrt{7} = rational$ 1

But, this contradicts the ct that $\sqrt{7}\ \overline{\text{is}}$ irrational.

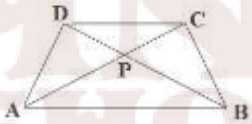
so, 16-5$\sqrt{7}$ is irrational.

16\. 1

Given that

to prove

fig.



$\frac{CP}{AP} = \frac{1}{2}$ 1

In Δ, ABP and ΔCDP,

$\angle$ABP=$\angle$CDP (alt. $\angle$s)

$\angle$BAP=$\angle$DCP (alt. $\angle$s)

$\therefore \Delta$ABP ~ ΔCDP (BY AA-Similarity)

⇨ $\frac{AB}{DC} = \frac{AP}{CP} = \frac{2}{1}$ 1

⇨ AB=2DC Proved.

17\. Given: $\frac{Sin\theta}{cot\theta+cosec\theta} = 2 + \frac{Sin\theta}{cot\theta-cosec\theta}$ ½

⇨ $\frac{Sin\theta}{cot\theta+cosec\theta} - \frac{Sin\theta}{cot\theta-cosec\theta} = 2$

L.H.S.=Sin$\theta \left[\frac{1}{cot\theta+cosec\theta} - \frac{1}{cot\theta-cosec\theta}\right]$ ½

=sin$\theta \left[\frac{-2cosec\theta}{cot^2\theta-cosec^2\theta}\right]$ 1

=$\frac{-2sin\theta \cdot \frac{1}{sin\theta}}{-(cosec^2\theta-cot^2\theta)}$ = 2 Proved. 1

18\. $Let\ the\ numerator\ be\ x\ and\ denominator\ be\ y, then\ fraction = \frac{x}{y}$ ½

As per question, 1

$x + y = 8$

$\frac{x+3}{y+3} = \frac{3}{4}$

solving, $x = 3,\ y = 5$ 1

fraction=$\frac{x}{y} = \frac{3}{5}$ ½

OR

Let the unit digit be x $and\ tens\ digit\ be\ y$ ½
$then\ no. = 10y + x$
As per question 1

$$7(10y + x) = 4(10x + y)$$

and $x - y = 3$
solving $x = 6, \quad y = 3$ 1
so No. = 36 ½

19. Let zeroes of the given polynomial $p(x) be\ \alpha\ and\ \beta$ ½
then, as per question

$$\beta = 7\alpha$$

sum of zeroes $\alpha + \beta = \frac{-b}{a}$ 1

⇨ $\alpha + 7\alpha = \frac{8}{3}$

⇨ $\alpha = \frac{1}{3}$

also, Product of zeroes $\alpha\beta = \frac{c}{a}$ ½

⇨ $7\alpha\ X\ \alpha = \frac{2k+1}{3}$

solving, $k = \frac{2}{3}$ 1

zeroes are $\frac{1}{3}, \frac{7}{3}$

20. We have, ½

$\sin\theta + \sin^2\theta + \sin^3\theta = 1$

$\sin\theta(1+\sin^2\theta) = 1-\sin^2\theta$

⇨ $\sin^2\theta(1+\sin^2\theta) = \cos^2\theta$ squaring both sides ½

⇨ $(1-\cos^2\theta)(1+1-\cos^2\theta)^2 = \cos^4\theta$ ½

solving, $1\frac{1}{2}$

$\cos^6\theta - 4\cos^4\theta + 8\cos^2\theta = 4$

21. $1\frac{1}{2}$

Class –Interval	Mid value (x_i)	frequency (f_i)	$u_i = \frac{x_i - d}{h}$	$f_i u_i$
0-20	10	7	-2	-14
20-40	30	8	-1	-8
40-60	A=50	12	0	0
60-80	70	10	1	10
80-100	90	8	2	16
100-120	110	5	3	15
		$\sum f_i = 50$		$\sum f_i u_i = 19$

Here, A=50, h=20, $\sum f_i = 50, \sum f_i u_i = 19$ ½

Mean($\bar{x}$)= A + $\left[\frac{\sum f_i u_i}{\sum f_i}\right] X\ h$

$Mean(\bar{x})$= $50 + \frac{19}{50}\ X\ 20$ 1

= 57.6

OR

$1\frac{1}{2}$

Class	frequency (f_i)	class – mark(x_i)	$f_i x_i$
0-20	17	10	170
20-40	28	30	840
40-60	32	50	1600
60-80	p	70	70p
80-100	19	90	1710
	$\sum f_i = 96 + p$		$\sum f_i x_i = 4320 + 70p$

½

Mean($\bar{x}$)= $\left[\frac{\sum f_i x_i}{\sum f_i}\right]$

Given $\bar{x} = 50$

⇨ $50 = \frac{4320+70p}{96+p}$ 1

$Solving\ p = 24$

22. L.H.S. $= \tan\theta - cot\theta$

$= \frac{sin\theta}{cos\theta} - \frac{cos\theta}{sin\theta}$ ½

$= \frac{sin^2\theta - cos^2\theta}{sin\theta cos\theta}$ ½

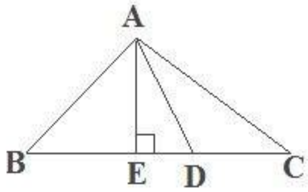
$= \frac{sin^2\theta - 1 + sin^2\theta}{sin\theta cos\theta}$ 1

$= \frac{2sin^2\theta - 1}{sin\theta cos\theta}$ 1

23. construction:-Draw AE ⊥BC ½

In ΔADE, $AD^2=AE^2+ED^2$ (Pythagoras theorem)

⇨ $AE^2=AD^2-DE^2$ (I)



In ΔABE, $AB^2=AE^2+BE^2$ (∠B=90^0, Pythagoras theorem) 1

⇨ $AB^2=AD^2-DE^2+(BD-DE)^2$ Using (I)

⇨ $AB^2=AD^2+BD^2-2BD.DE$ -----------(II)

In ΔACE, $AC^2=AE^2+EC^2$ 1

⇨ $AC^2 = AD^2 - DE^2+(CD+DE)^2$

⇨ $AC^2 = AD^2+CD^2+2CD.DE$

⇨ $AC^2 = AD^2+BD^2+2BD.DE$ (As BD=CD) ------------(III)

Adding (II) and (III) ½

$AB^2+AC^2=2[AD^2+BD^2]$ Proved

24. 1

Class – Interval	Frequency (f_i)	Cumulative frequency
10-20	12	12
20-30	30	42
30-40	34	76
40-50	65	141
50-60	46	187
60-70	25	212
70-80	18	230
	N=$\sum f_i = 230$	

$\frac{N}{2} = \frac{230}{2} = 115$

Median class is 40-50 ½

$l = 40, \quad cf = 76, \quad f = 65, \quad h = 10$

$\therefore Median = l + \frac{\frac{N}{2} - cf}{f}\ X\ h$ ½ 1

= 46

25. correct given that, to prove, construction, fig. 2

correct proof 2

OR

Correct fig., given that, to prove, construction 2

correct proof 2

26. Given, $x = asin\theta, \quad y = btan\theta$ 1

⇨ $\frac{a}{x} = \frac{1}{sin\theta} \quad and \quad \frac{b}{y} = \frac{1}{tan\theta}$

⇨ $\frac{a}{x} = Cosec\theta\ and \quad \frac{b}{y} = cot\theta$ 1

LHS=$\frac{a^2}{x^2} - \frac{b^2}{y^2} = cosec^2\theta - cot^2\theta = 1 = RHS$ $Proved.$ 1+1

27. Here, $Dividend\ p(x) = 3x^3 + 4x^2 + 5x - 13$ ½

$Quotient\ q(x) = 3x + 10$

$Remainder\ r(x) = 16x - 43$

By division algorithm ½

$$p(x) = g(x)q(x) + r(x)$$

$$g(x) = \frac{p(x) - r(x)}{q(x)}$$ 1

$$= \frac{(3x^3+4x^2+5x-13)-(16x-43)}{3x+10}$$

$$= \frac{3x^3+4x^2-11x+30}{3x+10}$$

Correct division 2

$\therefore g(x) = x^2 - 2x + 3$

28. Let the fraction be $\frac{x}{y}$ ½

As per question

$\frac{x+2}{y+2} = \frac{9}{11}$ 1

$\Rightarrow$ 11x – 9y + 4 = 0

$\frac{x+3}{y+3} = \frac{5}{6}$ 1

$\Rightarrow$6x – 5y + 3 = 0

Solving above, x = 7, y = 9 1

$\therefore$ Fraction = $\frac{7}{9}$ ½

29. 1

Age (in years)	No. Of Patients (f_i)	$Class\ Mark(x_i)$	$f_i x_i$
5-15	6	10	60
15-25	11	20	220
25-35	21	30	630
35-45	23	40	920
45-55	14	50	700
55-65	5	60	300
	$\sum f_i = 80$		$\sum f_i x_i = 2830$

The modal class is 35-45 1/2

Here l= 35, f_1 = 23, f_2 = 14, f_0 = 21, h = 10

Mode = l + $\left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) * h$ $1\frac{1}{2}$

= 36.8

Mean($\bar{x}$) = $\frac{\sum f_i x_i}{\sum f_i}$ 1

= 35.68

30. Given that, figure, to prove. ½

Proof:- ½

$\because$DB = 3 CD

$\therefore$DB = ¾ BC

and CD = ¼ BC

In right ΔADB, 1

$AB^2 = AD^2 + DB^2$

In right ΔACB,

$AC^2 = AD^2 + CD^2$

$\therefore$ $AB^2 - Ac^2 = DB^2 - CD^2$ 1

$= \left(\frac{3}{4}BC\right)^2 - \left(\frac{1}{4}BC\right)^2 = \frac{\cancel{8}\ 1}{\cancel{16}\ 2}BC^2 = \frac{1}{2}BC^2$

⇨ $AB^2 - AC^2 = \frac{1}{2}BC^2$ 1

⇨ $2AB^2 = 2AC^2 + BC^2$ Proved.

31. Given Equations are 1

$2x + 3y = 12 \ -----(i)$

$x - y = 1 ------(ii)$

$from\ (i)\ y = \frac{12-2x}{3}$

x	0	3	6
y	4	2	0

$From\ (ii)\ \ y = x - 1$ 2

Graph for above equations

x	0	1	3
y	-1	0	2

Vertices of triangle are (1,0), (6,0), (3,2) 1

32. LHS $= \frac{cos\theta}{1-tan\theta} + \frac{sin^2\theta}{sin\theta-cos\theta}$ $1\frac{1}{2}$

$= \frac{cos^2\theta}{cos\theta-sin\theta} + \frac{sin^2\theta}{sin\theta-cos\theta}$

$= \frac{cos^2\theta-sin^2\theta}{cos\theta-sin\theta}$ 1

$= \frac{(cos\theta+sin\theta)\ (cos\theta-sin\theta)}{cos\theta-sin\theta}$ 1

$= cos\theta + sin\theta$ 1/2

OR

$\frac{sin^2\theta+sin^2(90^0-\theta)}{3(sec^2 61^0-cot^2 29^0)} - \frac{3cot^2 30^0 sin^2 54^0 sec^2 36^0}{2(cosec^2 65^0-tan^2 25^0)}$ 2

$= \frac{sin^2\theta+cos^2\theta}{3(sec^2 61^0-tan^2 61)} - \frac{3\ X\ (\sqrt{3})^2 sin^2 54^0 cosec^2 54^0}{2(cosec^2 65^0-cot^2 65^0)}$

$= \frac{1}{3\ X\ 1} - \frac{3\ X\ 3\ \ X\ 1}{2\ X\ 1}$ 1

$= \frac{1}{3} - \frac{9}{2} = -\frac{25}{6}$ 1

33. Prime ctorization of 48=$2^4\ X$ 3 1

Prime ctorization of 60=$2^2\ X$ 3 X 5

Prime ctorization of 132=$2^2\ X$ 3 X 11

HCF of (48, 60, 132)=2 X 2 X3 = 12 1

$\therefore In\ each\ room\ 12\ players\ of\ same\ sports\ can\ be\ accomodated.$ 1

$\therefore number\ of\ roms\ required = \frac{Total\ number\ of\ players}{Number\ of\ players\ in\ a\ room}$

⇨ $number\ of\ roms\ required = \frac{48+60+132}{12}$ 1

= 20

34. correct table 2

correct graph 2

DETAILS OF THE CONCEPTS TO BE MASTERED BY EVERY CHILD OF CLASS X WITH EXCERCISES AND EXAMPLES OF NCERT TEXT BOOK

SYMBOLS USED
* : Important Questions, **: Very important questions, ***: Very, Very Important questions

01	Quadratic Equation	Standard form of quadratic equation	*	NCERT Text book Q.1.2, Ex 4.1
		Solution of quadratic equation by ctorization	***	Example 3,4,5, Q.1, 5 Ex. 4.2
		Solution of quadratic equation by completing the square	**	Example 8,9 Q.1 Ex. 4.3
		Solution of quadratic equation by quadratic formula	***	Example. 10,11,13,14,15 , Q2,3(ii) Ex.4.3
		Nature of roots	***	Example 16 Q.1.2, Ex. 4.4
02	Arithmetic progression	General form of an A.P.	*	Exp-1,2, Ex. 5.1 Q.s2(a), 3(a),4(v)
		nth term of an A.P.	***	Exp. 3,7,8 Ex. 5.2 Q.4,7,11,16,17,18
		Sum of first n terms of an A.P.	** * ** ***	Exp.11,13,15 Ex. 5.3, Q.No.1(i, ii) Q3(i,iii) Q.7,10,12,11,6, Ex5.4, Q-1
03	Coordinate geometry	Distance formula	**	Exercise 7.1, Q.No 1,2,3,4,7,8
		Section formula Mid point formula	** ***	Example No. 6,7,9 Exercise 7.2, Q.No. 1,2,4,5 Example 10. Ex.7.2, 6,8,9. Q.No.7
		Area of Triangle	** ***	Ex.12,14 Ex 7.3 QNo-12,4 Ex.7.4, Qno-2
04	Some application of Trigonometry	Heights and distances		Example-2,3,4 Ex 9.1 Q 2,5,10,12,13,14,15,16
05	Circles	Tangents to a circle		Q3(Ex10.1) Q 1,Q6,Q7(Ex 10.2),4
		Number of tangents from a point to a circle	***	Theorem 10.1,10.2 Eg 2.1 Q8,9,,10,12,13 (Ex 10.2)
06	Constructions	Division of line segment in the given ratio	*	Const 11.1 Ex 11.1 Qno 1
		Construction of triangle similar to given triangle as per given scale	***	Ex 11.1 Qno-2,4,5,7
		Construction of tangents to a circle	***	Ex 11.2 Qno 1,4
07	Area related to circles	Circumference of a circle	*	Example 1 Exercise 12.1 Q.No 1,2,4
		Area of a circle	*	Example 5,3

		Length of an arc of a circle	*	Exercise 12.2 Q No 5
		Area of sector of a circle	**	Example 2 Exercise 12.2 QNo 1.2
		Area of segment of a circle	**	Exercise 12.2 Qno 4,7,9,3
		Combination of figures	***	Ex 12.3 Example 4.5 1,4,6,7,9,12,15
08	Surce area and volumes	Surce area of a combination of solids	**	Example 1,2,3 Exercise 13.1 Q1,3,6,7,8
		Volume of combination of a solid	**	Example 6 Exercise 13.2 Q 1,2,5,6
		Conversion of solids from one shape to another	***	Example 8 & 10 Exercise 13.3 Q 1,2,6,4,5
		Frustum of a cone	***	Example 12& 14 Exercise 13.4 Q 1,3,4,5 Ex-13.5, Q. 5
09	Probability	Events	*	Ex 15.1 Q4,8,9
		Probability lies between 0 and1	**	Exp- 1,2,4,6,13
		Performing experiment	***	Ex 15 1,13,15,18,24

QUADRATIC EQUATIONS

KEY POINTS

1. The general form of a quadratic equation is $ax^2+bx+c=0$, a≠o. a, b and c are real numbers.
2. A real number x is id to be a root of the quadratic equation $ax^2+bx+c=0$ where a≠o if $ax^2+bx+c=0$. The zeroes of the quadratic equation polynomial $ax^2+bx+c=0$ and the roots of the corresponding quadratic equation $ax^2+bx+c=0$ are the me.
3. Discriminant:- The expression b^2-4ac is called discriminant of the equation $ax^2+bx+c=0$ and is usually denoted by D. Thus discriminant D= b^2-4ac.
4. Every quadratic equation has two roots which may be real , co incident or imaginary.
5. IF α and β are the roots of the equation $ax^2+bx+c=0$ then

   $\alpha = \frac{-b+\sqrt{b^2-4ac}}{2a}$ And $\beta = \frac{-b-\sqrt{b^2-4ac}}{2a}$

6. Sum of the roots , $\alpha + \beta$ = $-\frac{b}{a}$ and product of the roots, $\alpha\beta = \frac{c}{a}$
7. Forming quadratic equation, when the roots α and β are given.

   $x^2-(\alpha + \beta)x+ \alpha.\beta = 0$
8. Nature of roots of $ax^2+bx+c=0$
   i. If D>0, then roots are real and unequal.
   ii. D=0, then the equation has equal and real roots.
   iii. D<0, then the equation has no real roots

LEVEL-I

1. IF ½ is a root of the equation x^2+kx-5/4=0, then the value of K is [Ans(d)]
   (a) 2
   (b) -2
   (c) ¼
   (d) ½
2. IF D>0, then roots of a quadratic equation $ax^2+bx+c=0$ are
   (a) $\frac{-b\pm\sqrt{D}}{2a}$ (b) $\frac{-b+\sqrt{D}}{2a}$ (c) $\frac{-b-\sqrt{D}}{2a}$ (d) None of these [Ans(a)]
3. Discriminant of x^2 +5x+5=0 is
   (a)5/2 (b) -5 (c) 5 (d)-4 [Ans(c)]
4. The sum of roots of a quadratic equation x^2+4x-320=0 is [Ans(a)]
   (a)-4 (b)4 (c)1/4 (d)1/2
5. The product of roots of a quaradatic equation 2 x^2+7x-4=0 is [Ans(d)]
   (a)2/7 (b)-2/7 (c)-4/7 (d)-2
6. Values of K for which the equation 9 x^2+2kx-1=0 has real roots are: [Ans(b)]
   (a)k≥ $\pm$3 (b)k≥3 or K≤-3 (c)K≥-3 (d) k≤ $\pm$3

LEVEL-II

1. For what value of k, x=a is a solution of equation x^2-(a+b)x+k =0 ?

Ans. K=ab

2. Represent the situation in the form of quadratic equation:-
Rohan 's mother is 26 years older than him . the product of their ages (in years) 3 years from now will be 360. We would like to find Rohan's present age.

Ans . x^2+32x -273 = 0 where x(in years) is Rohan's present age

3. Find the roots of x^2 -3x-10 = 0

Ans . -2 ,5

4. Find two consecutive positive integers , sum of whose squares is 365.

Ans .13,14

5. Find the roots of Quadratic equation $4x^2$ +4$\sqrt{3}x$+3 = 0 by using the quadratic formula.

Ans . - $\sqrt{\frac{3}{2}}$, - $\sqrt{\frac{3}{2}}$

6. Find the discriminant of the Quadratic equation $2x^2$-4x+3 = 0 and hence find the nature of its roots .

Ans . D= -8<0 its no real roots.

LEVEL - 3

1. If $x = 2\ and\ x = 3$ are roots of the equation $3x^2 - 2kx + 2m = 0$ find the value of k and m.

Ans. $K = \frac{15}{2}, m = 9$

2. Solve the equation:
$$\frac{x}{x+1} + \frac{x+1}{x} = \frac{34}{15}, x \neq 0, x \neq -1$$

Ans. $x = \frac{3}{2}\ or\ x = \frac{-5}{2}$

3. Solve the equation $2x^2 - 5x + 3 = 0$ by the method of completing square.

Ans. $x = \frac{3}{2}\ or\ x = 1$

4. Using quadratic formula, solve the equation: $p^2x^2 + (p^2 - q^2)x - q^2 = 0$.

Ans. $x = -1, or\ x = \frac{q^2}{p^2}$

5. The sum of two numbers is 15, if the sum of their reciprocals is $\frac{3}{10}, find\ the\ numbers$.

Ans. 10 and 5

[LEVEL - 4]

1. In a class test , the sum of sheli's marks in maths and English is 30. Had she got 2 marks more in maths and 3 marks less in English, the product of their marks would have been 210. Find her marks in the two subjects.

Ans. Marks in maths = 12 , marks in English =18 or ,marks in maths = 13 , marks in English = 17

2. Two water taps together can fill a tank in $9\frac{3}{8}$ hours. The tap of larger diameter takes 10 hours less than the smaller one to fill the tank separately. Find the time in which each tap can separately fill the tank.

Ans . 15 hours , 25 hours.

3. Find the roots of equation $\frac{1}{x+4}$ - $\frac{1}{x-7}$ = $\frac{11}{13}$, x≠-4 , 7

Ans . 1 , 2

4. Solve the following equation for 'x' $9x^2$ - 9(a+b)x + ($2a^2$+5ab+$2b^2$) = 0

Ans .$\frac{2a+b}{3}$, $\frac{a+2b}{3}$

5. If the roots of the equation (a-b) x^2+(b-c)x + (c-a) = 0 are equal , prove that 2a =b+c.

Self Evaluation

1. Find the value of p so that the equation $3x^2 - 5x + 2p = 0$ has equal roots. Also find the roots.
2. The sum of two numbers is 15. If the sum of their reciprocals is $\frac{3}{10}, find\ the\ two\ numbers$.
3. Find a and b such that x+1 and x+2 are ctors of the polynomials $x^3 + ax^2 - bx + 10$.
4. Find the quadratic equation whose roots are 2 + $\sqrt{3}$ and 2 - $\sqrt{3}$
5. A person on tour has Rs. 360 for his daily expenses. If he exceeds his tour program me by four days, he must cut down his daily expenses by Rs 3 per day. Find the number of days of his tour program me.
6. Divide 29 into two parts so that the sum of squares of the parts is 425.
7. Solve for x: $9x^2 - 6ax + (a^2 - b^2) = 0$
8. If the equation $(1 + m^2)x^2 + \ 2mcx + c^2 - a^2 = 0\ has\ equal\ roots$, show that $c^2 = a^2(1 + m^2)$

ARITHMETIC PROGRESSION

(Key Points)

- Arithmetic progression (A.P.) :- An A.P. is a list of numbers in which each term is obtained by adding a fixed number to the preceding term except the first term.
- This fixed number is called the common difference of the A.P.
- If a is first term and d is common difference of an A.P. , then the A.P is a , a+d , a+2d , 2+3d …..
- The n^{th} term of an a.p is denoted by a_n and a_n = a+(n-1) d , where a = first term and d = common difference.
- n^{th} term from the end = l – (n-1) d , where l = last term.
- Three terms a-d , a , a+d are in A.P with common difference d.
- Four terms a-3d , a-d , a+d ,a+3d are in A.P with common diff. 2d .
- The sum of first n natural number is $\frac{n(n+1)}{2}$
- The sum of n terms of an A.P with first term a and common difference d is denoted by $s_n = \frac{n}{2}\{2a+(n-1)d\}$ also , $s_n = \frac{n}{2}(a+l)$ where , l = last term.
- $a_n = s_n - s_{n-1}$. Where $a_n = n^{th}$ term of an A.P
- D = $a_n - a_{n-1}$. Where d = common difference of an A.P.

[LEVEL -1]

1. Find n^{th} term of – 15 , -18 , -21 , ……….

Ans .-3 (n+4)

2. Find the common diff. of A.P 1 , -2 ,-5 ,-8 ,………

Ans . -3

3. Find the A.P whose first term is 4 and common difference is – 3

Ans . a.p = 4 , 1 -2, -5, -8…………

4. Find 5^{th} term from end of the AP : 17 , 14 ,11…………-40.

Ans . -28

5. If 2p, p+10 , 3p+2 are in AP then find p.

Ans . p= 6

6. If arithmetic mean between 3a and 2a-7 is a+4 , then find a.

Ans . a= 5

7. Find sum of all odd numbers between 0 & 50.

Ans . 625

8. If a = 5 , d = 3 and a_n = 50 , then find n.

Ans .n =16

9. For what value of n are the n^{th} term of two AP , 63 , 65 , 67 ,…… and 3 , 10 , 17 ,…….equal?

Ans . n = 13.

10. If sum of n terms of an AP is $2n^2$+5n , then find its n^{th} term.

Ans. 4n+3.

[LEVEL - 2]

1. Find n^{th} term of an AP is 7-4n. find its common difference.

Ans. -4.

2. Which term of an AP 5,2,-1,….will be -22 ?

Ans . 10^{th} term .

3. Write the next term of an AP $\sqrt{8}$, $\sqrt{18}$, $\sqrt{32}$,…….

Ans. $5\sqrt{2}$.

4. Determine 27^{th} term of an AP whose 9^{th} term is -10 and common difference is1 $\frac{1}{4}$

Ans. 927 = $\frac{25}{2}$.

5. Find the sum of series 103=+101+99+…..49.

Ans. 2128.

6. Which term of the AP 3,15,27,39,….will be 132 more than its 54^{th} term ?

Ans. 65^{th} term .

7. How many three digit numbers are divisible by 7 ?

Ans. 128.

8. Given a = 2 , d = 8 , s_n = 90 , find n and a_n.

Ans. N = 5 & a_n = 34

(LEVEL- 3)

1. Which term of the sequence -1, 3, 7, 11 …………. Is 95?

Ans. 25^{th} term

2. How many terms are there in the sequence 3, 6, 9, 12, ……111?

Ans. 37 terms

3. The first term of an AP is -7 and the common difference 5, find its 18^{th} term and the general term.

Ans. a_{18} =78n & a_n = 5n – 12

4. How many numbers of two digits are divisible by 3?

Ans. 30

5. If the n^{th} term of an AP is (2n+1), find the sum of first n terms of the AP.

Ans. S_n= n(n+2)

6. Find the sum of all natural numbers between 250 and 1000 which are exactly divisible by 3.

Ans. 156375.

Problems for self evaluation.

1. Show that the sequence defined by t_n=4_n+7 is an AP.
2. Find the number of terms for given AP :7,13 ,19,25,…..,205.
3. The 7^{th} term of an AP is 32 and it 13^{th} term is 62. Find AP.
4. Find the sum of all two digit odd positive nos.
5. Find the value of 'x' for AP. 1+6+11+16+….+X=148.
6. Find the 10^{th} term from the end of the AP 8,10,12,…126.
7. The sum of three numbers of AP is 3 and their product is -35.Find the numbers.
8. A man repays a loan of Rs3250 by paying Rs20 in the first month and then increase the payment by Rs15 every month .How long will it take him to clear the loan ?

9. The ratio of the sums of m and n terms of an AP is m^2 :n^2 .show that the ratio of the mth and nth terms is (2m-1) : (2n-1).

10. In an AP , the sum of first n terms is $\frac{3n^2}{2} + \frac{5n}{2}$, Find it 25th term.

CO-ORDINATE GEOMETRY

IMPORTANT CONCEPTS
TAKE A LOOK

1. **Distance Formula**:-

The distance between two points $A(x_1,y_1)$ and $B(x_2,y_2)$ is given by the formula.
$AB=\sqrt{(X_2-X_1)^2+(Y_2-Y_1)^2}$

COROLLARY:- The distance of the point P(x,y) from the origin 0(0,0) is give by
$OP=\sqrt{(X-0)^2+(Y-0)^2}$ ie $OP=\sqrt{X^2+Y^2}$

2. **Section Formula :-**

The co-ordinates of the point P(x,y) which divides the line segment joining $A(x_1,y_1)$ and $B(x_2,y_2)$ internally in the ratio m:n are given by .

$X=\frac{mx_2+nx_1}{m+n}$ $\qquad y=\frac{my_2+ny_1}{m+n}$

3. **Midpoint Formula:-**

If R is the mid-point, then $m_1=m_2$ and the coordinates of R are

$R\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

4. **Co-ordinates of the centroid of triangle:-**

The co-ordinates of the centroid of a triangle whose vertices are $P(x_1,y_1)$, $Q(x_2,y_2)$ and $R(x_3,y_3)$ are

$$\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$$

5. **Area of a Triangle:-**

The area of the triangle fromed a by the points $P(x_1,y_1)$ $Q(x_2,y_2)$ and $R(x_3,y_3)$ is the numerical value of the expression.

$$ar(\Delta PQR)=1/2\left[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\right]$$

LEVEL- 1

1. If the coordinates of the points P and Q are (4,-3) and (-1,7). Then find the abscis of a point R on the line segment PQ such that $\frac{PR}{PQ}=\frac{3}{5}$

Ans. 1

2. If P $(\frac{a}{3},4)$ is the midpoint of the line segment joining the points Q (-6 , 5) and R (-2 , 3) , then find the value of a .

Ans . -12

3.A line intersects y –axis and x-axis at the points P and Q respectively . If (2 ,-5) is the midpoint of PQ , then find the coordinates of P and Q respectively .

Ans. (0,-10) and (4,0)

4. If the distance between the points (4,p)&(1,0) is 5,then find the value of p

Ans.±4

5. If the point A(1,2), B(0,0) and C(a,b)are collinear, then find the relation between a and b.

Ans. 2a=b

6. Find the coordinate of the point on x-axis which is equidistant from (2,-5)and (-2,9).

Ans. (-7,0)

7. Find the coordinates of a point A, where AB is diameter of a circle whose centre is (2, -3) and B is (1, 4)

Ans. (3, -10)

8. Find the centroid of triangle whose vertices are (3, -7), (-8, 6) and (5, 10).

Ans. (0, 3)

LEVEL-2

1. Point P (5, -3) is one of the two points of trisection of the line segment joining the points A (7, -2) and B (1, -5) near to A. Find the coordinates of the other point of trisection.

   Ans. (3, -4)
2. Show that the point P (-4, 2) lies on the line segment joining the points A (-4 , 6) and B (-4, -6).
3. If A (-2, 4) ,B (0, 0) , C (4, 2) are the vertices of a ΔABC, then find the length of median through the vertex A.

   Ans. 5 units
4. Find the value of x for which the distance between the points P (4, -5) and $Q(12, x)$ Is 10 units .

   Ans. 1, -11
5. If the points A (4,3) and B (x,5) are on the circle with centre O(2,3) then find the value of x.

   Ans. 2
6. What is the distance between the point A (c, 0) and B (0, -c)?

   Ans. $\sqrt{2}$ c
7. For what value of p, are the points (-3, 9) , (2, p) and (4, -5) collinear?

   Ans. $p = -1$

LEVEL-3

1. Show that the points (3, 2) , (0, 5) , (-3,2) and (0, -1) are the vertices of a square.
2. Point P divides the line segment joining the points A(2,1) and B(5,-8) such that AP:AB=1:3.If P lies on the line 2x-y+k=0,then find the value of k.

   Ans. k = -8
3. Points P, Q , R, and S in that order are dividing a line segment joining A (2, 6) and B (7, -4) in five equal parts. Find the coordinates of point P and R ?

   Ans. P (3, 4) , R (5, 0)
4. Find a relation between x and y if the points (2, 1) , (x, y) and (7, 5) are collinear.

   Ans. 4x - 5y + 3 = 0
5. If A (-4, -2) , B (-3, -5) , C (3, -2) and D (2, 3) are the vertices of a quadrilateral, then find the area of the quadrilateral.

   Ans. 28 sq. units
6. Find the values of x for which the distance between the points P(2, -3) and $Q(x, 5)$ is 10 units

   Ans. $x = 8 \text{ or } x = -4$
7. Find the point on y- axis which is equidistant from the points (5, -2) and (-3, 2)

   Ans. (0, -2)

LEVEL-4

1. A (6, 1), B (8, 2), C (9, 4) are the three vertices of a parallelogram ABCD. If E is the midpoint of DC, then find the area of ΔADE .

Ans. $\frac{3}{4} sq.unit$

2. In each of following , find the value of 'k' for which the points are collinear .
   (a) (7, -2) , (5, 1) , (3, k) (b) (8, 1) , (k, -4) ,(2,-5)

Ans. $(a)\ k = 4 \qquad (b)\ k = 3$

3. Find the area of the triangle formed by joining the mid points of the sides of the triangle whose vertices are (0, -1) , (2,1) and (0,3). Find the ratio of this area to the area of the given triangle.

Ans. 1:4

4. Find the coordinates of the points which divides the line segment joining the points (-2,0) and (0,8) in four equal parts.

Ans. $(\frac{-3}{2}, 2), (-1,4), (-\frac{1}{2}, 6)$

5. Find the area of the quadrilateral whose vertices taken in order are (-4, -2), (-3, -5), (3, -2) and (2,3)

Ans. 28 sq. units

6. Find the area of the rhombus, if its vertices are (3,0), (4,5), (-1,4) and (-2,-1) taken in order.

Ans. 24 sq. units

HOTS /SELF EVALUATION

1. Two opposite vertices of a square are (-1,2) and (3, 2). Find the coordinates of the other two vertices.

[Ans. (1,0) and (1,4)]

2. Find the centre of a circle passing through the points (6,-6), (3, 7) and (3, 3). [Ans.3,-2]

3. If the distance between the points (3,0) and (0,y) is 5 units and y is positive, then what is the value of y? [Ans.4]
4. If the points (x,y) ,(-5,-2) and (3,-5) are collinear, then prove that 3x+8y+31 = 0.
5. Find the ratio in which the Y-axis divides the line segment joining the points (5, -6) and (-1, -4). Also find the coordinates of the point of division.

Ans. 5:1; (0,-13/3)

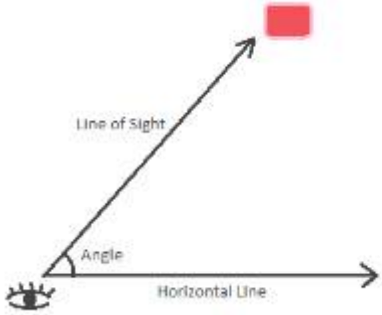
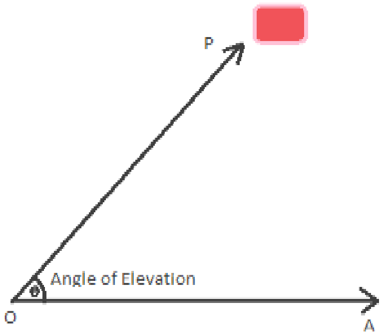
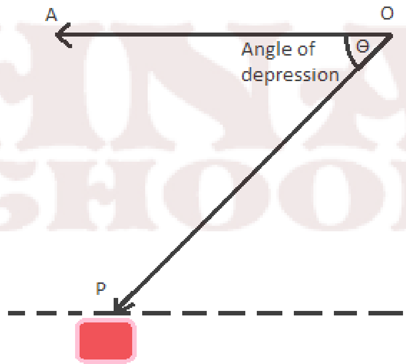
6. Find k so that the point P(-4,6) lies on the line segment joining A (k,0) and B (3, -8). Also find the ratio in which P divides AB.

[Ans. 3:7 externally; k=-1]

7. By distance formula, show that the points (1, -1), (5,2) and (9,5) are collinear.

APPLICATIONS OF TRIGONOMETRY
(HEIGHT AND DISTANCES)

KEY POINTS

Line of sight Line segment joining the object to the eye of the observer is called the line of sight.	
Angle of elevation When an observer sees an object situated in upward direction, the angle formed by line of sight with horizontal line is called angle of elevation.	
Angle of depression When an observer sees an object situated in downward direction the angle formed by line of sight with horizontal line is called angle of depression.	

LEVEL- 1

1. A *ploe* 6cm high casts a shadow $2\sqrt{3}$m long on the ground, then find the sun's elevation?

Ans. 60^0

2. If $\sqrt{3}tan\vartheta = 1, then\ find\ the\ value\ of\ sin^2\theta - cos^2\vartheta$

Ans. -1/2

3. An observer 1.5m tall is 20.5 metres away from a tower 22m high. Determine the angle of elevation of the top of the tower from the eye of the observer.

Ans. 45°

4. A ladder 15m long just reaches the top of vertical wall. If the ladder makes an angle 60^0 with the wall, find the height of the wall

Ans. 15/2 m

5. In a rectangle ABCD, AB =20cm ∠BAC=60^0 then find the length of the side AD.

Ans. $20\sqrt{3}$cm

6. Find the angle of elevation of the sun's altitude when the height of the shadow of a vertical pole is equal to its height:

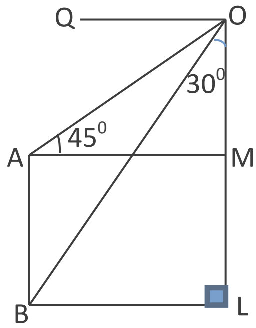
Ans. 45^0

7. From a point 20m away from the foot of a tower ,the angle of elevation of top of the tower is 30°,find the height of the tower.

Ans.$\frac{20}{\sqrt{3}}$m

8. In the adjacent figure, what are the angles of depression of the top and bottom of a pole from the top of a tower h m high:

Ans45^0, 60^0



LEVEL -2

1. In $\Delta ABC, \angle B = 45^0,\ \angle C = 45^0, A\,B = 5cm\ then\ find\ the\ length\ of\ the\ other\ two\ sides.$

Ans.$5cm, 5\sqrt{2}cm$

2. From a point 20 m away from the foot of the tower, the angle of elevation of the top of the tower is 30^0, find the height of the tower.

Ans.$\frac{20\sqrt{3}}{3}m$

3. A ladder 50m long just reaches the top of a vertical wall. If the ladder makes an angle of 60^0 with the wall, find the height of the wall.

Ans. 25 m

4. A circus artist is climbing a 20m long rope, which is tightly stretched and tied from the top of a vertical pole to the ground. Find the height of the pole, if the angle made by the rope with the ground level is 30^0.

Ans. 10 m

5. A tree breaks due to storm and the broken part bends so that the top of the tree touches the ground making an angle 30^0 with it. The distance between the foot of the tree to the point where the top touches the ground is 8m. Find the height of the tree.

Ans.$8\sqrt{3}m$

<u>**LEVEL - 3**</u>

1. The shadow of a tower standing on a level plane is found to be 50m longer when sun's elevation is 30^0 then when it is 60^0. Find the height of the tower.

Ans. $25\sqrt{3}m$

2. The angle of depression of the top and bottom of a tower as seen from the top of a 100m high cliff are 30^0 and 60^0 respectively. Find the height of the tower. [Ans.66.67m]

3. From a window (9m above ground) of a house in a street, the angles of elevation and depression of the top and foot of another house on the opposite side of the street are 30^0 and 60^0 respectively. Find the height of the opposite house and width of the street.

[Ans.12m,$3\sqrt{3}$m]

4. From the top of a hill, the angle of depression of two consecutive kilometer stones due east are found to be 30^0 and 45^0. Find the height of the hill.

Ans.1.37 km

5. Two poles of equal heights are standing opposite each other on either side of the road ,which is 80m wide . From a point between them on the road the angles of elevation of the top of the poles are 60° and 30°. Find the heights of pole and the distance of the point from the poles.

[Ans; h=34. 64m; 20m , 60m] .

6. The angle of elevation of a jet fighter from a point A on the ground is 60^0 . After a flight of 15 seconds, The angle of elevation changes to 30°. If the jet is flying at a speed of 720km/ hr, find the constant height at which the jet is flying.

[Ans;1500m]

7. A window in a building is at a height of 10m above the ground . The angle of depression of a point P on the ground from the window is 30^0. The angle of elevation of the top of the building from the point P is 60^0 . Find the height of the building .

[Ans; 30m]

8. A boy , whose eye level is 1.3m from the ground , spots a ballon moving with the wind in a horizontal line at me height from the ground. The angle of elevation of the ballon from the eyes of the boy at any instant is 60^0. After 2 seconds , the angle of elevation reduces to 30^0 If the speed of the wind at that moment is $29\sqrt{3}$ m/s , then find the height of the ballon from the ground .

[Ans; 88.3m]

9. A man on the deck on a ship 14m above water level , observes that the angle of elevation of the top of a cliff is $60^0$and the angle of depression of the base of the cliff is 30^0. Calculate the distance of the cliff from the ship and the height of the cliff .

[Ans ; h= 56m , distance 24.25m]

10. A straight highway leads to the foot of a tower . A man standing at the top of the tower observes

a car at an angle of depression of 30°, which is approaching the foot of tower with a uniform speed Six minutes later , the angle of depression of the car is found to be 60° . Find the time taken by the car to

reach the foot of the tower . [Ans.3 minutes]

SELF EVALUATION/HOTS

1. An aeroplane when flying at a height of 3125m from the ground passes vertically below another

plane at an instant when the angle of elevation of the two planes from the me point on the ground are 30°and 60° respectively . Find the distance between the two planes at that instant .

[Ans ; 6250m]

2. From the top of a building 60m high , the angels of depression of the top and botton of a vertical lamp post are observed to be 30° and 60°respectively. Find [i] horizontal distance between the building and the lamp post [ii] height of the lamp post .

[Ans. 34.64m h=40m]

3. A vertical tower stands on a horizontal plane and is surmounted by a vertical flag staff of height h m. At a point on the plane , the angles of elevation of the bottom and the top of the flag staff are $\propto$ and β, respectively. Prove that the height of the tower is $\frac{htan\propto}{tan\beta - tan\propto}$

4. The angle of elevation of a cloud from a point 60m above a lake is 30◦ and the angle of depression of the reflection of the cloud in the lake is 60° . Find the height of the cloud from the surce of the lake.

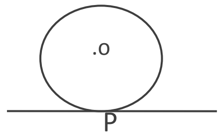
[Ans 120m]

Circle

KEY POINTS

Tangent to a circle :

A tangent to a circle is a line that intersect the circle at only one point.



P= point of contact

- There is only one tangent at a point on a circle.
- There are exactly two tangents to a circle through appoint lying out side the circle.
- The tangent at any point of a circle is perpendicular to the radius through the point of contact.
- The length of tangents drown from an external point to a circle are equal.

(1 Mark Questions)

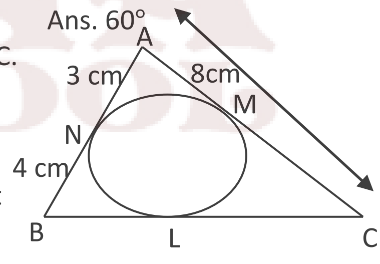
1. If radii of the two concentric circles are 15cm and 17cm , then find the length of each chord of one circle which is tangent to one other.

Ans. 16cm

2. If two tangents making an angle of 120^0 with each other , are drawn to a circle of radius 6cm, then find the angle between the two radii, which are drawn to the tangents.

Ans. 60°

3. In the adjoining figure ,Δ ABC is circumscribing a circle , then find the length of BC.



Ans. 9cm

4. PQ is a chord of a circle and R is point on the minor arc. If PT is a tangent at point P such that ∠QPT = 60° then find <PRQ.

Ans. 120°

5. If a tangent PQ at a point P of a circle of radius 5cm meets a line through the centre O at a point Q such that OQ = 12 cm then find the length of PQ.

Ans. $\sqrt{119}$cm

6. From a point P, two tangents PA and PB are drawn to a circle C(O,r) . If OP =2r ,then what is the type of Δ APB.

Ans. Equilateral triangle

7. If the angle between two radii of a circle is 130°,then find the angle between the tangents at the end of the radii.

Ans. 50°.

8. ABCD is a quadrilateral. A circle centred at O is inscribed in the quadrilateral. If AB = 7cm , BC = 4cm , CD = 5cm then find DA.

Ans. 8 cm

9. In a Δ ABC , AB = 8cm , ∠ABC = 90°. Then find the radius of the circle inscribed in the triangle.

Ans. 2cm

(Two Marks Questions)

1. Two tangents PA and PB are drawn from an external point P to a circle with centre O. Prove that OAPB is a cyclic quadrilateral.

2. If PA and PB are two tangents drawn to a circle with centre O , from an external point P such that PA=5cm and $\angle APB = 60°$, then find the length of the chord AB.

Ans. 5cm

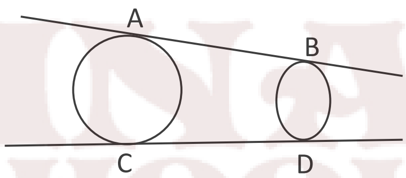
3. CP and CQ are tangents from an external point C to a circle with centre O .AB is another tangent which touches the circle at R and intersects PC and QC at A and B respectively . If CP = 11cm and BR = 4cm, then find the length of BC.

Ans. 7cm

4. If all the sides of a parallelogram touch a circle, show that the parallelogram is a rhombus.

5. Prove that the perpendicular at the point of contact to the tangent to a circle passes through the centre of the circle.

6. In adjacent figure; AB & CD are common tangents to two circles of unequal radii. Prove that AB=CD.



(Three Marks Questions)

1. If quadrilateral ABCD is drawn to circumscribe a circle then prove that AB+CD=AD+BC.

2. Prove that the angle between the two tangents to a circle drawn from an external point, is supplementary to the angle subtended by the line segment joining the points of contact to the centre.

3. AB is a chord of length 9.6cm of a circle with centre O and radius 6cm.If the tangents at A and B intersect at point P then find the length PA.

Ans. 8cm

4. The incircle of a ΔABC touches the sides BC, CA &AB at D,E and F respectively. If AB=AC, prove that BD=CD.

5. Prove that the intercept of a tangent between two parallel tangents to a circle subtends a right angle at the centre of the circle.

6. PQ and PR are two tangents drawn to a circle with centre O from an external point P. Prove that $\angle QPR = 2\angle OQR$.

(Four Marks Questions)

1. Prove that the length of tangents drawn from an external point to a circle are equal. Hence, find BC, if a circle is inscribed in a ΔABC touching AB,BC &CA at P,Q &R respectively, having AB=10cm, AR=7cm &RC=5cm.

   Ans. 8cm

2. Prove that the tangent at any point of a circle is perpendicular to the radius through the point of contact. Using the above, do the following: If O is the centre of two concentric circles, AB is a chord of the larger circle touching the smaller circle at C, then prove that AC=BC.

3. A circle touches the side BC of a ΔABC at a point P and touches AB and AC when produced, at Q & R respectively. Show that AQ=1/2 (perimeter of ΔABC).

4. From an external point P, a tangent PT and a line segment PAB is drawn to circle with centre O, ON is perpendicular to the chord AB. Prove that $PA.PB=PN^2-AN^2$.

5. If AB is a chord of a circle with centre O, AOC is diameter and AT is the tangent at the point A, then prove that ∠BAT=∠ACB.

6. The tangent at a point C of a circle and diameter AB when extended intersect at P. If ∠PCA=110^0, find ∠CBA.

   Ans. 70^0

[Self Evaluation/HOTS Questions]

1. If PA and PB are tangents from an external point P to the circle with centre O, the find ∠AOP+∠OPA.

   Ans. 90^0

2. ABC is an isosceles triangle with AB=AC, circumscribed about a circle . Prove that the base is bisected by the point of contact.

3. AB is diameter of a circle with centre O. If PA is tangent from an external point P to the circle with ∠POB=115^0 then find ∠OPA.

   Ans. 25^0

4. PQ and PR are tangents from an external point P to a circle with centre . If ∠RPQ=120^0, Prove that OP=2PQ.

5. If the common tangents AB and CD to two circles C(O,r) and C'(O'r') intersect at E, then prove that AB=CD.

6. If a, b, c are the sides of a right triangle where c is the hypotenuse , then prove that radius r of the circle touches the sides of the triangle is given by r= (a+b-c)/2.

CONSTRUCTION

KEY POINTS

1. Division of line segment in the given ratio.
2. Construction of triangles:-
   a. When three sides are given.
   b. When two sides and included angle given.
   c. When two angles and one side given.
   d. Construction of right angled triangle.
3. Construction of triangle similar to given similar to given triangle as per given scale.
4. Construction of triangles to a circle.

LEVEL - I

1. Divide a line segment in given ratio.
2. Draw a line segment AB=8cm and divide it in the ratio 4:3.
3. Divide a line segment of 7cm internally in the ratio 2:3.
4. Draw a circle of radius 4 cm. Take a point P on it. Draw tangent to the given circle at P.
5. Construct an isosceles triangle whose base 7.5 cm and altitude is 4.2 cm.

LEVEL –II

1. Construct a triangle of sides 4cm , 5cm and 6cm and then triangle similar to it whose side are 2/3 of corresponding sides of the first triangle.

2. Construct a triangle similar to a given ΔABC such that each of its sides is $2/3^{rd}$ of the corresponding sides of ΔABC. It is given that AB=4cm BC=5cm and AC=6cm also write the steps of construction.

3. Draw a right triangle ABC in which $\angle B=90^0$ AB=5cm, BC=4cm then construct another triangle ABC whose sides are 5/3 times the corresponding sides of ΔABC.

4. Draw a pair of tangents to a circle of radius 5cm which are inclined to each other at an angle of 60^0.

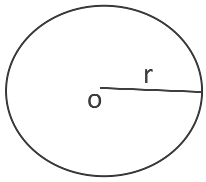
5. Draw a circle of radius 5cm from a point 8cm away from its centre construct the pair of tangents to the circle and measure their length.

6. Construct a triangle PQR in which QR=6cm $\angle Q=60^0$ and $\angle R=45^0$. Construct another triangle similar to ΔPQR such that its sides are 5/6 of the corresponding sides of ΔPQR.

AREAS RELATED TWO CIRCLES

KEY POINTS

1. Circle: The set of points which are at a constant distance of r units from a fixed point o is called a circle with centre o.



2. Circumference: The perimeter of a circle is called its circumference.

3. Secant: A line which intersects a circle at two points is called secant of the circle.

4. Arc: A continuous piece of circle is called and arc of the circle..

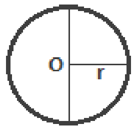
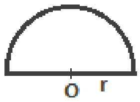
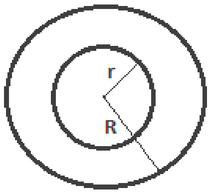
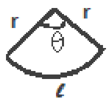
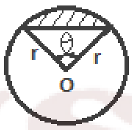
5. Central angle:- An angle subtended by an arc at the center of a circle is called its central angle.

6. Semi Circle: - A diameter divides a circle into two equal arc. Each of these two arcs is called a semi circle.

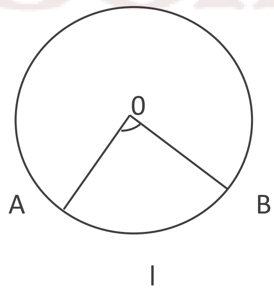
7. Segment :- A segment of a circle is the region bounded by an arc and a chord, including the arc and the chord.

8. Sector f of a circle: The region enclosed by and an arc of a circle and its two bounding radii is called a sector of the circle.

9. Quadrant:- One fourth of a circle disc is called a quadrant. The central ang of a quadrant is 90^0.

S.N	NAME	FIGURE	PERIMETER	AREA
1.	Circle		$2\pi r$ or πd	πr^2
2.	Semi- circle		πr + 2r	½ πr^2
3.	Ring (Shaded region)		2 $\pi(r$ + R)	$\pi(R^2-r^2)$
4.	Sector of a circle		l+2r=$\frac{\pi r\theta}{180°} + 2r$	$\frac{\pi r^2\theta}{360°}$ or $\frac{1}{2}lr$
5.	Segment of a circle		$\frac{\pi r\theta}{180°}$+2r Sin$\frac{\theta}{2}$	$\frac{\pi r^2\theta}{360°}$ - $\frac{1}{2}r^2\sin\theta$

a. Length of an arc AB= $\frac{\theta}{360}$ 2 πr



b. Area of major segment= Area of a circle – Area of minor segment

c. Distance moved by a wheel in
1 rotation=circumference of the wheel

d. Number of rotation in 1 minute

=Distance moved in 1 minute / circumference

LEVEL-I

1. If the perimeter of a circle is equal to that of square, then the ratio of their areas is
   i. 22/7
   ii. 14/11
   iii. 7/22
   iv. 11/14 [Ans-ii]

2. The area of the square that can be inscribed in a circle of 8 cm is
   i. 256 cm^2
   ii. 128 cm^2
   iii. $64\sqrt{2}$ cm^2
   iv. 64 cm^2 [Ans-ii]

3. Area of a sector to circle of radius 36 cm is 54 πcm^2 . Find the length arc of the corresponding arc of the circle is
   i. 6 πcm
   ii. 3 πcm
   iii. 5 πcm
   iv. 8 πcm

   [Ans –ii]

4. A wheel has diameter 84 cm. The number of complete revolution it will take to cover 792 m is.
   i. 100
   ii. 150
   iii. 200
   iv. 300 [Ans-iv]

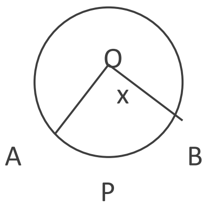
5. The length of an arc of a circle with radius 12cm is 10 π cm. The central angle of this arc is .
   i. 120^0 [Ans-iv]
   ii. 60^0
   iii. 75^0
   iv. 150^0

6. The area of a quadrant of a circle whose circumference is 22 cm is
   i. 7/2 cm^2
   ii. 7 cm^2
   iii. 3 cm^2
   iv. 9.625 cm^2 [Ans-iv]

LEVEL-II

1. In figure 'o' is the centre of a circle. The area of sector OAPB is 5/18 of the area of the circle find x.

   [Ans 100]



2. If the diameter of a semicircular protractor is 14 cm, then find its perimeter . [Ans-36 cm]

3. The radius of two circle are 3 cm and 4 cm . Find the radius of a circle whose area is equal to the sum of the areas of the two circles.

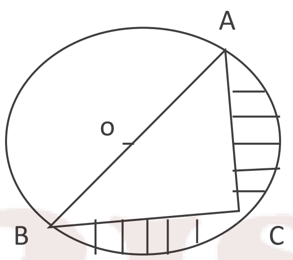
[Ans: 5 cm]

4. The length of the minute hand of a clock is 14 cm. Find the area swept by the minute hand in 5 minutes.
[Ans: 154/3 cm]

5. The radii of two circle are 3 cm and 4 cm . Find the radius of a circle whose area is equal to the sum of the areas of the two circles.
[Ans 5cm]

LEVEL-III

1. Find the area of the shaded region in the figure if AC=24 cm ,BC=10 cm and o is the center of the circle (use $\pi = 3.14$)



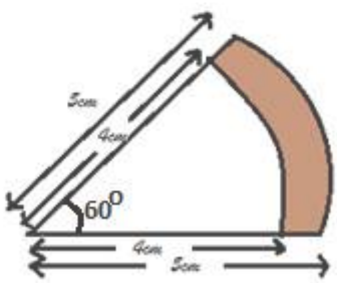
[Ans- 145.33 cm^2]

2. The inner circumference of a circular track is 440m. The track is 14m wide. Find the diameter of the outer circle of the track. [Take π =22/7]

[Ans-168]

3. Find the area of the shaded region.

[Ans: 4.71 cm^2]



4. A copper wire when bent in the form of a square encloses an area of 121 cm^2 . If the me wire is bent into the form of a circle, find the area of the circle (Use π=22/7)

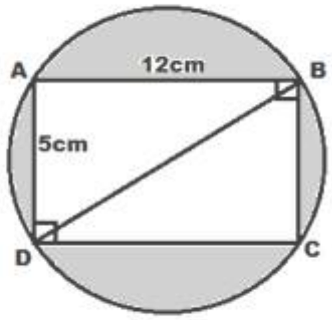
[Ans 154 cm^2]

5. A wire is looped in the form of a circle of radius 28cm. It is rebent into a square form. Determine the side of the square (use $\pi = 22/7$)

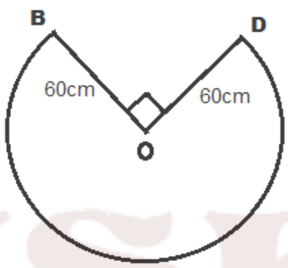
[Ans-44cm]

LEVEL-IV

1. In fig, find the area of the shaded region [use $\pi = 3.44$]



2. In fig find the shape of the top of a table in restaurant is that of a sector a circle with centre 0 and $\angle$bod=90^0. If OB=OD=60cm fnd
   i. The area of the top of the table [Ans 8478 cm^2]
   ii. The perimeter of the table top (Take $\pi = 3.44$) [Ans 402.60 cm]



3. An arc subtends an angle of 90^0 at the centre of the circle of radius 14 cm. Write the area of minor sector thus form in terms of π.

   [Ans 49π cm^2]

4. The length of a minor arc is 2/9 of the circumference of the circle. Write the measure of the angle subtended by the arc at the center of the circle.

   [Ans 80^0]

5. The area of an equilateral triangle is 49√3 cm^2. Taking each angular point as center, circle are drawn with radius equal to half the length of the side of the triangle. Find the area of triangle not included in the circles. [Take π √3=1.73] [Ans 777cm^2]

SELF EVALUATION

1. Two circles touch externally the sum of the areas is 130 π cm^2 and distance between there center is 14 cm. Find the radius of circle.

2. Two circle touch internally. The sum of their areas is 116 π cm^2 and the distance between there centers is 6 cm. Find the radius of circles.

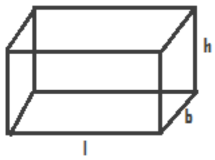
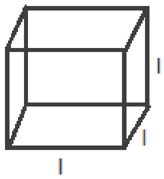
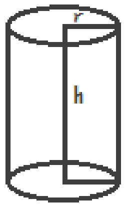
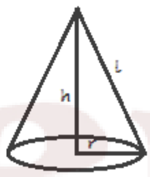
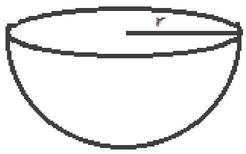
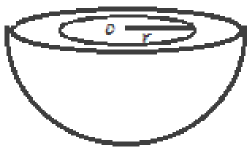
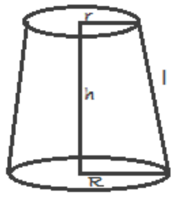
3. A pendulum swings through an angle of 30^0 and describes and arc 8.8 cm in length. Find length of pendulum.

4. What is the measure of the central angle of a circle?

5. The perimeter and area of a square are numerically equal. Find the area of the square.

SURCE AREAS AND VOLUMES

IMPORTANT FORMULA
TAKE A LOOK

SNo	NAME	FIGURE	LATERAL CURVED SURCE AREA	TOTAL SURCE AREA	VOLUME	NOMENCLATURE
1	**Cuboid**		**2(l+b)xh**	**2(lxb + bxh + hx l)**	**l x b x h**	**L=length, b=breadth, h=height**
2	**Cube**		**$4l^2$**	**$6l^2$**	**l^3**	**l=edge of cube**
3	**Right Circular Cylinder**		**$2\pi rh$**	**$2\pi r(r+h)$**	**$\pi r^2 h$**	**r= radius h=height**
4	**Right Circular Cone**		**πrl**	**$\pi r(l+r)$**	**$\frac{1}{3}\pi r^2 h$**	**r=radius of base, h=height , l=slant height = $\sqrt{r^2 - h^2}$**
5	**Sphere**		**$4\pi r^2$**	**$4\pi r^2$**	**$\frac{4}{3}\pi r^3$**	**r=radius of the sphere**
6	**Hemisphere**		**$2\pi r^2$**	**$3\pi r^2$**	**$\frac{2}{3}\pi r^3$**	**r=radius of hemisphere**
7	**Spherical shell**		**$2\pi(R^2 + r^2)$**	**$3\pi(R^2 - \pi r^2)$**	**$\frac{4}{3}\pi(R^3 - r^3)$**	**R=External radius, r=internal radius**
8	**Frustum of a cone**		**$\pi l(R+r)$ where $l^2=h^2+(R-r)^2$**	**$\pi[R^2 + r^2 + l(R+r)]$**	**$\pi h/3[R^2 + r^2 + Rr]$**	**R and r = radii of the base, h=height, l=slant height.**

9. Diagonal of cuboid = $\sqrt{l^2 + b^2 + h^2}$
10. Diagonal of Cube = $\sqrt{3}l$

(LEVEL - 1)

[1] The height of a cone is 60 cm.A small cone is cut off at the top by a plane parallel to the base and its volume is $\frac{1}{64}^{th}$ the volume of original cone. Find the height from the base at which the section is made?

ANS :- 45 cm

[2] Find the volume of the largest right circular cone that can be cut out from a cube of edge 4.2 cm?

ANS:- 19.4 cm^3.

[3] A cubical ice cream brick of edge 22cm is to be distributed among some children by filling ice cream cones of radius 2cm and height 7cm up to its brim.how many children will get ice cream cones?

ANS :-363.

[4] Find the volume of the largest right circular cone that can be cut out from a cube of edge 4.9 cm is?

ANS :- 30.8cm^3.

[5] The slant height of a frustum of a cone is 4 cm and the perimeter of its circular ends are18cm and 6cm. Find the curved surce area of the frustum [use $\pi = \frac{22}{7}$].

ANS :- 48cm^2.

[6] A plumbline is a combination of which geometric shapes?

ANS :-A cone with hemisphere.

LEVEL - 2

[1] The slant height of the frustum of a cone is 5 cm . If the difference between the radii of its two circular ends is 4cm . write the height of the frustum.

ANS :- 3cm

[2] A cylinder, a cone and a hemisphere are of me base and of me height . Find the ratio of their volumes?

ANS :- [3:1:2].

[3] A cone of radius 4cm is divided into two parts by drawing a plane through the midpoint of its axis and parallel to its base, compare the volume of the two parts.

ANS :- 1:7

[4] How many spherical lead shots each having diameter 3cm can be made from a cuboidal lead solid of dimensions 9cm X 11cm X 12cm .

ANS :- 84

[5] Three metallic solid cubes whose edges are 3cm, 4cm, and 5cm are melted and converted into a single cube .Find the edge of the cube so formed?

ANS :- 6cm .

(LEVEL-3)

[1] How many shots each having diameter 4.2 cm can be made from a cuboidal lead solid of dimensions 66cm X 42cm X 21cm?

ANS:-1500

[2] Find the number of metallic circular disk with 1.5cm base diameter and of height 0.2 cm to be melted to form a right circular cylinder of height 10cm and diameter 4.5cm ?

ANS:-450

[3] From a solid cube of side 7cm,a conical cavity of height 7cm and radius 3cm is hollowed out . Find the volume of remaining solid?

ANS:-277cm^3.

[4] A cubical block of side 7cm is surmounted by a hemisphere. what is the greatest diameter of the hemisphere can have? Find the surce area of the solid?

ANS:- 7cm,332.5cm^2.

[5] A heap of rice is in the form of a cone of diameter 9m and height 3.5m .Find the volume of the rice .How much canvas cloth is required to just cover the heap?

ANS:-74.25m^3, 80.61 m^2 .

[6] A square field and an equilateral triangle park have equal perimeter .If the cost of ploughing the field at the rate of Rs 5/m^2 is Rs 720. Find the cost of maintain the park at the rate of Rs10/m^2?

ANS:-Rs1108.48

(LEVEL -4)

[1] A well of diameter 3cm and 14m deep in dug. The earth, taken out of it, has been evenly spread all around it in the shape of a circular ring of width 4m to form an embankment.find the height of embankment?

ANS:-$\frac{9}{8}$ m.

[2] 21 glass spheres each of radius 2cm are packed in a cuboidal box of internal diamenions 16cmX8cmX8cmand then the box is filled with water. Find the volume of water filled in the box?

ANS:-320cm^3.

[3] The slant height of the frustum of a cone is 4cm and the circumferences of its circular ends are 18cm and 6cm. Find curved surce area and total surce area of the frustum.

ANS:-48cm^2, 76.63cm^2.

[4] A rmer connects a pipe of internal diameter 25cm from a canal into a cylindrical tank in his field, which is 12m in diameter and 2.5m deep. If water flows through the pipe at the rate of 3.6km/hr, in how much time will the tank be filled? Also find the cost of water, if the canal department charges at the rate of Rs0.07/m^3?

ANS:-96min, Rs19.80

[5] A spherical glass vessel has a cylindrical neck 7cm long and 4cm in diameter . The diameter of the spherical part is 21cm Find the quantity of water it can hold.

ANS:-4939cm^3.

[6] The surce area of a solid metallic sphere is 616cm^2. It is melted and recast into a cone of height 28cm. Find the diameter of the base of the cone so formed.

ANS:-14cm.

SELF EVALUTION/HOTS QUESTIONS

[1] A spherical copper shell , of external diameter 18cm,is melted and recast into a solid cone of base radius 14cm and height 4cm. Find the inner diameter of the shell.

ANS:-16cm.

[2] A bucket is in the form of a frustum of a cone with a capacity of 12308.8cm^3. The radii of the top and bottom circular ends of the bucket are 20cm and 12cm respectively. Findthe height of the bucket and also the area of metal sheet used in making it [take π 3.14]?

ANS:- $l = 14cm$, $AREA = 2160.32cm2$.

[3] The volume of a solid metallic sphere is 616cm^3.its is melted and recast into a cone of height 28cm. Find the diameter of the base of the cone so formed?

ANS:-21cm.

[4] From a solid cylinder whose height is 8cm and radius 6cm , a conical cavity of height 8cm and of base radius 6cm , is hollowed out. Find the volume of the remaning solid correct to two places of decimals. Also find the total surce area of the remaining solid [take π=3.14] ?

ANS:-603.19cm^3, 603.19cm^2 .

[5] A cylindrical vessel, with internal diameter10cm and height 10.5 cm is full of water. A solid cone of base diameter 7cm and height 6cm is completely immersed in water. Find the volume of :-

(i) water displaced out of the cylindrical vessel.

(ii) water left in the cylindrical vessel.

ANS:- (i): 77cm^3 , (ii) 748cm^3.

[6] A wooden article was made by scooping out a hemisphere from each ends of a solid cylinder. If the height of the cylinder is 20cm, and radius of the base is 3.5cm , find the total surce area of the article.

ANS:-544cm^2.

[7] A building is in the form of a cylinder surmounted by a hemishperical vaulted dome and contains $41\frac{19}{21}$ m^{3}of air. If the internal diameter of the building is equal to its total height above the floor, find the height of the building?

ANS:-4m .

[8] A shuttle cock used for playing badminton has the shape of a frustum of a cone mounted on a hemisphere. The external diameters of the frustum are 5cm and 2cm , the height of the entire shuttle cock is 7cm . Find the external surce area.

ANS:-74.38cm^2.

PROBABLITY

KEY POINTS

1. **Probability:-** The theoretical probability of an event E, written as P(E) is defined as.

$$P(E)=\frac{\text{Number of outcomes vorable to E}}{\text{Number of all possible outcomes of the experiment}}$$

Where we assume that the outcomes of the experiment are equally likely.

2. The probability of a sure event (or certain event) is 1.

3. The probability of an impossible event is 0.

4. The probability of an Event E is number P (E) such that $0\leq P(E)\leq 1$.

5. Elementary events:- An event having only one outcome is called an elementary event. The sum of the probabilities of all the elementary events of an experiment is 1.

6. For any event E,P(E)+P($\bar{E}$)=1, where$\bar{E}$ stands for not E, E and $\bar{E}$ are called complementary event.

7. Performing experiments:-
   a. Tossing a coin.
   b. Throwing a die.
   c. Drawing a card from deck of 52 cards.

8. **mple space:-**The set of all possible outcomes in an experiment is called mple space.

LEVEL-1

1. The probability of getting bad egg in a lot of 400 is 0.035.Then find the no. of bad eggs in the lot. [ans.14]

2. Write the probability of a sure event. [ans.1]

3. What is the probability of an impossible event. [ans.0]

4. When a dice is thrown, then find the probability of getting an odd number less than 3. [ans. $\frac{1}{6}$]

5. A girl calculates that the probability of her winning the third prize in a lottery is 0.08.If 6000 tickets are sold, how many ticket has she brought. [Ans.480]

6. What is probability that a non-leap year selected at random will contain 53 Sundays. [Ans.$\frac{1}{7}$]

7. A bag contains 40 balls out of which some are red, some are blue and remaining are black. If the probability of drawing a red ball is $\frac{11}{20}$ and that of black ball is $\frac{1}{5}$, then what is the no. of black ball. [Ans.10]

8. Two coins are tossed simultaneously. Find the probability of getting exactly one head. [Ans.$\frac{1}{2}$]

9. A card is drawn from a well suffled deck of 52 cards. Find the probability of getting an ace. [Ans.$\frac{1}{13}$]

10. In a lottery, there are 10 prizes and 25 blanks. Find the probability of getting a prize. [Ans.$\frac{2}{7}$]

LEVEL-2

1. Find the probability that a no. selected at random from the number 3,4,5,6,..........25 is prime. [Ans.$\frac{8}{23}$]

2. A bag contains 5 red,4 blue and 3 green balls. A ball is taken out of the bag at random. Find the probability that the selected ball is (a) of red colour (b) not of green colour. [Ans.$\frac{5}{12}$,$\frac{3}{4}$]

3. A card is drawn at random from a well-shuffled deck of playing cards. Find the probability of drawing

(a) A ce card (b)card which is neither a king nor a red card [Ans. $\frac{3}{13},\frac{6}{13}$]

4. A dice is thrown once. What is the probability of getting a number greater than 4? [Ans.$\frac{1}{3}$]

5. Two dice are thrown at the me time. Find the probability that the sum of two numbers appearing on the top of the dice is more than 9. [Ans.$\frac{1}{6}$]

6. Two dice are thrown at the me time. Find the probability of getting different numbers on both dice. [Ans.$\frac{5}{6}$]

7. A coin is tossed two times. Find the probability of getting almost one head. [Ans.$\frac{3}{4}$]

8. Cards with numbers 2 to 101 are placed in a box. A card selected at random from the box. Find the probability that the card which is selected has a number which is a perfect square. [Ans.$\frac{9}{100}$]

9. Find the probability of getting the letter M in the word "MATHEMATICS". [Ans.$\frac{2}{11}$]

LEVEL-3

1. Cards bearing numbers 3,5,…………..,35 are kept in a bag. A card is drawn at random from the bag.Find the probability of getting a card bearing (a)a prime number less than 15 (b)a number divisible by 3 and 5.

[Ans.$\frac{5}{17},\frac{2}{17}$]

2. Two dice are thrown at the me time. Find the probability of getting (a)me no. on the both side (b)different no. on both sides.

[Ans.$\frac{1}{6},\frac{5}{6}$]

3. A child game has 8 triangles of which three are blue and rest are red and ten squares of which six are blue and rest are red. One piece is lost at random. Find the probability of that is (a) A square (b) A triangle of red colour.

[Ans.$\frac{5}{9},\frac{5}{18}$]

4.Two dice are thrown simultaneously. What is the probability that:
(a)5 will not come up either of them? (b)5 will come up on at least one? (C)5 will come at both dice?

[Ans.$\frac{25}{36},\frac{11}{36},\frac{1}{36}$]

5. The king, queen and jack of clubs are removed from a deck of 52 playing cards and remaining cards are suffled. A card is drawn from the remaining cards. Find the probability of getting a card of (a)heart (b)queen (c)clubs

[Ans.$\frac{13}{49},\frac{3}{49},\frac{10}{49}$]

6. A game consist of tossing a one-rupee coin 3 times and noting its outcome each time. Hanif wins if all the tosses give the me result, i.e., 3 heads or three tails and looses otherwise. Calculate the probability that hanif will lose the game. [Ans.$\frac{3}{4}$]

7. Cards bearing numbers 1,3,5,…………..,37 are kept in a bag. A card is drawn at random from the bag. Find the probability of getting a card bearing
(a)a prime number less than 15

[Ans. $\frac{5}{19}$]

(b)a number divisible by 3 and 5.

[Ans. $\frac{2}{19}$]

8. A dice has its six ces marked 0,1,1,1,6,6.Two such dice are thrown together and total score is recorded.(a)how many different scores are possible? (b)what is the probability of getting a total of seven?

[Ans.{a} 5 scores(0,1,2,6,7,12) {b }$\frac{1}{3}$]

Self Evaluation/Hots

1. Three unbiased coins are tossed together. find the probability of getting
   (i) all heads

   Ans. $\frac{1}{8}$

   (ii) two heads

   Ans. $\frac{3}{8}$

   (iii) one heads

   Ans. $\frac{3}{8}$

   (iv) at least two heads

   Ans. $\frac{1}{2}$

2. Two dice are thrown simultaneously .Find the probability of getting an even number as the sum.

   Ans. $\frac{1}{2}$

3. Cards marked with the number 2 to 101 are placed in a box and mixed thoroughly . One card is drawn from the box . Find the probability that the number on the card is:
   (i) An even number Ans. $\frac{1}{2}$
   (ii) A number less than 14 Ans. $\frac{3}{25}$
   (iii) A number is perfect square Ans. $\frac{9}{100}$
   (iv) A prime number less than 20 Ans. $\frac{2}{25}$
4. Out of the milies having three children, a mily is chosen random. Find the probability that the mily has
   (i) Exactly one girl Ans. $\frac{3}{8}$
   (ii) At least one girl Ans. $\frac{7}{8}$
   (iii) At most one girl Ans. $\frac{1}{2}$
5. Five card the ten, jack, queen, king, and ace of diamonds are well shuffled with their ce downward . One card is picked up at random
   (i) What is the probability that the card is the queen? Ans. $\frac{1}{5}$
   (ii) If the queen is drawn and put aside what is the probability that the second card picked up is

   (a) an ace (b) a queen Ans. $\frac{1}{4}, 0$

CLASS – X
SUB : MATHS

TIME ALLOWED : 3 HRS **M.M. =**

General Instructions:

(i) All questions are compulsory.
(ii) The question paper consists of 34 questions divided into four sections A, B, C and D.
(iii) Section A contains 8 questions of 1 marks each, which are MCQ. Section B contains 6 questions of 2 marks each, Section C contains 10 questions of 3 marks each and Section D contains 10 questions of 4 marks each.
(iv) There is no overall choice in the paper. However, internal choice is provided in one question of 2 marks, three question of 3 marks and two questions of 4 marks.
(v) Use of calculator is not permitted.

SECTION – A

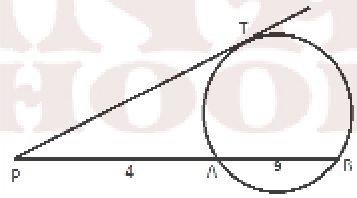
Choose the correct option

1. The roots of a quadratic equation $px^2 + 6x + 1 = 0$ have real roots then value of p is
(A) $p \geq 9$ (B) $p < 9$ (C) $p \leq 9$ (D) None of these

2. The number of terms in the AP 7,13, 19, …………., 205 are
(A) 35 (B) 36 (C) 38 (D) 34

3. For what value of k, 10, k-2 are in A.P.
(A) k=4 (B) k=3 (C) k=2 (D) k=1

4. In the figure given, PA= 4 cm, AB= 9 cm, then value of PT is



(A) 9 cm (B) 4 cm (C) 6 cm (D) None of these

5. The height of a tower is $\sqrt{3}$ times of its shadow. The angle of elevation of the source of height is
(A) 30^0 (B) 60^0 (C) 45^0 (D) None of these

6. The probability of selecting a queen of hearts is
(A) $\frac{1}{4}$ (B) $\frac{1}{52}$ (C) $\frac{1}{13}$ (D) $\frac{12}{13}$

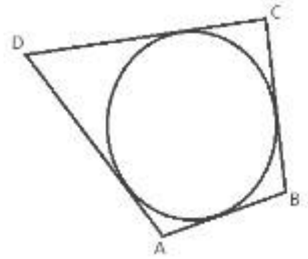
7. If the points P(1,2), Q(0,0) and R(a,b) are collinear, then
(A) a=b (B) a=2b (C) 2a=b (D) a= - b

8. A cone , a hemisphere and a cylinder stand on equal bases and have the me height then their volumes are in the ratio of
(A) 3:1:2 (B) 1:2:3 (C) 2:1:3 (D) 3:2:1

SECTION – B

9. Find the value of k, so that the quadratic equation kx(x-2) + 6 = 0 has two equal roots.

10. In the figure, a circle touches all the four sides of a quadrilateral ABCD whose sides are AB= 6 cm, BC = 9 cm and CD = 8 cm. Find the length of side AD.

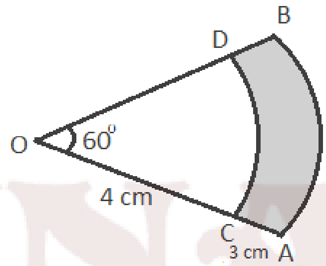


11. Prove that the tangents drawn at the ends of a diameter of a circle are parallel.

12. Draw a line segment AB of length 7 cm. Using ruler and compasses, find a point P on AB such that $\frac{AP}{AB} = \frac{3}{5}$

13. Two cubes each of volume 64 cm3 are joined end to end. Find the surce area of the resulting cuboid.
OR
A sphere of radius 8 cm is melted and recast into a right circular cone of height 32 cm. Find the radius of the base of the cone.

14. Calculate the area of the shaded region shown in the figure.

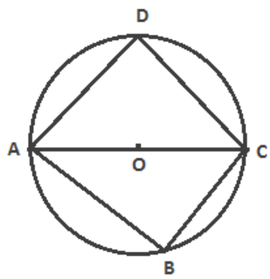


SECTION – C

15. Find the roots of the quadratics equation $3x^2 - 4\sqrt{3}x + 4 = 0$

16. The sum of three numbers of AP is 3 and their product is -35. Find the numbers.
OR
Which term of the AP 3, 10, 17, ………. will be 84 more than its 13th term?

17. In the given figure, AOC is a diameter of the circle. If AB= 7cm, BC = 6 cm and CD = 2cm. Find the perimeter of the cyclic quadrilateral ABCD.



18. Draw a pair of tangents to a circle of radius 3 cm, which are inclined to each other at an angle of 60^0.
OR
Draw a right triangle in which the sides(other than hypotenuse) are of lengths 4 cm and 3 cm. Then construct another triangle whose sides are $\frac{3}{5}$ times the corresponding sides of the given triangle.

19. The shadow of a tower standing on a level ground is found to be 40m longer when the sun's altitude is 30^0 than when it is 60^0. Find the height of the tower.

20. A bag contains 5 red balls and some blue balls. If the probability of drawing a blue ball is double that of a red ball, determine the number of blue balls in the bag.
OR
What is the probability that a leap year, selected at random will contain 53 Sundays?

21. Find the ratio in which the segment joining the points (-3,10) and (6,-8) is divided by (-1,6)

22. Find the area of the quadrilateral whose vertices taken in order are (-4,-2); (-3,-5); (3,-2);(2,3)

23. The circumference of a circle is 88 cm. Find the area of the sector, whose angle at the centre is 45^0.

24. A drinking glass is in the shape of a frustum of a cone of height 14 cm. The diameters of its two circular ends are 4 cm and 2 cm. Find the capacity of the glass.

SECTION – D

25. Solve for x.

$$\frac{1}{a+b+x}=\frac{1}{a}+\frac{1}{b}+\frac{1}{x},\ \ a+b\neq 0$$

OR

A plane left 30 minutes later than the schedule time and in order to reach its destination 1500 km away in time, it has to increase its speed by 250 km/hr from its usual speed. Find its usual speed.

26. Find the sum of all natural numbers between 250 and 1000 which are exactly divisible by 3.

27. Which term of the sequence 20, $19\frac{1}{4}$, $18\frac{1}{2}$, $17\frac{3}{4}$,… is the first negative term?

28. A circle is touching the side BC of ∆ABC at P and touching AB and AC produced at Q and R respectively. Prove that AQ = $\frac{1}{2}$ (Perimeter of ∆ABC)

OR

If all the side of a parallelogram touch a circle, show that the parallelogram is a rhombus.

29. From the top of a building 60m. high the angles of depression of the top and the bottom of a tower are observed to be 30^0 and 60^0. Find the height of the tower.

30. The king, queen and jack of clubs are removed from a deck of 52 playing cards and the well shuffled. One card is selected from the remaining cards. Find the probability of getting
(i) a king (ii) a heart (iii) a club (iv) the '10' of hearts.

31. Find the value of 'k' for the points (7,-2);(5,1);(3,k); are collinear

32. A gulab jamun, contains sugar syrup up to about 30% of its volume . Find approximately, how much syrup would be found in 45 gulab jamuns, each shaped like a cylinder with two hemispherical ends with length 5 cm and diameter 2.8 cm.

33. Water is flowing at the rate of 5 km/hr through a pipe of diameter 14 cm into a rectangular tank which is 50 m long and 44 m wide. Determine the time in which the level of the water in the tank will rise by 7 cm.

34. A toy is in the form of a cone mounted on hemisphere of diameter 7 cm. The total height of the toy is 14.5 m. Find the volume and the total surce area of the toy.

MARKING SCHEME

CLASS-X (MATHS)

EXPECTED ANSWERS/VALUE POINTS

MARKING SCHEME FOR -2

SECTION-A

Q. No.	Solution	Marks
1.	(C)	1
2.	(D)	1
3.	(A)	1
4.	(C)	1
5.	(B)	1
6.	(B)	1
7.	(C)	1
8.	(B)	1
	SECTION - B	
9.	Since, we know that for equal roots	1
	D=0	
	Or, $b^2-4ac=0$	
	Or, $(-2k)^2-4 \times k \times 6=0$	
	Or, $4k^2-24k=0$	1
	Or, $4k(k-6)=0$	
	Or, $4k=0$, or $k-6=0$	
	Or, $k=0$, or $k=6$	
	Or, $k=0, 6$ Ans.	
10.	Here the circle touches the all sides of the Quadrilateral	1
	So, AB+CD=AD+BC	
	Or, 6+8=AD+9	1
	Or, AD=14 – 9 = 5cm Ans.	
11.	Required Fig., Given and to prove	1
	Proof:	1
12.		1
	Drawing $\overline{AB}$=7cm	
	Correct division by any method	
	Correct location of point i.e; AP/AB=3/5	1
13.	$\because$ vol. of the cube=side^3	1
	or, 64 = side^3	
	$\therefore$ side of the cube= $\sqrt[3]{64}$ =4cm	
	Now S.A of the resultant cuboid=2(lb+bh+hl)	1
	=2(8x4+4x4+4x8)	
	=2(32+16+32)	
	=2(80)	
	=160 cm^2 Ans.	
	Or	1
	By question	
	Vol.of the cone = vol.of the sphere	
	Or, $1/3\pi r^2 h = 4/3\pi R^3$	
	Or, $r^2 \times 32 = 4 \times 8 \times 8 \times 8$	1
	$\therefore$ r = 8cm	
	so, the radius of the base of the cone=8cm Ans.	
14.	Ar. of the shaded portion $=\frac{\theta}{360} \times \pi(R^2-r^2)$	1
	$=(60/360) \times (22/7)\ (7^2-4^2)$	

$=1/6 \times 22/7 \times 33$ 1

$= 17.28 cm^2$ Ans.

15. $\because x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$ and putting the correct value 1

$= \frac{-(-4\sqrt{3}) \pm \sqrt{(-4\sqrt{3})^2 - 4 X 3 X 4}}{2 X 3}$ 1

$= \frac{4\sqrt{3} \pm 0}{6}$ 1

$= \frac{2}{\sqrt{3}}, \frac{2}{\sqrt{3}}$ Ans

16. Let the three nos. of the AP are 1

$\alpha-\beta, \alpha, \alpha+\beta$

By queston, 1

$\alpha-\beta+\alpha+\alpha+\beta = 3$

or, $3\alpha = 3$

$\therefore \alpha = 1$

And $(\alpha-\beta) \times \alpha \times (\alpha+\beta) = -35$ 1

or, $\alpha(\alpha^2-\beta^2) = -35$

Putting the value of $\alpha=1$ then

$1(1-\beta^2) = -35$

or, $-\beta^2 = -36$

or, $\beta = \pm 6$

hence the no. are 7,1,-5,or,-5,1,7 respectively. Ans.

Or

Here $t_{13} = a+12d$ 1

$= 3+12(7)$

$= 87$

Let $t_n = t_{13}+84$ 1

or, $a+(n-1)d = 87+84$

or, $3 + (n-1)7 = 171$

or, $(n-1) = 168/7 = 24$ 1

or, $n = 25$

$\therefore$ the required term $= 25^{th}$ Ans.

17. Since, AOC is a diameter of the circle. 1

$\therefore \angle ABC = 90^0$

so, in right triangle ABC

$AC^2 = 7^2+6^2$

$= 85$

Similarly, $\angle ADC = 90^0$ 1

So, in right triangle ADC

$AD^2 = AC^2 - CD^2$

$= 85-4$

$= 81$

$\therefore$ AD=9 cm

So, the perimeter of the cyclic Quad.ABCD=(7+6+2+9) cm 1

=24cm Ans.

18. Constructing 120^0 at the centre with radii 1

Drawing tangents at the end of radii 1

Angle 60^0 between both tangents at the intersection point 1

Or

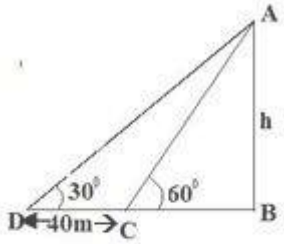
For drawing correct triangle 1

For correct construction steps for making similar triangle 1

Required triangle whose sides are 3/5 times the corresponding sides 1

19. 1

For correct figure.



In triangle ABC, $\tan 60^0 \frac{AB}{BC}$ 1

Or $\sqrt{3} = \frac{h}{BC}$

$\sqrt{3} = \frac{h}{x}$

$\therefore$ h $= \sqrt{3}\,x$

Now in triangle ABD 1

$\tan 30^0 = \frac{h}{40+x}$

or, $\frac{1}{\sqrt{3}} = \frac{\sqrt{3}x}{40+x}$

or, $x = 20$

$\therefore$ h=$20\sqrt{3}$m Ans.

20 Here , no. Of red balls=5 1

let no. Of blue balls = x

$\therefore Total\ no.of\ balls = (5 + x)$

By question, 1

P(B) = 2P(R)

or, $\frac{x}{5+x} = 2\left(\frac{x}{5+x}\right)$

or, $x = 10$ 1

so, No. Of blue balls=10 Ans.

or

In a leap year =366 days= 52 weeks and 2 days 1

The remaining two days can be

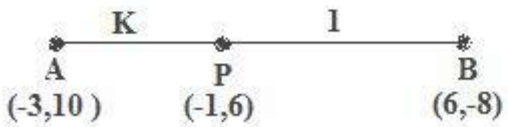
(i)	SUN, MON	(v) THU, FRI
(ii)	MON, TUE	(vi) FRI, T
(iii)	TUE, WED	(vii) T, SUN
(iv)	WED, THU	

There are total seven possibilities i.e. n (s) = 7 1

and n(E) = 2 i.e. SUN, MON & T, SUN

$\therefore$ P(E)= $\frac{n(E)}{n(s)} = \frac{2}{7}$ Ans. 1

21. 1



By question, $-1 = \frac{6k-3\ X1}{k+1}$ 1

k = $\frac{2}{7}$ 1

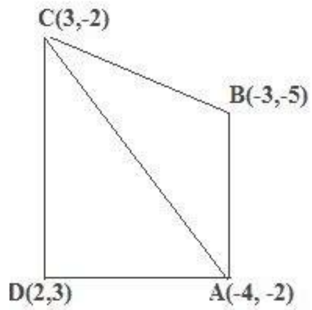
Hence required ration is 2:7

22. 1

Ar. Of ΔABC $= \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

$= \frac{1}{2}[-4(-5+2) + (-3)(-2+2) + (-4)(-2-3]$

$= \frac{21}{2}\ unit^2$



Ar. Of ΔCDA $= \frac{1}{2}[3(3+2) + 2(-2+2) + (-4)(-2-3]$ 1

$= \frac{35}{2}\ unit^2$

so, Ar. Of qua. ABCD = $\frac{21}{2} + \frac{35}{2} = 28\ unit^2$ Ans. 1

23. Since, the Circumference of the circle = 88 cm 1

or, $2\pi r = 88$

$\therefore r = \frac{88\ X\ 7}{44} = 14\ cm$

So, Ar. of the required sector = $\frac{\theta}{360}\pi r^2$ 1

$= \frac{45}{360}\ X\ \frac{22}{7}\ X\ 14\ X\ 14$ 1

$= 77\ cm^2$

24. Vol. Of Glass (Shaped frustum of a cone) = $\frac{1}{3}\pi(R^2 + r^2 + Rr)h$ 1

$= \frac{1}{3}\ X\ \frac{22}{7}\ (2^2 + 1^2 + 2\ X\ 1)14$ 1

$= \frac{1}{3}\ X\ \frac{22}{7}\ X\ 7\ X\ 14$ 1

$=102.67\ cm^3$

SECTION - D

25. $\frac{1}{a+b+x} - \frac{1}{x} = \frac{1}{a} + \frac{1}{b}$ 1

or, $\frac{x-(a+b+x)}{x\,(a+b+x)} = \frac{a+b}{ab}$ 1

or, $\frac{-(a+b)}{x\,(a+b+x)} = \frac{a+b}{ab}$ 1

or, $x(a+b+c) + ab = 0$

or, $x^2 + ax + bx + ab = 0$

or, $x(x+a) + b(x+a) = 0$ 1

or, $(x+a)(x+b) = 0$

$\therefore$ $x = -a$ or $x = -b$ Ans.

Or

Let the usual speed of the plane be x km/hr. 1

Then, By question,

$\frac{1500}{x} - \frac{1500}{x+250} = \frac{1}{2}$

or, $x^2 + 250x - 750000 = 0$ 1

or, $(x + 1000)(x - 7500 = 0$ 1

Or, $x = -1000\ (rejected)\ or,\ x = 750$ 1

Hence, the usual speed of the plane is 750 km/hr. Ans.

26. Required nos. are 252,255, 258, ...999 1

Here, a + (n-1)d=999 1

or, 252 + (n-1)3=999 1

$\therefore n = 250$

So, Required sum = $s_n = \frac{n}{2}\{a + l\} = \frac{250}{2}(252 + 999) = 156375.$ Ans. 1

27. Let the n^{th} term of the given AP be the first negative term. 1

Then $a_n < 0$

or, a +(n-1)d<0 1

or, 20 +(n-1) $(-\frac{3}{4}) < 0$

or, 83 -3n <0 1

or, 3n > 83

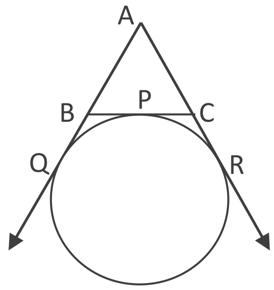
or, n > $\frac{83}{3}$

or, n > $27\frac{2}{3}$ 1

$\therefore$ n $\geq$ 28

Thus, 28[th] term of the given sequence is the first negative term. Ans.

28. Required fig. 1



Since, tangents from an external point to a circle are equal in length 1

$\therefore$ BP = BQ -----------(i)

CP = CR -----------(ii)

And, AQ =AR -----------(iii)

or, AB + BQ = AC + CR 1

or, AB + BP = AC + CP

Now, Perimeter of $\Delta\, ABC = AB + BC + AC$ 1

= AB + (BP +PC) + AC

= (AB + BP) + (AC +PC)

= 2 (AB+BP)

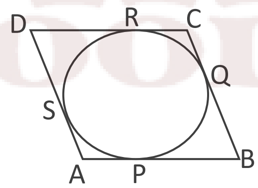
= 2(AB+BQ)

= 2AQ

$\therefore AQ = \frac{1}{2}(Perimeter\ of\ \Delta ABC)$ Proved.

Or

Required fig. 1



We know that the tangents to a circle from an external point are equal in length. 1

$\therefore AP = AS$ --------------(i)

BP = BQ -------------(ii)

CR = CQ ------------(iii)

DR =DS ------------(iv)

Adding (i), (ii), (iii) & (iv), we get 1

(AP+BP) + (CR+DR)= (AS+DS) + (BQ+CQ)

or, AB + CD = AD +BC

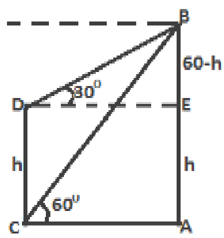
or, 2AB = 2BC 1

or, AB = BC

so, AB = BC = CD = AD

⇨ ABCD is a rhombus. Proved.

29. For correct fig.



1

Let AB = Building, CD = Tower

In, $\Delta\, DEB$,

tan30^0= $\frac{BE}{DE}$

or, $\frac{1}{\sqrt{3}} = \frac{60-h}{x}$

$\therefore\ x = (60-h)\sqrt{3} - - - - - - -(i)$

In, ΔCAB, 1

$\tan 60^0 = \frac{AB}{CA}$

or, $\sqrt{3} = \frac{60}{x}$

$\therefore x = \frac{60}{\sqrt{3}}$ -------------(ii)

By (i) & (ii) 1

$(60-h)\sqrt{3} = \frac{60}{\sqrt{3}}$

h=40 m

Thus, the height of the tower is 40m. Ans.

30 Here, n(s) =49 1

(i) $P(E_1) = \frac{n(E_1)}{n(s)} = \frac{3}{49}$

(ii) $P(E_2) = \frac{n(E_2)}{n(s)} = \frac{13}{49}$ 1

(iii) $P(E_3) = \frac{n(E_3)}{n(s)} = \frac{10}{49}$ 1

(iv) $P(E_4) = \frac{n(E_4)}{n(s)} = \frac{1}{49}$

31 Three points are collinear if $\frac{1}{2}[x_1(y_2-y_3) + x_2(y_3-y_1) + x_3(y_1-y_2)] = 0$ 1

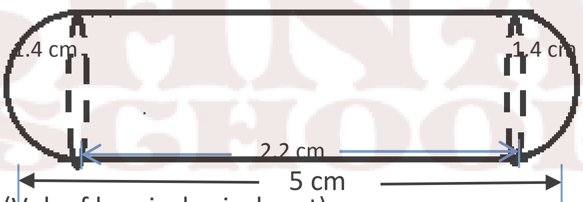
or, $\frac{1}{2}[7(1-k) + 5(k+2) + 3\,(-2-1)] = 0$ 1

or, -2k + 8 =0 1

or, k=4 1

$\therefore k = 4 \quad Ans.$

32. For correct Fig.



Vol. of 1 Gulab Jamun= Vol. of cylindrical part + 2(Vol. of hemispherical part) 1

$= \pi r^2 h + 2\left(\frac{2}{3}\pi r^3\right)$

$= \pi r^2 + \left(h + \frac{4}{3}r\right)$

$= \frac{22}{7}$x 1.4 x 1.4 [2.2 + $\frac{4}{3}$x 1.4]

= 25.05 cm^3

So, vol. of 45 gulab jamuns = 45 x 25.05 = 1127.28 cm^3

Hence, Vol. of sugar syrup = 30/100 x 1127.28 = 338.18 cm^3 = 338 cm^3 (approx.) 1

33 Let the level of the water in the tank will rise by 7cm in x hrs

So, vol of the water flowing through the cylindrical pipe in x hrs = $\pi r^2 h$ 1

$= \frac{22}{7} \times \left(\frac{7}{100}\right)^2 \times 5000\ x$ m^3

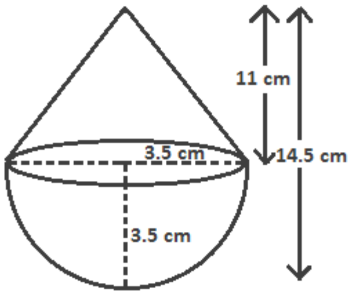
= 77 x m^3 1

Also, Vol of water that lls into the tank in x hrs = 50 x 44 x $\frac{7}{100}$ m^3 = 154 m^3

By ques 77 x = 154

x = 2

So, the level of the water in the tank will rise by 7 cm in 2 hours 1

34 

for correct figure 1 marks

Radius of hemisphere = 7/2 = 3.5 cm

Height of cone = (14.5 – 3.5)

=11cm

Slant height of cone = $\sqrt{r^2 + h^2}$

=$\sqrt{(3.5)^2 + (11)^2}$

=11.55 cm 1

Now, vol of toy = Vol of hemisphere + Vol of cone

= $\frac{2}{3}\pi r^3 + \frac{1}{3}\pi r^2 h$

=$\frac{1}{3}\pi r^2(2r + h)$

=$\frac{1}{3} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}(2 \ \times \frac{7}{2} + 11)$ cm^3

= 231 cm^3 1

And, T of the Toy = of hemisphere + of cone

= 2 πr^2 + $\pi r l$

= $\pi r(2r + l)$

=$\frac{22}{7} \times \frac{7}{2}(2 \ \times \frac{7}{2} + 11.55)$

=204.05 cm^2 1

ACTIVITES (TERM-I)
(Any Eight)

Activity1: To find the HCF of two Numbers Experimentally Based on Euclid Division Lemma

Activity2: To Draw the Graph of a Quadratic Polynomial and observe:

i. The shape of the curve when the coefficient of x^2 is positive

ii. The shape of the curve when the coefficient of x^2 is negative

iii. Its number of zero

Activity3: To obtain the zero of a linear Polynomial Geometrically

Activity4: To obtain the condition for consistency of system of linear Equations in two variables

Activity5: To Draw a System of Similar Squares, Using two intersecting Strips with nails

Activity6: To Draw a System of similar Triangles Using Y shaped Strips with nails

Activity7: To verify Basic proportionality theorem using parallel line board

Activity8: To verify the theorem: Ratio of the Areas of Two Similar Triangles is Equal to the Ratio of the Squares of their corresponding sides through paper cutting.

Activity9: To verify Pythagoras Theorem by paper cutting, paper folding and adjusting (Arranging)

Activity10: Verify that two figures (objects) having the me shape (and not Necesrily the me size) are similar figures. Extend the similarity criterion to Triangles.

Activity11: To find the Average Height (in cm) of students studying in a school.

Activity12: To Draw a cumulative frequency curve (or an ogive) of less than type .

Activity13: To Draw a cumulative frequency curve (or an ogive) of more than type.

ACTIVITES (TERM-II)
(Any Eight)

Activity1: To find Geometrically the solution of a Quadratic Equation $ax^2+bx++c=0$, $a\neq0$ (where a=1) by using the method of computing the square.

Activity2: To verify that given sequence is an A.P (Arithmetic Progression) by the paper Cutting and Paper Folding.

Activity3: To verify that $\sum n = \frac{n(n+1)}{2}$ by Graphical method

Activity4: To verify experimentally that the tangent at any point to a circle is perpendicular to the Radius through that point.

Activity5: To find the number of Tangent from a point to the circle

Activity6: To verify that lengths of Tangents Drawn from an External Point, to a circle are equal by using method of paper cutting, paper folding and pasting.

Activity7: To Draw a Quadrilateral Similar to a given Quadrilateral as per given scale ctor (Less than 1)

Activity8: (a) To make mathematical instrument clinometer (or sextant) for measuring the angle of elevation/depression of an object

(b) To calculate the height of an object making use of clinometers(or sextant)

Activity9: To get miliar with the idea of probability of an event through a double color card experiment.

Activity10: To verify experimentally that the probability of getting two tails when two coins are tossed simultaneously is ¼=(o.25) (By eighty tosses of two coins)

Activity11: To find the distance between two objects by physically demonstrating the position of the two objects y two Boys in a Hall, taking a set of reference axes with the corner of the hall as origin.

Activity12: Division of line segment by taking suitable points that intersects the axes at some points and then verifying section formula.

Activity13: To verify the formula for the area of a triangle by graphical method .

Activity14: To obtain formula for Area of a circle experimentally.

Activity15: To give a suggestive demonstration of the formula for the surce Area of a circus Tent.

Activity16: To obtain the formula for the volume of Frustum of a cone.

PROJECTS

Project 1 : Efficiency in packing

Project 2 : Geometry in Daily Life

Project 3: Experiment on probability

Project 4: Displacement and Rotation of a Geometrical Figure

Project 5: Frequency of letters/ words in a language text.

Project 6: Pythagoras Theorem and its Extension

Project 7: Volume and surce area of cube and cuboid.

Project 8: Golden Rectangle and golden Ratio

Project 9 : Male-Female ratio

Project 10 : Body Mass Index(BMI)

Project 11 : History of Indian Mathematicians and Mathematics

Project 12 : Career Opportunities

Project 13 : π(Pie)

Project Work Assignment (Any Eight)

ACTIVITY- 1

TOPIC:- Prime ctorization of composite numbers.

OBJECTIVE:- To verify the prime ctorization 150 in the form
5^2x3x2 i.e 150=5^2x3x2.

PRE-REQUISITE KNOWLEDGE:- For a prime number P, P^2 can be represented by the area of a square whose each side of length P units.

MATERIALS REQUIRED:-

i. A sheet of graph paper (Pink / Green)

ii. Colored (black) ball point pen.

iii. A scale

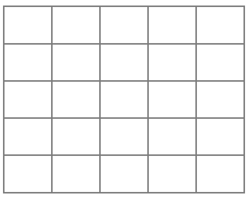
TO PERFORM THE ACTIVITY:-

Steps:-

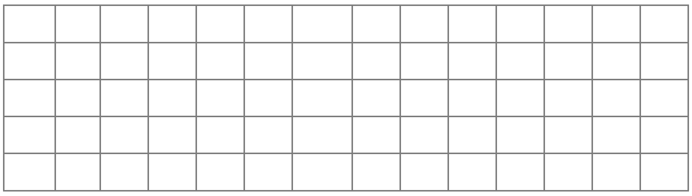
1. Draw a square on the graph paper whose each side is of length 5 cm and then make partition of this square into 25 small squares as shown in fig 1.1 each small square has its side of length 1cm.

   Here, we observe that the area of the square having side of length 5 cm =5^2 cm^2=25cm^2

2. As shown in Fig 1.2 draw there equal squares where each square is of me size as in figure 1.1 then the total area in the fig1.2
   =5^2+5^2+5^2 cm^2
   =5^2x3cm^2 ie,75 cm^2



Fig=1.1



Fig=1.2

3. As shown in fig 1.3 draw six equal square where each square is as me size as in Fig 1.1 Here , three squares are in one row and three squares in the second row.

   We observe that the total area of six squares

   =5^2x(3+3)cm^2
   = 5^2x3x2 cm^2

Also observe that the total area

$=75cm^2+75cm^2=150cm^2$

Hence, we have verified that

$150=5^2x3x2$

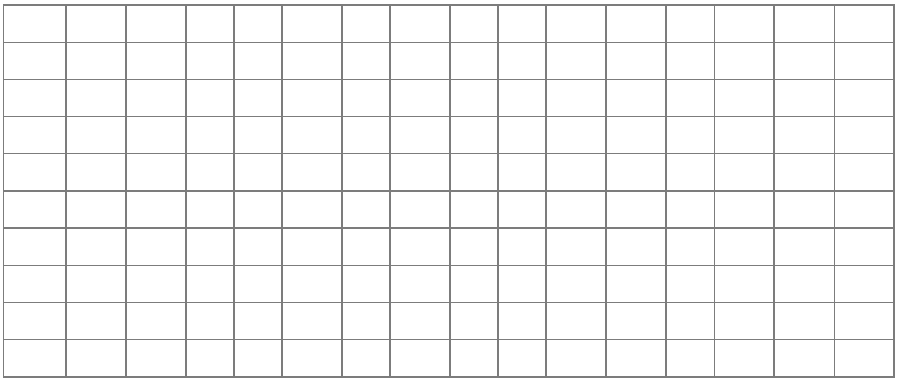


Fig-1.3

ACTIVITY-2

TOPIC:- Ratio of the areas of two similar triangles

STATEMENT:- The ratio of the area of two similar triangle is equal to the ratio of the squares of their corresponding sides.
OBJECTIVE:- To verify the above statement through activity.
PRE-REQUISITE KNOWLEDGE:-

1. The concept of similar triangles.
2. Division of a line segment into equal parts.
3. Construction of lines parallel to given line.

MATERIAL REQUIRED:-

1. White paper sheet
2. Scale /Rubber
3. Paint box
4. Black ball point pen or pencil

TO PERFORM THE ACTIVITY:-

STEPS:-

1. On the poster paper sheet, draw two similar triangle ABC and DEF. We have the ratio of their corresponding sides me and let as have
   AB: DE= BC: EF=CA: FD=5:3

   ie , AB/DE=5/3 , BC/EF=5/3 , CA/FD =5/3,

   ie DE =3/5 AB, EF=3/5 BC,FD=3/5 CA

2. Divide each side of ΔABC into 5 equal parts and those of ΔDEF into 3 equal parts as shown in Fig (i) and (ii).
3. By drawing parallel lines as shown in Fig (i) and (ii)., we have partition ΔABC into 25 smaller triangle of me size and also each smaller triangle in fig (i) has me size and as that of the smaller triangle fig (ii).
4. Paint the smaller triangle as shown in Fig (i) and (ii)..

OBSERVATION:-

1. Area of ΔABC= Area of 25 smaller triangle in fig (i)=25 square unit

Where area of one smaller triangle in fig (i)=P (square unit)

2. Area of ΔDEF=Area of a smaller triangle in Fig (ii)=9p where area of one smaller triangle in fig (ii)=P square units.

3. $\frac{\text{Area of } \Delta \text{ ABC}}{\text{Area of } \Delta\text{DEF}} = \frac{25\text{ P}}{9\text{P}} = \frac{25}{9}$

4. $\frac{(AB)^2}{(DE)^2} = \frac{(AB)^2}{(3/5AB^2)} = \frac{(AB)^2}{9/25(AB)^2} = \frac{25}{9}$

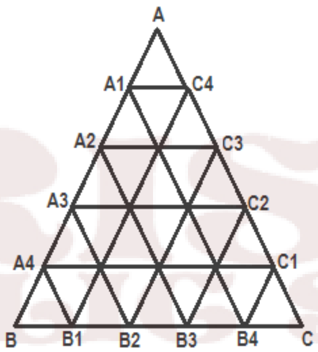
Similarly

$\frac{(BC)^2}{(EF)^2} = \frac{25}{9}$ and $\frac{(CA)^2}{(FD)^2} = \frac{25}{9}$

5. From steps (3) and (4) , we conclude that

$$\frac{\text{Area of } \Delta \text{ ABC}}{\text{Area of } \Delta\text{DEF}} = \frac{(AB)^2}{(DE)^2} = \frac{(BC)^2}{(EF)^2} = \frac{(CA)^2}{(FD)^2}$$

Hence the ratio of the areas of two similar triangles is equal to the ratio of the squares of their corresponding sides.



Fig(i)

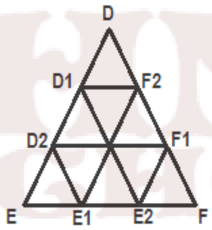


Fig (ii)

ACTIVITY-3

TOPIC:- Trigonometric identities.

STATEMENT:- $\sin^2\theta + \cos^2\theta = 1, 0^0 < \theta < 90^0$

OBJECTIVE: - To verify the above identity

PRE-REQUISITE KNOWLEDGE:- In a right angled triangle.

$$\sin\theta = \frac{\text{Side opposite to angle }\theta}{\text{Hypotenuse of the triangle}}$$

$$\cos\theta = \frac{\text{Side adjacent to angle }\theta}{\text{Hypotenuse of the triangle}}$$

MATERIAL REQUIRED:-

1. Drawing sheet
2. Black ball point pen
3. Geometry box
4. Scale

TO PERFORM THE ACTIVITY

Step:-

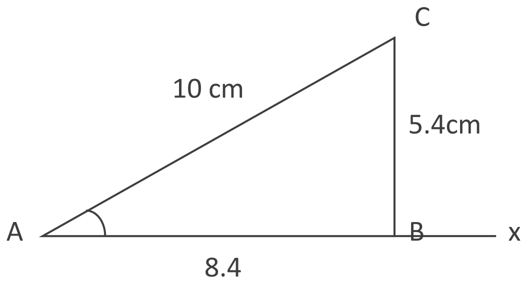
1. On the drawing sheet, draw horizontal ray AX .
2. Construct any arbitrary $\angle CAX = O(y)$
3. Construct AC=10 cm.
4. From C draw $CB \perp AX$.
5. Measure the length sides of sides AB and BC of the right angled Δ ABC (see fig)
6. We measure that AB=8.4 cm (approx) and BC=5.4 cm (approx)

OBSERVATION

1. Sin θ= BC/AC=5.4/10=.54 (Approx)
2. Cosθ=AB/AC=8.4/10=.84(approx)
3. $\text{Sin}^2\,\theta + \cos^2\,\theta = (.54)^2 + (.84)^2$

$$= .2916 + .7056$$
$$= .9972 \text{(Approx)}$$

Ie. $\text{Sin}^2\,\theta + \text{Cos}^2\,\theta$ is nearly equal to 1. Hence the identity is verified.



ACTIVITY-4

Topics:- Measure of the central tendencies of a data.

STATEMENT:- We have an empirical relationship for statistical data as 3 x median=Mode+2 x mean.

OBJECTIVE :- To verify the above statement for a data.

PRE-REQUISITE KNOWLEDGE:-

Method to find central tendencies for grouped data.

MATERIAL REQUIRED:-

1. A data about the heights of students of a class and arranged in grouped form.
2. A ball point pen.
3. A scale.

TO PERFORM THE ACTIVITY:-

Step:-

1. Count the number of girl students in the class. The number is 51
2. Record the data about their height in centimeter.
3. Write the data in grouped form as below:-

Height in cm	135-140	140-145	145-150	150-155	155-160	160-165	Total no of girls
Number of girls	4	7	18	11	6	5	51

4. On three different sheets of paper find mean height on the sheet of paper , median height on the second and the modal height on the third sheet of paper.

5. Let us find mean by step deviation method:-

Class of heights (in cm)	Frequency p	Class mark x_i	U1=$\frac{x1-147.5}{5}$ a= 147.5,h=5	Fi x ui
135-140	4	137.5	-2	-8
140-145	7	142.5	-1	-7
145-150	18	147.5	0	0
150-155	11	152.5	1	11
155-160	6	157.5	2	12
160-165	5	162.5	3	13
	$\sum fi = 51$			$\sum fiui = 51$

Mean=a+h x $\frac{\sum fiui}{\sum fi}$ =147.5+5 x 23/51 =147.5+115/51

=(147.5+2.255)cm=149.755cm

6. Let us find median of the data:-

Class of height (in cm)	Frequency number of girls	Cumulative frequency
135-140 140-145 145-150 150-155 155-160 160-165	4 7 18=f 11 6 5	4 11=cf 29 40 46 51
Total	n$\sum fi = 51$	

n/2=25.5

we have median class (145-150) it gives l=145,h=5,f=18,cf=11

median=l+$\left\{\frac{\frac{n}{2}-cf}{f}\right\}$ x h=145 +$\left\{\frac{25.5-11}{18}\right\}$ x5

=145+$\frac{14.5 \text{ x5}}{18}$

=145+4.028

=149.028cm

7. Let us find mode of the data:-

	Class of heights (in cm)	FREQUENCY (No of Girls)
	135-140	4
	140-145	7=f1
(Modal class)	145-150	18=fm
	150-155	11=f2
	155-160	6
	160-165	5
	Total	51

Modal class is 145-150

Thus l=145, h=5, fm=18, f1=7, f2=11

Mode=H $\left\{\frac{fm-f1}{2fm-f1-f2}\right\}$ xh=145 +$\left\{\frac{18-7}{36-7-11}\right\}$ x 5

=145+55/18 =145+3.055

=148.055 cm

8. CONCLUSION:-

Mean=149.755, median=149.028 and mode=148.055

3x median=3x149.028=447.084

Mode + 2 x mean=148.055+2x149.755

=148.055+299.510=447.565

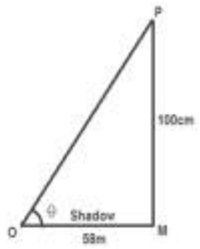
Thus we have verified that 3x median =mode + 2 x mean(Approx)

ACTIVITY – 5

TOPIC : Angle of Elevation

OBJECTIVE : To find the angle of elevation of the sun at a particular time on a sunny day.

PRE-REQUISITE KNOWLEDGE: knowledge of trigonometric ratios.



MATERIAL REQUIRED :

1. A metre rod
2. Measuring tape
3. Table for tangent of angles .

TO PERFORM THE ACTIVITY:

STEPS :

1. On the particular sunny day at the given time, put the metre rod on the level ground with one end on the ground and the other vertically upward.
2. Measure the length of the shadow of the metre rod from the beginning to the end. Let the length of the shadow be 58cm = 0.58m.
3. The length of the metre rod = 1m or 100cm.

OBSERVATION:

1. If θ be the angle of elevation of the sun at the given moment, then we have the following figure on a sheet of paper by taking a suitable scale.
2. From the right angle ΔOMP drawn in figure, we have

   Tan $\theta = \frac{MP}{OM} = \frac{100}{58} = 1.724(approx)$

   Tan $\theta = \sqrt{3}$ (approx.)

   i.e. tan θ = tan 60^{o}

   $\theta = 60^{o}$

Hence, the required angle of elevation of the sun is 60^{o}. For better result, we can take the help of the table of tangent of angles.

ACTIVITY – 6

TOPIC - Probability of events of a random experiment.

STATEMENT: For an event E of a random experiment, P(not E) = 1 – P(E).

OBJECTIVE: To verify the above statement by tossing three coins of different denominations simultaneously for head and tail. Event E happens if we get at least two heads and the event not-E happens if we do not get two or more than two heads.

PRE-REQUISITE KNOWLEDGE:

1. Probability of an event : $\dfrac{\text{Number of outcome which vour the happening of the event E}}{\text{Total number of outcome}}$
2. Event not-E happens when the outcome is not vourable for the event E to happen.

TO PERFORM THE ACTIVITY:
STEPS:

1. Take three ir coins of different denominations and toss these coins simultaneously.
2. We imaging about the possible outcomes as below.
   HHH, HHT, HTH, THH, HTT, TTH, TTT
   i.e. there can be 8 possible outcomes
   vourable outcomes to the event E are
   HHH, HHT, HTH, THH
   Then P(E) = 4/8= ½
   Now, vourable outcomes to the event not-E are HTT, THT, TTH, TTT
   Then P(not-E) = 1- ½ = 1-P(E)

3. Repeating above random experiment, we record the observation of 20 trials as below:

Number of Heads:	0	1	2	3
Number of times out of 20 trials :	4	7	5	4

4. From table in step 3, we observe that for 2 heads or for 3 heads, the event E happens i.e. there are 5+4=9 chances out of 20 which vour E
   Thus, we have P(E) = $\frac{9}{20}$
   Also we observe that for 0 head or for 1 head the event not-E happens. There are 4+7=11 chances out of 20 which vour not-E.
   So, P(not-E) = 11/20 = 1- 9/20 = 1-P(E).

QUIZ

(REAL NUMBERS)

Answer the following questions

1. What is a lemma?
2. State Euclid's Division Lemma?
3. What does HCF stand for?
4. Give the full form of LCM.
5. State Euclid's division algorithm.

ORAL TEST

(REAL NUMBERS)

Answer the following questions:

1. Euclid's division algorithm is a technique to compute the ___________ of two given positive integers.
2. HCF(124, 24) is ___________.
3. "Every composite number can be expressed(ctorised) as a product of primes, and this ctorition is unique, apart from the order in which the prime ctors occurs". The above statement is called ___________.
4. For any two positive integers a and b,
   a x b = HCF(a, b) x ______
5. If a number cannot be written in the form p/q, where p and q are integers and q $\neq$ 0, then it is called ____________.

QUIZ

(POLYNOMIALS)

Answer the following questions:

1. What is a quadratic polynomial?
2. What is the degree of a quadratic polynomial?
3. What are the zeros of a polynomial?
4. What is the shape of curve of a quadratic polynomial graph?
5. State remainder theorem.

ORAL TEST

1. If P(x) is a polynomial in x, the highest power of x in P(x) is called the ________ of the polynomial P(x).
2. A polynomial of degree 2 is called a __________.
3. The linear polynomial ax + b, a$\neq$ 0, has exactly one zero, namely, the x-coordinate of the point where the graph of y = ax + b intersects the _________.
4. A polynomial P(x) of degree n has atmost ________ zeroes.
5. The sum and the product of the zeroes of a quadratic polynomial $x^2 + 7x + 10$ is ____ and _______.

QUIZ

(Pair of linear equations in two variables)

Answer the following questions:

1. What is a pair of linear equations in two variables?
2. Give the general form of a pair of linear equation?
3. What are the methods of solving a pair of linear equation in two variables?
4. What is the condition for inconsistent solution?
5. What is the shape of curve in graph of a linear equation?

Oral Test

1. Every solution (x, y) of a linear equation in two variables, ax+by +c = 0 corresponds to a ____ on the line representing the equation, and vice ver.
2. If the pair of linear equations in two variables have only one common point on both the lines, then we have a _______ solution.
3. A pair of equations which has no solution is called a/an ________ pair of linear equations.
4. Half the perimeter of a rectangular garden, whose length is 4 m more than its width is 36 m. The dimension of the garden are ________ and ___________.

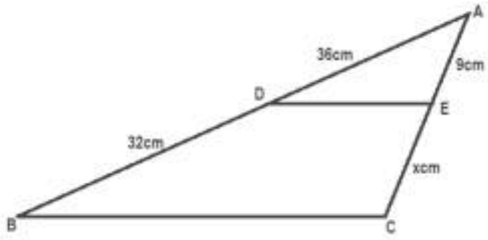
5. A pair of linear equations in two variables can be represented and solved by the graphical method and ________ method.

QUIZ
(Triangles)

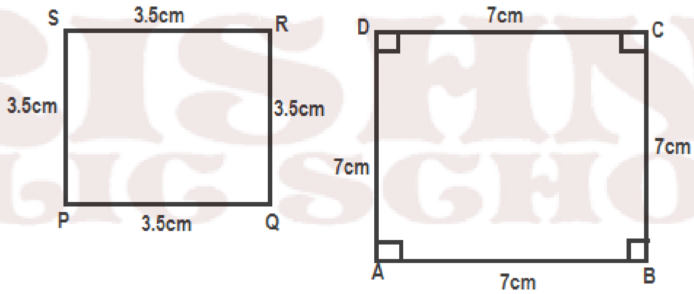
1. What is S similarity criterion?
2. What is the relationship between congruency and similarity of figures?
3. What is the criteria for the similarity of two triangles?
4. For what types of triangles is Pythagoras theorem applicable?
5. What is the another name of Basic Proportionality Theorem?

ORAL TEST

1. All __________ triangles are similar(equilateral/ isosceles/Scalene)
2. The longest side of a right angled triangle is called __________.
3. In a ___________ the square of the hypotenuse is equal to the sum of squares of the other two sides.
4. In the given figure, if DE|| BC, then the value of x is __________

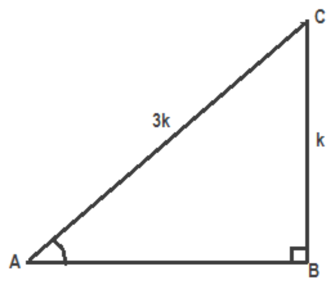


5. State whether the following quadrilateral are similar or not.



QUIZ
(Introduction to Trigonometry)

1. What is trigonometry?
2. What are trigonometric ratios of an acute angle in a right triangle?
3. From the figure find the value of cos A.



4. Write the trigonometric ratios of 60^o.
5. Evaluate tan 70^o / cot 20^o.

ORAL TEST

1. In a right triangle ABC, right angles at B, sin A = _______.
2. Sec(90o –A) = ___________
3. Sec^2 A - __________ = 1 , for $0^o \leq A \leq 90^o$.
4. If cot θ= 7/8, then $(1+ \sin \theta)(1 - \sin \theta)/(1 + \cos \theta)(1 - \cos \theta)$
5. $(1 - \tan^2 45^o)/(1 +\tan^2 45^o)$= ____________

QUIZ
(STATISTICS)
1. Name the measures of central tendency.
2. What is cumulative frequency?
3. How will you represent the cumulative frequency distribution graphically?
4. How will you find the median of a grouped data graphically with the help of one ogive?
5. How will you find the median of a grouped data graphically with the help of both ogives (i.e of the less than type and of more than type)?

ORAL TEST
1. __________ is the sum of the values of all the observations divided by the total number of observations.
2. Class mark = _____ /2.
3. The formula for finding the mean using the step deviation method is _________.
4. The formula for finding the mode in a grouped frequency distribution is _________.
5. The formula for finding the median of grouped data is ___________.

FORMATIVE ASSESSMENT
QUIZ
1. Define the fundamental theorem of arithmetic.
2. Define euclid's division lemma.
3. What is a quadratic polynomial.
4. What is the relationship between zeros and coefficients of a quadratic polynomial.
5. Give the condition for a pair of linear equations to be inconsistent.

ORAL TEST
1. For any two positive integers a and b, HCF(a,b) x LCM(a, b) = _________
2. $5 - \sqrt{3}$ is a/an ________ number.
3. A polynomial of degree 3 is called a ______ polynomial.
4. A quadratic polynomial having the sum and product of its zeroes respectively 5 and 6 is ________ .
5. All _______ triangles are similar. (equilateral/isosceles/scalene).

QUIZ
QUADRATIC EQUATION
1. What is a quadratic equation?
2. How many roots can a quadratic equation have?
3. Give the formula for finding the roots of $ax^2 + bx + c = 0$ $(a \neq 0)$
4. Give the nature of roots of the equation $ax^2 + bx + c = 0$ $(a \neq 0)$
5. Find the nature of the roots of the equation $3x^2 - 2x + 1/3 = 0$

ORAL TEST
1. A real number α is id to be a root of the quadratic equation $ax^2 + bx + c = 0$, if $a\alpha^2 + b\alpha + c =$ ______.
2. A quadratic equation $ax^2 + bx + c = 0$ has two roots, if $b^2 - 4ac > 0$.
3. The quadratic equation $3x^2 - 4\sqrt{3}x + 4 = 0$ has two _______ roots.
4. The roots of a quadratic equation $2x^2 - 7x + 3 = 0$ are _____ and _______.
5. Two numbers whose sum is 27 and product is 182 are _______ and _______.

QUIZ
(ARITHMETIC PROGRESSIONS)
1. What is an A.P.?
2. What is meant by common difference in an A.P. ?
3. What is the formula for the nth term of an A.P.?

4. What is the formula for the sum of first n terms of an A.P. ?
5. What is the formula for the sum of first n natural numbers?

ORAL TEST
1. The common difference of a sequence of multiples of 7 is ________.
2. The difference of consecutive terms in an A.P. is always _______.
3. The sum of first 20 natural numbers is _______.
4. The sum of first eight odd natural numbers is ________.
5. The sum of first ten even natural numbers is _______.

QUIZ
(Coordinate geometry)
1. What is abscis?
2. What is ordinate?
3. What is distance formula?
4. What is the distance of a point p(x,y) from origin?
5. Give the section formula.

ORAL TEST
1. If the area of a triangle is 0 square units, then its vertices are _______.
2. The area of a triangle whose vertices are (1 , -1), (-4, 6) and (-3, -5) is ________ square units.
3. The distance between the points (-5, 7) and (-1, 3) is ______ units.
4. _______ has been developed as an algebraic toll for studying geometry of figures.
5. The distance between the points (a,b) and (-a, -b) is ________ units.

QUIZ
(Some applications of trigonometry or heights and distance
1. Why trigonometry was invented? Give its uses.
2. What is the line of sight?
3. What is the angle of elevation?
4. What is the angle of depression?
5. What is a theodolite?

ORAL TEST
1. The other name of clinometer is _________.
2. If height of clinometer is 1 m, distance between object and clinometer is 40m and angle of elevation of object is 45^{o}, then the height of object is ____________.
3. A tower stands vertically on the ground. From the point on the ground, which is 25m away from the foot of the tower, the angle of elevation of the top of the tower is found to be 60^{o} . The height of the tower is _____.
4. The angles of elevation of the top of a tower from two points at distances a and b from the base and on the me straight line with it are complementary. The height of the tower is __________.
5. A ladder 15m long just reaches the top of a vertical wall. If the ladder makes an angle of 60^{o} with the wall, then the height of the wall is __________.

QUIZ
(CIRCLES)
1. Define tangent to a circle.
2. How many tangent(s) is/are there at a point of circle?
3. How many tangent can be drawn to a circle from a point outside the circle?
4. Define length of a tangent.
5. What is the relation between the lengths of tangents drawn from an external point to a circle?

ORAL TEST

1. A tangent to a circle intersects it in _____________ point(s).
2. A line intersecting a circle in two points is called a ___________.
3. A circle can have _________ parallel tangents at the most.
4. The common point of a tangent to a circle and the circle is called __________.
5. The tangent at any point of a circle is ____________ to the radius through the point of contact.

QUIZ

(Constructions)

1. What is scale ctor?
2. How will you draw a tangent at a point of a circle?
3. How will you locate the centre of a circle, if it is not given?
4. How many tangents can be drawn from a point outside the circle?
5. Is it possible to draw a tangent from a point inside a circle?

ORAL TEST

1. To divide a line segment AB in the ratio m:n (m, n are positive integers), draw a ray AX so that $\angle$ BAX is an acute angle and then mark point on ray AX at equal distances such that the minimum number of these points is _______.
2. To draw a pair of tangents to a circle which are inclined to each other at an angle of 45^o, it is required to draw tangents at the end point of those two radii of the circle, the angle between which is _____.
3. To divide a line segment AB in the ration 4:5, a ray AX is drawn first such that $\angle$BAX is an acute angle and them points A1, A2, A3… are located at equal distance on the ray AX and the point B is joined to ________.
4. To construct a triangle similar to a given ΔABC with its sides 3/5 of the corresponding sides of ΔABC, first draw a ray BX such that $\angle$CBX is an acute angle and X lies on the opposite side of A with respect to BC. To locate points B1, B2, B3, ____ on BX at equal distances and next step is to join _______ to ________.
5. State 'True' or 'lse'
    a. By geometrical construction, it is possible to divide a line segment in the ratio 3+√5: 3-√5.
    b. A pair of tangents can be drawn from a point P to a circle of radius 4.5 cm situated at a distance of 4 cm from the centre.
    c. By geometrical construction, it is possible to divide a line segment in the ratio √5 : 1/√5.
    d. A pair of tangents can be constructed to a circle inclined at an angle of 175^o.
    e. From a point P outside the circle we can draw only one tangent.
    f. We cannot locate the centre of a circle if it is not given.

QUIZ

(AREAS RELATED TO CIRCLES)

1. What is circumference of a circle? Give its formula.
2. Name the great Indian mathematician who gave an approximate value of π.
3. Give the formula for the area of a circle of radius r cm.
4. Give the formula for area of a sector of a circle having radius r and measuring an angle θ at the centre.
5. How will you find the area of a segment of a circle?

ORAL TEST

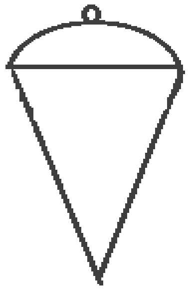
1. If the area of a circle is 154 cm^2, then its perimeter is ________.
2. Area of the largest triangle that can be inscribed in a semicircle of radius r is ________.
3. The diameter of a circle whose area is equal to the sum of the areas of the two circles of radii 24 cm and 7 cm is _____.
4. If the areas of two circles are equal, then their circumferences are ________.
5. The circles which have the me centre are called ________ circles.

QUIZ
(SURCE AREAS AND VOLUMES)

1. A cone of height 24cm and radius of base 6cm is made up of modeling clay. A child reshapes it in the form of a sphere. Find the radius of the sphere.
2. A shuttle cork used for playing badminton has the shape of the combination of which basic solids?
3. What is a frustum of a right circular cone?
4. Does a frustum has two circular ends with equal radii?
5. Give the formula for the volume of the frustum of a cone.

ORAL TEST

1. A plumbline(hul) shown in the figure is the combination of a ________ and a cone.



2. If the radii of the circular ends of a conical bucket which is 45cm high, are 28cm and 7cm then the capacity of the bucket is _______ cm^3.
3. The volume of the solid formed by joining two basic solids will actually be the _____ of the volumes of the constituents.
4. The curved surce area of the frustum of a cone is _________, where l=$\sqrt{h^2 + (r_1 - r_2)^2}$
5. If two cubes each of volumes 64cm3 are joined end to end then the surce area of the resulting cuboid is ________.

QUIZ
(PROBABILITY)

1. Define the theoretical probability of an event E.
2. What is the probability of a sure event?
3. What is an elementary event?
4. What are complementary events?
5. One card is drawn from a well shuffled deck of 52 cards. Calculate the probability that the card will be a king.

ORAL TEST

1. The probability of an impossible event is ________.
2. The probability of an event lies between ________ and __________.
3. The sum of the probabilities of all the elementary events of an experiment is ________.
4. A die is thrown once, the probability of getting a prime number is ________.
5. Two coins are tossed simultaneously. The probability of at most one tail is ______.

PUZZLES

1. Catching Fish
   If Five fishermen catch 5 fishes in 5 minutes, how long will it take fifty fishermen to catch fifty fish?

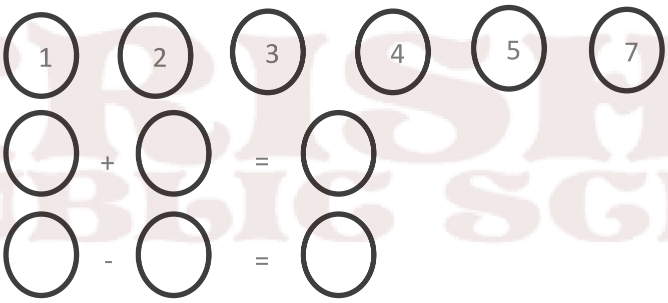
2. Look at the Division
   One day professor Agarwal went to the blackboard and demonstrated to his astonished class that one half of eight was equal to three! What did the professor do?

3. How Big
   Can you guess how big the number : ninth power nine?

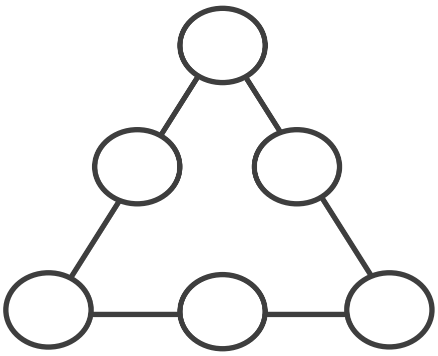
4. Counting Street Lights
   On two sides of a street, there are 35 street lights, each one is at a distance of 30 metres from the other. The street lights on one side are arranged so that each lamp fills a gap between the two other street lights on the opposite. How long is the street?

5. Who covered more distance
   Two friends Vijay and Ajay walk with constant speed of 100m/min. Vijay takes rest for 1 min after walking 100metres while Ajay takes rest for 3 min after walking 300 metres on a square path of side 400m. Both of them start from the me corner in opposite direction. Who covered more distance and when they meet?

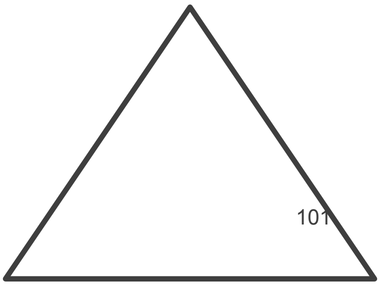
6. The missing Six
   Place the six numbers below into empty circles, so that both the equation are true. Use each number once and only once.



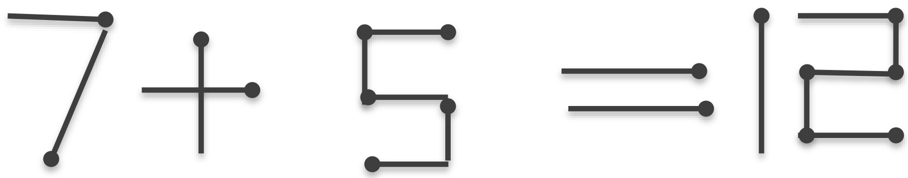
7. Magic Triangle
   Place the numbers 4 through 9 in the circles in such a way that every side of the triangle add up to 21.



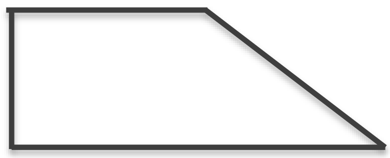
8. Add up
   Here is an equilateral triangle. Add another equilateral triangle to it in such a way that you get five equilateral triangles.



9. Magic Sticks
   Just by moving one stick, make another equation.



10. Identical Four
    Divide the adjoining figure into four identical pieces.



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Higher Order Thinking Skills (H.O.T.S.)

"God is a child; and when he began to play, he cultivated mathematics. It is the most godly of man's games"

Good Education is defined as acquiring skills. There are many different ways to be educated and many subjects that can be studied. A good education is one that teaches a student to think.

Mathematics develops logic and skill of reasoning among students. Focus of this material is primarily to strengthen the mind to absorb the concepts and bring in the students the required self-confidence while learning the subject. Math should be learnt with interest and it is made simple and approachable.

The material is a supplement to the curriculum and arranged in a chronological manner as published in textbook. As per CBSE examination pattern Higher Order Thinking Skills questions with solutions can be found in each chapter. This will definitely cilitate students to approach examinations with ease and confidence.

And finally, let the students remember that success is 1% inspiration and 99% perspiration. Hard work can never il and will certainly help them reap rich rewards. Success will then become a habit for them.

"Seeing much, suffering much, and studying much, are the three pillars of learning"

"Learning is a treasure that will follow its owner everywhere."

CONTENTS

UNIT-1

NUMBER SYSTEMS

Numbers are intellectual witnesses that belong only to mankind.

1. If the H C F of 657 and 963 is expressible in the form of 657x + 963**x** - 15 find x. (Ans:x=22)

Ans: Using Euclid's Division Lemma

$a= bq+r$, $o \leq r < b$

$963=657\times1+306$

$657=306\times2+45$

$306=45\times6+36$

$45=36\times1+9$

$36=9\times4+0$

$\therefore$ HCF (657, 963) = 9

now $9 = 657x + 963\times(-15)$

$657x=9+963\times15$

$=9+14445$

$657x=14454$

$x=14454/657$

$\boxed{x =22}$

2. Express the GCD of 48 and 18 as a linear combination. (Ans: Not unique)

$A=bq+r$, where $o \leq r < b$

48=18x2+12

18=12x1+6

12=6x2+0

$\therefore$ HCF (18,48) = 6

now 6= 18-12x1

6= 18-(48-18x2)

6= 18-48x1+18x2

6= 18x3-48x1

6= 18x3+48x(-1)

i.e. 6= 18x +48y

$\therefore$ $\boxed{x=3 , y=-1}$

$6 = 18\times3 + 48\times(-1)$
$= 18\times3 + 48\times(-1) + 18\times48 - 18\times48$
$= 18(3+48) + 48(-1-18)$
$= 18\times51 + 48\times(-19)$
$6 = 18x + 48y$

$\therefore \quad \boxed{x = 51, \ y = -19}$

Hence, x and y are not unique.

3. Prove that one of every three consecutive integers is divisible by 3.

Ans:
n,n+1,n+2 be three consecutive positive integers
We know that n is of the form 3q, 3q +1, 3q + 2
So we have the following cases

Case – I when n = 3q

In the this case, n is divisible by 3 but n + 1 and n + 2 are not divisible by 3

Case - II When n = 3q + 1
Sub n = 2 = 3q +1 +2 = 3(q +1) is divisible by 3. but n and n+1 are not divisible by 3

Case – III When n = 3q +2
Sub n = 2 = 3q +1 +2 = 3(q +1) is divisible by 3. but n and n+1 are not divisible by 3

Hence one of n, n + 1 and n + 2 is divisible by 3

4. Find the largest possible positive integer that will divide 398, 436, and 542 leaving remainder 7, 11, 15 respectively.
(Ans: 17)

Ans: The required number is the HCF of the numbers

Find the HCF of 391, 425 and 527 by Euclid's algorithm

$\therefore$ HCF (425, 391) = 17

Now we have to find the HCF of 17 and 527
527 = 17 x 31 +0

$\therefore$ HCF (17,527) = 17

$\therefore$ HCF (391, 425 and 527) = 17

5. Find the least number that is divisible by all numbers between 1 and 10 (both inclusive).

(Ans:2520)

Ans: The required number is the LCM of 1,2,3,4,5,6,7,8,9,10

$\therefore$ LCM $= 2 \times 2 \times 3 \times 2 \times 3 \times 5 \times 7 = 2520$

6. Show that 571 is a prime number.

Ans: Let $x=571 \Rightarrow \sqrt{x}=\sqrt{571}$

Now 571 lies between the perfect squares of $(23)^2$ and $(24)^2$
Prime numbers less than 24 are 2,3,5,7,11,13,17,19,23
Since 571 is not divisible by any of the above numbers
571 is a prime number

7. If d is the HCF of 30, 72, find the value of x & y tisfying d = 30x + 72y.

(Ans:5, -2 (Not unique)

Ans: Using Euclid's algorithm, the HCF (30, 72)

$72 = 30 \times 2 + 12$
$30 = 12 \times 2 + 6$
$12 = 6 \times 2 + 0$

HCF (30,72) = 6
$6=30-12\times2$
$6=30-(72-30\times2)2$
$6=30-2\times72+30\times4$
$6=30\times5+72\times-2$
$\therefore$ x = 5, y = -2

Also $6 = 30 \times 5 + 72 (-2) + 30 \times 72 - 30 \times 72$

Solve it, to get

x = 77, y = -32

Hence, x and y are not unique

8. Show that the product of 3 consecutive positive integers is divisible by 6.

Ans: Proceed as in question sum no. 3

9. Show that for odd positive integer to be a perfect square, it should be of the form 8k +1.

Let a=2m+1

Ans: Squaring both sides we get

$a^2 = 4m\ (m + 1) + 1$

$\therefore$ product of two consecutive numbers is always even

$m(m+1)=2k$

$a^2=4(2k)+1$

$a^2 = 8\ k + 1$

Hence proved

10. Find the greatest number of 6 digits exactly divisible by 24, 15 and 36. (Ans:999720)

Ans: LCM of 24, 15, 36

LCM = $3 \times 2 \times 2 \times 2 \times 3 \times 5 = 360$

Now, the greatest six digit number is 999999

Divide 999999 by 360

$\therefore$ Q = 2777 , R = 279

$\therefore$ the required number = 999999 – 279 = 999720

11. If a and b are positive integers. Show that √2 always lies between $\frac{a}{b}\ and\ \frac{a-2b}{a+b}$

Ans: We do not know whether $\frac{a^2-2b^2}{b(a+b)}$ or $\frac{a}{b} < \frac{a+2b}{a+b}$

$\therefore$ to compare these two number,

Let us comute $\frac{a}{b} - \frac{a+2b}{a+b}$

=> on simplifying , we get $\frac{a^2-2b^2}{b(a+b)}$

$\therefore\ \frac{a}{b}-\frac{a+2b}{a+b}>0$ or $\frac{a}{b}-\frac{a+2b}{a+b}<0$

now $\frac{a}{b}-\frac{a+2b}{a+b}>0$

$\frac{a^2-2b^2}{b(a+b)}>0$ solve it , we get , a > √2b

Thus , when a > √2b and
$\frac{a}{b}<\frac{a+2b}{a+b}$,

We have to prove that $\frac{a+2b}{a+b}<\sqrt{2}<\frac{a}{b}$

Now a $>\sqrt{2}$ b$\Rightarrow 2a^2+2b^2>2b^2+a^2+2b^2$
On simplifying we get

$\sqrt{2}>\frac{a+2b}{a+b}$
Also a>√2
$\Rightarrow\frac{a}{b}>\sqrt{2}$
Similarly we get √2, $<\frac{a+2b}{a+b}$
Hence $\frac{a}{b}<\sqrt{2}<\frac{a+2b}{a+b}$

12. Prove that ($\sqrt{n-1}+\sqrt{n+1}$) is irrational, for every n∈N

Self Practice

UNIT-2

POLYNOMIALS

It is not once nor twice but times without number that the me ideas make their appearance in the world.

1. Find the value for K for which $x^4 + 10x^3 + 25x^2 + 15x + K$ exactly divisible by x + 7.

(Ans : K= - 91)

Ans: Let $P(x) = x^4 + 10x^4 + 25x^2 + 15x + K$ and $g(x) = x + 7$

Since P(x) exactly divisible by g(x)

$\therefore$ r (x) = 0

now

$$
\begin{array}{r|l}
 & x^3+3x^2+4x-13 \\
x+7 & x^4+10x^3+25x^2+15x+K \\
 & x^4+7x^3 \\
 & \text{-------------} \\
 & 3x^3+25x^2 \\
 & 3x^3+21x^2 \\
 & \text{--------------------} \\
 & 4x^2+15x \\
 & 4x^2+28x \\
 & \text{------------------} \\
 & -13x+K \\
 & -13x-91 \\
 & \text{----------------} \\
 & K+91 \\
 & \text{------------}
\end{array}
$$

$\therefore$ K + 91 = 0

$\boxed{K = -91}$

2. If two zeros of the polynomial $f(x) = x^4 - 6x^3 - 26x^2 + 138x - 35$ are $2 \pm \sqrt{3}$.Find the other zeros. (Ans:7, -5)

Ans: Let the two zeros are $2 + \sqrt{3}$ and $2 - \sqrt{3}$

Sum of Zeros $= 2 + \sqrt{3} + 2 - \sqrt{3}$

$= 4$

Product of Zeros $= (2+\sqrt{3})(2-\sqrt{3})$

$= 4 - 3$

$= 1$

Quadratic polynomial is x^2 – (sum) x + Product

$$
\begin{array}{r l}
 & x^2 - 2x - 35 \\
x^2 - 4x + 1 \enclose{longdiv}{} & x^4 - 6x^3 - 26x^2 + 138x - 35 \\
 & x^4 - 4x^3 + x^2 \\
 & \overline{\quad -2x^3 - 27x^2 + 138x} \\
 & \quad -2x^3 + 8x^2 - 2x \\
 & \overline{\qquad -35x^2 + 140x - 35} \\
 & \qquad -35x^2 + 140x - 35 \\
 & \overline{\qquad\qquad 0} \\
\end{array}
$$

$\therefore\ x^2 - 2x - 35 = 0$

$(x - 7)(x + 5) = 0$

$x = 7, -5$

$\boxed{\text{other two Zeros are 7 and -5}}$

3. Find the Quadratic polynomial whose sum and product of zeros are $\sqrt{2} + 1$, $\dfrac{1}{\sqrt{2}+1}$.

Ans: sum = $2\sqrt{2}$

Product = 1

Q.P =

X^2 – (sum) x + Product

$\therefore\ x^2 - (2\sqrt{2})\,x + 1$

4. If α,β are the zeros of the polynomial $2x^2 - 4x + 5$ find the value of a) $\alpha^2 + \beta^2$ b) $(\alpha - \beta)^2$.

(Ans: a) -1 , b) –6)

Ans: $p(x) = 2x^2 - 4x + 5$

$\alpha + \beta = \dfrac{-b}{a} = \dfrac{4}{2} = 2$

$\alpha\beta = \dfrac{c}{a} = \dfrac{5}{2}$

$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

Substitute then we get, $\alpha^2 + \beta^2 = -1$

$(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$

Substitute, we get $= (\alpha - \beta)^2 = -6$

5. If α,β are the zeros of the polynomial $x^2 + 8x + 6$ frame a Quadratic polynomial whose zeros are a) $\frac{1}{\alpha}$ and $\frac{1}{\beta}$ b) $1 + \frac{\beta}{\alpha}$, $1 + \frac{\alpha}{\beta}$.

(Ans: $x^2+\frac{4}{3}x + \frac{1}{6}$, $x^2-\frac{32}{3}x + \frac{32}{3}$)

Ans: $p(x) = x^2 + 8x + 6$

$\alpha + \beta = -8$ and $\alpha\beta = 6$

a) Let two zeros are $\frac{1}{\alpha}$ and $\frac{1}{\beta}$

Sum $= \frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha+\beta}{\alpha.\beta} = \frac{-8}{6} = \frac{-4}{3}$

Product $= \frac{1}{\alpha} \times \frac{1}{\beta} = \frac{1}{\alpha.\beta} = \frac{1}{6}$

Required Q.P is

$$x^2 + \frac{4}{3}x + \frac{1}{6}$$

b) Let two Zeros are $1+\frac{\beta}{\alpha}$ and $1 + \frac{\alpha}{\beta}$

sum $= 1+\frac{\beta}{\alpha}+1+\frac{\alpha}{\beta}$

$= 2 + \frac{\alpha}{\beta} + \frac{\beta}{\alpha}$

$= 2+\frac{\alpha^2+\beta^2}{\alpha\beta}$

$= 2+\frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta}$ after solving this problem,

We get $= \frac{32}{3}$

Product $= (1+\frac{\beta}{\alpha})(1+\frac{\alpha}{\beta})$

$= 1+\frac{\alpha}{\beta}+\frac{\beta}{\alpha}+1$

$= 2+\frac{\alpha^2+\beta^2}{\alpha\beta}$

Substitute this sum,

We get = $\frac{32}{3}$

Required Q.P. is $x^2 - \frac{32}{3}x + \frac{32}{3}$

6. On dividing the polynomial $4x^4 - 5x^3 - 39x^2 - 46x - 2$ by the polynomial g(x) the quotient is $x^2 - 3x - 5$ and the remainder is $-5x + 8$.Find the polynomial g(x). (Ans:$4x^2+7x+2$)

Ans: $p(x) = g(x)\,q(x) + r(x)$

$g(x) = \frac{p(x) - r(x)}{q(x)}$

let $p(x) = 4x^4 - 5x^3 - 39x^2 - 46x - 2$

$q(x) = x^2 - 3x - 5$ and $r(x) = -5x + 8$

now $p(x) - r(x) = 4x^4 - 5x^3 - 39x^2 - 41x - 10$

when $\frac{p(x) - r(x)}{q(x)} = 4x^2 + 7x + 2$

$\therefore g(x) = 4x^2 + 7x + 2$

7. If the squared difference of the zeros of the quadratic polynomial $x2 + px + 45$ is equal to 144 , find the value of p. (Ans: $\pm$ 18).

Ans: Let two zeros are α and β where $\alpha > \beta$

According given condition

$(\alpha - \beta)^2 = 144$

Let $p(x) = x^2 + px + 45$

$\alpha + \beta = \frac{-b}{a} = \frac{-p}{1} = -p$

$\alpha\beta = \frac{c}{a} = \frac{45}{1} = 45$

now $(\alpha - \beta)^2 = 144$

$(\alpha + \beta)^2 - 4\alpha\beta = 144$

$(-p)^2 - 4(45) = 144$

Solving this we get $p = \pm 18$

8. If α,β are the zeros of a Quadratic polynomial such that $\alpha + \beta = 24$, $\alpha - \beta = 8$. Find a Quadratic polynomial having α and β as its zeros. (Ans: $k(x^2 - 24x + 128)$)

Ans: $\alpha+\beta = 24$

$\alpha - \beta = 8$

$2\alpha = 32$

$\alpha = \dfrac{32}{2} = 16, \therefore \alpha = 16$

Work the me way to $\alpha+\beta = 24$

So, $\beta = 8$

Q.P is $x^2 - (\text{sum})\ x + \text{product}$
$= x^2 - (16+8)\ x + 16 \times 8$
Solve this,
it is $k\ (x^2 - 24x + 128)$

9. If α & $ß$ are the zeroes of the polynomial $2x^2 - 4x + 5$, then find the value of
   a. $\alpha^2 + ß^2$ b. $1/\alpha + 1/ß$ c. $(\alpha - ß)^2$ d. $1/\alpha^2 + 1/ß^2$ e. $\alpha^3 + ß^3$

(Ans:-1, $\frac{4}{5}$,-6, $\frac{-4}{25}$,-7)

Ans: Let $p(x) = 2x^2 - 4x + 5$

$\alpha+\beta = \dfrac{-b}{a} = \dfrac{4}{2} = 2$

$\alpha\beta = \dfrac{c}{a} = \dfrac{5}{2}$

a) $\alpha^2+\beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$
Substitute to get $= \alpha^2+\beta^2 = -1$

b) $\dfrac{1}{a} + \dfrac{1}{\beta} = \dfrac{\alpha+\beta}{\alpha\beta}$

substitute , then we get $= \dfrac{1}{a} + \dfrac{1}{\beta} = \dfrac{4}{5}$

b) $(\alpha-\beta)^2 = (\alpha+\beta)^2 - 4\alpha\beta$
Therefore we get, $(\alpha-\beta)^2 = -6$

d) $\dfrac{1}{\alpha^2} + \dfrac{1}{\beta^2} = \dfrac{\alpha^2+\beta^2}{\alpha\beta^2} = \dfrac{-1}{\left(\dfrac{5}{2}\right)^2}$

$\therefore \dfrac{1}{\alpha^2} + \dfrac{1}{\beta^2} = \dfrac{-4}{25}$

e) $\alpha^3+\beta^3 = (\alpha+\beta)(\alpha^2+\beta^2 - \alpha\beta)$
Substitute this,

to get, $\alpha^3+\beta^3 = -7$

10. Obtain all the zeros of the polynomial p(x) = $3x^4 - 15x^3 + 17x^2 + 5x - 6$ if two zeroes are $-1/\sqrt{3}$ and $1/\sqrt{3}$. (Ans:3,2)

11. Give examples of polynomials p(x), g(x), q(x) and r(x) which tisfy the division algorithm.
a. deg p(x) = deg q(x) b. deg q(x) = deg r(x) c. deg q(x) = 0.

12. If the ratios of the polynomial $ax^3+3bx^2+3cx+d$ are in AP, Prove that $2b^3-3abc+a^2d=0$

Ans: Let $p(x) = ax^3 + 3bx^2 + 3cx + d$ and α , β , r are their three Zeros
but zero are in AP
let $\alpha = m - n$, $\beta = m$, $r = m + n$
sum = $\alpha+\beta+ r = \frac{-b}{a}$
substitute this sum , to get = $m= \frac{-b}{a}$

Now taking two zeros as sum $\alpha\beta + \beta r + \alpha r = \frac{c}{a}$

$(m-n)m + m(m+n) + (m + n)(m - n) = \frac{3c}{a}$

Solve this problem , then we get

$\frac{3b^2 - 3ac}{a^2} = n^2$

Product $\alpha\beta r = \frac{d}{a}$

$(m-n)m (m+n) = \frac{-d}{a}$

$(m^2 - n^2)m = \frac{-d}{a}$

$[(\frac{-b}{a})^2 - (\frac{3b^2 - 3ac}{a^2})] (\frac{-b}{a}) = \frac{-d}{a}$

Simplifying we get

$2b^3 - 3abc + a^2 d = 0$

13. Find the number of zeros of the polynomial from the graph given.

(Ans:1)

14. If one zero of the polynomial $3x^2 - 8x + 2k+1$ is seven times the other, find the zeros and the value of k (Ans k= 2/3)

Self Practice

14. If (n-k) is a ctor of the polynomials x^2+px+q & $x^2 + m\,x+n$. Prove that

$$k = n + \frac{n-q}{m-p}$$

Ans : since (n – k) is a ctor of $x^2 + px + q$

$\therefore\ (n-k)^2 + p(n-k) + q = 0$

And $(n-k)^2 + m(n-k) + n = 0$

Solve this problem by yourself,

$$\therefore\ k = n + \frac{n-q}{m-p}$$

<u>SELF PRACTICE</u>

16. If 2, ½ are the zeros of px^2+5x+r, prove that p= r.
17. If m, n are zeroes of ax^2-5x+c, find the value of a and c if m + n = m.n=10 (Ans: a=1/2 ,c=5)
18. What must be subtracted from $8x^4 + 14x^3 - 2x^2 + 7x - 8$ so that the resulting polynomial is exactly divisible by $4x^2+3x-2$. (Ans: 14x – 10)
19. What must be added to the polynomial $p(x)= x^4 + 2x^3 - 2x^2 + x - 1$ so that the resulting polynomial is exactly divisible by x^2+2x-3. (Ans: x-2)

UNIT-3

PAIR OF LINEAR EQUATIONS IN TWO VARIABLES

Like the crest of a peacock so is mathematics at the head of all knowledge.

1. At a certain time in a deer park, the number of heads and the number of legs of deer and human visitors were counted and it was found there were 39 heads & 132 legs. Find the number of deer and human visitors in the park.
(Ans:27,12)

Ans: Let the no. of deers be x
And no. of humans be y

ASQ :

$x + y = 39$ ---- (1)
$4x + 2y = 132$ ----- (2)

Multiply (1) and (2)

On solving, we get …

$x = 27$ and $y = 12$

$\therefore$ No. of deers = 27 and No. of humans = 12

2. Solve for x , y

a. $\dfrac{x+y-8}{2} = \dfrac{x+2y-14}{3} = \dfrac{3x+y-12}{11}$ (Ans: x=2, y=6)

Ans: $\dfrac{x+y-8}{2} = \dfrac{x+2y-14}{3} = \dfrac{3x+y-12}{11}$

$\dfrac{x+y-8}{2} = \dfrac{x+2y-14}{3}$

On solving, we will get….y= 6

$$\frac{x+y-8}{2} = \frac{x-2}{2} = \frac{x+2y-14}{3}$$

On solving , we will get….

$x = 2$

b. 7(y + 3) − 2(x + 2) = 14, 4 (y − 2) + 3(x − 3) = 2

Ans: $7(y + 3) - 2(x+ 2) = 14$ --------- (1)

$4(y-2) + 3(x - 3) = 2$ ----------(2)

From (1) $7y + 21 - 2x - 4 = 14$

On solving, we will get….

$2x - 7y - 3 = 0$ ------------- (3)

From (2) $4y - 8 + 3x - 9 = 2$

On solving, we will get….

$3x + 4y - 19 = 0$ ----------------- (4)

$2x - 7y - 3$

$3x + 4y - 19$

Substitute this, to get $y = 1$ and $x = 5$

$\therefore$ $x = 5$ and $y = 1$

c. (a+2b)x + (2a− b)y = 2, (a − 2b)x + (2a +b)y = 3

(Ans: $\frac{5b-2a}{10ab}, \frac{a+10b}{10ab}$)

Ans:

$2ax + 4ay = y$

, we get $4bx - 2by = -1$

$2ax + 4ay = 5$ $4bx - 2by = -1$

Solve this, to get $y = \frac{10b+a}{10ab}$

Similarly, we can solve for x

d. $\frac{x}{a} + \frac{y}{b} = a + b,\ \frac{x}{a^2} + \frac{y}{b^2} = 2\ ;\ a \neq 0, b \neq 0$ **(Ans: x=a²,y=B²)**

Ans: $\frac{x}{a}+\frac{y}{b} = a + b$

$$\frac{x}{a^2}+\frac{y}{b^2} = 2$$

$$\frac{xb+ya}{ab} = a + b$$

$$\frac{xb^2+ya^2}{a^2b^2} = 2$$

On solving , we get … $x= a^2$ and $y= b^2$

e. $2^x + 3^y = 17,\ 2^{x+2} - 3^{y+1} = 5$

Ans: $2^x + 3^y = 17,\ 2^{x+2} - 3^{y+1} = 5$

Let 2^x be a and 3^y be b

$2^x + 3^y = 17$

$a + b = 17$ ----(1)

$2^{x+2} - 3^{y+1} = 5$

$4a – 3b = 5$ ------(2)

on solving , we get….. a = 8

from (1)

$a + b = - 17$

$\therefore\ b = 9, a = 8$

$\Rightarrow x = 3, y = 2$

f. **If** $\frac{4x-3y}{7x-6y}$ **=** $\frac{4}{13}$**, Find** $\frac{x}{y}$ Ans: $\frac{4x-3y}{7x-6y}=\frac{4}{13}$

On dividing by *y*, we get $\frac{x}{y}=\frac{5}{8}$

g. **41x + 53y = 135, 53x +41y =147**

Ans: 41x + 53 y = 135, 53 x + 41 y = 147

Add the two equations :

Solve it, to get … x + y = 3 -------(1)

Subtract :

Solve it , to get, …. x – y = 1 --------(2)

From (1) and (2)

x + y = 3
x – y = 1

on solving , we get … x = 2 and y = 1

3. Find the value of p and q for which the system of equations represent coincident lines 2x +3y = 7, (p+q+1)x +(p+2q+2)y = 4(p+q)+1

Ans: $a_1 = 2, b_1 = 3, c_1 = 7$

$a_2 = p + q + 1$, $b_2 = p + 2q + 2$, $c_2 = (p + q) + 1$

For the following system of equation the condition must be

$$\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$$

=> $\frac{2}{p+q+1}=\frac{3}{p+q+2}=\frac{7}{4(p+q)+1}$

=> $\frac{2}{p+q+1}=\frac{7}{4(p+q)+1}$

7p +14q + 14 = 12p + 12q + 3

$= 5p - 2q - 11 = 0$ ----------------(2)

$p + q + - 5 = 0$
$5p - 2q - 11 = 0$

From (1) and (2)

$5p + 5q - 25 = 0$
$5p - 2q - 11 = 0$

Solve it, to get $q = 2$
Substitute value of q in equation (1)

$p + q - 5 = 0$

On solving we get, $p = 3$ and $q = 2$

4. Students are made to stand in rows. If one student is extra in a row there would be 2 rows less. If one student is less in a row there would be 3 rows more. Find the number of students in the class.

Ans: No. of rows be y
Let the number of students be x
Number of students in the class will be = xy

One student extra, 2 rows less

$(x + 1)(y - 2) = xy$

$xy - 2x + y - 2 = xy$
$-(-2x + y - 2) = 0$
$+2x - y = -2$ -------------- (1)

One student less, three more rows

$(x - 1)(y + 3) = xy$

$xy + 3x - y - 3 = xy$
$3x - y = 3$ ---------------------(2)

From (1) & (2)

$2x - y = -2 \text{ X } 3$
$3x - y = 3 \text{ X } -2$

Solve it , to get … y = 12 and x = 5
$\therefore$ Number of student = xy
= 12 X 5
= 60 students

5. The larger of two supplementary angles exceeds the smaller by 18^0, find them. (Ans:99^0,81^0)

Ans: $x + y = 180^0$

$x – y = 18^0$

$2x = 198$

$x = 198 / 2 = x = 99^0$

$x + y = 180^0$

$y = 180 – 99$

$y = 81^0$

6. A train covered a certain distance at a uniform speed. If the train would have been 6km/hr ster, it would have taken 4hours less than the scheduled time. And if the train were slower by 6km/hr, it would have taken 6 hours more than the scheduled time. Find the distance of the journey.

Ans: Let the speed of the train by x km/hr

And the time taken by it by y

Now distance traveled by it is x x y = xy

APQ:

I--- $(x + 6) (y - 4) = xy$

$4x – 6y = -24$

$\Rightarrow 2x – 3y = -12$ ----------------(1)

II--- $(x - 6) (y+ 6) = xy$

$6x – 6y= 36$

$\Rightarrow x– y= 6$ ----------------(2)

Solving for x and y we get $y = 24$, $x = 30$

So the distance $= 30 \times 24$

$= 720$ km

7. A chemist has one solution which is 50% acid and a second which is 25% acid. How much of each should be mixed to make 10 litres of 40% acid solution. (Ans:6L,4L)

Ans: Let 50 % acids in the solution be x

Let 25 % of other solution be y

Total Volume in the mixture = x + y

A.P.Q:

$x + y = 10$ --------(1)

A.P.Q: $\frac{50}{100}x + \frac{25}{100}y = \frac{40}{100} \times 10$

$2x + y = 16$ --------(2)

So $x = 6$ & $y = 4$

8. The length of the sides of a triangle are $2x + \frac{y}{2}$, $\frac{5x}{3} + y + \frac{1}{2}$ and $\frac{2}{3}x + 2y + \frac{5}{2}$. If the triangle is equilateral. Find its perimeter.

Ans: $2x + \frac{y}{2}$

$= \frac{4x + y}{2}$ ---------(1)

$= \frac{10x + 6y + 3}{6}$ ---------(2)

$\frac{2}{3}x + 2y + \frac{5}{2}$

$= \frac{4x + 12y + 15}{6}$ ---------(3)

APQ:

$\frac{4x + y}{2} = \frac{10x + 6y + 3}{6} = \frac{4x + 12y + 15}{6}$

$24x + 6y = 20x + 12y + 6$

$2x - 3y = 3$ ---------(4)

$\frac{4x + y}{2} = \frac{4x + 12y + 15}{6}$

$24x + 6y = 8x + 24y + 30$

Solve it,

To get $8x - 9y = 15$ -----------(5)

Solve it ,

To get $x = 3$

Substitute value of x in (4)

$2x - 3y = 3$

Solve it ,

To get $y = 1$

So the values of $x = 3$ and $y = 1$

$$2x + \frac{y}{2} = 6.5 \text{ cm}$$

Perimeter $= 6.5$ cm $+ 6.5$ cm $+ 6.5$ cm
Perimeter $= 19.5$cm

$\therefore$ the perimeter of the triangle is 19.5 cm

8. In an election contested between A and B, A obtained votes equal to twice the no. of persons on the electoral roll who did not cast their votes & this later number was equal to twice his majority over B. If there were 18000 persons on the electoral roll. How many voted for B.

Ans: Let x and y be the no. of votes for A & B respectively.

The no. of persons who did not vote $= (18000 - x - y)$

APQ:

$x = 2(18000 - x - y)$
$\Rightarrow 3x + 2y = 36000$ ---------------(1)

&

$(18000 - x - y) = (2)(x - y)$

$\Rightarrow 3x - y = 18000$ ----------------(2)

On solving we get, $y = 6000$ and $x = 8000$

Vote for B = 6000

9. When 6 boys were admitted & 6 girls left the percentage of boys increased from 60% to 75%. Find the original no. of boys and girls in the class.

Ans: Let the no. of Boys be x
Girls be y

Total $= x + y$

APQ:

$$\frac{x}{x+y}=\frac{60}{100} \text{ ---------------(1)}$$

$$\frac{x+6}{(x+6)(y-6)}=\frac{75}{100}$$

On solving we get,

x = 24   and y = 16.

10. When the son will be as old as the ther today their ages will add up to 126 years. When the ther was old as the son is today, their ages add upto 38 years. Find their present ages.

Ans: let the son's present age be x
ther's age be y

Difference in age (y – x)
Of this difference is added to the present age of son, then son will be as old as the ther now and at that time, the ther's age will be [y + (y – x)]

APQ:

[x + (y – x)] + [y (y – x)] = 126
[y + (x – y)] + [x + (x – y)] = 38

Solving we get the value of   x and y

11. A cyclist, after riding a certain distance, stopped for half an hour to repair his bicycle, after which he completes the whole journey of 30km at half speed in 5 hours. If the breakdown had occurred 10km rther off, he would have done the whole journey in 4 hours. Find where the breakdown occurred and his original speed.
(Ans: 10km, 10km/hr)

Ans*:* Let x be the place where breakdown occurred

y be the original speed

$$\frac{x}{y}+\frac{30-x}{\frac{y}{2}}=5$$

$$\frac{x+10}{y}+\frac{30-(x+10)}{\frac{y}{2}}=4$$

$$\frac{x}{y}+\frac{60-2x}{y}=5$$

On solving, we get, x = 10 km and y = 10 km/h

12. The population of the village is 5000. If in a year, the number of males were to increase by 5% and that of a female by 3% annually, the population would grow to 5202 at the end of the year. Find the number of males and females in the village.

Let the number of Males be x and females be y

Ans: $x + y = 5000$

$x+\frac{5}{100}x+y+\frac{3y}{100}=5202$ …1

$\Rightarrow 5x+3y = 20200$ …2

On solving 1 & 2 we get $x= 2600$ $y=2400$

No. of males = 2600

No. of females = 2400

UNIT-4

QUADRATIC EQUATIONS

For the things of this world cannot be made known without a knowledge of mathematics.

1. Solve by ctorization

a. $\mathbf{4x^2 - 4a^2x + (a^4 - b^4) = 0}$

Ans: $4x^2 - 4a^2x + (a^4 - b^4) = 0.$

$4x^2 - [2(a^2 + b^2) + 2(a^2 - b^2)]\,x + (a^2 - b^2)(a^2 + b^2) = 0.$

$\Rightarrow 2x[2x-(a^2 + b^2)] - (a^2 - b^2)\,[2x - (a^2 + b^2) = 0.$

$\Rightarrow x = \dfrac{a^2 + b^2}{2}\quad x = \dfrac{a^2 - b^2}{2}$

b. $\mathbf{x^2 + \left(\dfrac{a}{a+b} + \dfrac{a+b}{a}\right)x + 1 = 0}$

Ans: $x^2 + \left(\dfrac{a}{a+b} + \dfrac{a+b}{a}\right)x + 1$

$\Rightarrow x^2 + \left(\dfrac{a}{a+b}x + \dfrac{a+b}{a}x + \dfrac{a}{a+b}.\dfrac{a+b}{a}\right)$

$\Rightarrow x\left[x + \dfrac{a}{a+b}\right] + \dfrac{a+b}{a}\left[x + \dfrac{a}{a+b}\right] = 0$

$\Rightarrow x = \dfrac{-a}{a+b}\quad x = \dfrac{(-a+b)}{a}\qquad a+b \neq 0.$

c. $\dfrac{1}{a+b+x} = \dfrac{1}{a} + \dfrac{1}{b} + \dfrac{1}{x}\qquad \mathbf{a + b \neq 0}$

Ans: $\dfrac{1}{a+b+x} = \dfrac{1}{a} + \dfrac{1}{b} + \dfrac{1}{x}$

$\Rightarrow \dfrac{1}{a+b+x} - \dfrac{1}{x} = +\dfrac{1}{a} + \dfrac{1}{b}$

$\Rightarrow \dfrac{x-(a+b+x)}{x(a+b+x)} = +\dfrac{a+b}{ab}$

$\Rightarrow (a+b)\{x(a+b+x)+ab\}=0$

$\Rightarrow x(a+b+x)+ab=0$

$\Rightarrow x^2 +ax+bx+ab=0$

$\Rightarrow (x+a)(x+b)=0$

$\Rightarrow x=-a\quad x=-b$

d. $(x-3)(x-4) = \frac{34}{33^2}$

Ans : $(x\text{-}3)(x\text{-}4) = \frac{34}{33^2}$

$\Rightarrow x^2\text{-}7x+12 = \frac{34}{33^2}$

$x^2\text{-}7x + \frac{13034}{33^2} = 0$

$x^2\text{-}7x + \frac{98}{33} x \frac{133}{33} = 0$

$x^2\text{-}\frac{231}{33}x + \frac{98}{33} x \frac{133}{33} = 0$

$x^2\text{-}\left(\frac{98}{33} + \frac{133}{33}\right)x + \frac{98}{33} x \frac{133}{33} = 0$

$\Rightarrow \left(x - \frac{98}{33}\right)\left(x - \frac{133}{33}\right) = 0$

$\Rightarrow x = \frac{98}{33}$ or $x = \frac{133}{33}$

e. $\mathbf{x} = \dfrac{1}{2 - \dfrac{1}{2 - \dfrac{1}{2-x}}}$ $\mathbf{x \neq 2}$

Ans: $x = \dfrac{2}{2 - \dfrac{1}{2 - \dfrac{1}{2-x}}}$ $x \neq 2$

$x = \dfrac{1}{2 - \dfrac{1}{2 - \dfrac{1}{2-x}}}$

$x = \dfrac{1}{2 - \dfrac{1}{2 - \dfrac{(2-x)}{4-2x-1}}}$

$$x = \frac{1}{2 - \frac{2-x}{3-2x}}$$

$$\Rightarrow \; x = \frac{3-2x}{2(3-2x)-(2-x)}$$

$$\Rightarrow \; x = \frac{3-2x}{4-3x}$$

$\Rightarrow 4x - 3x^2 = 3 - 2x$

$\Rightarrow 3x^2 - 6x + 3 = 0$

$\Rightarrow (x-1)^2 = 0$

$x = 1, 1.$

2. By the method of completion of squares show that the equation $4x^2+3x+5=0$ has no real roots.

Ans: $4x^2+3x+5=0$

$$\Rightarrow x^2 + \frac{3}{4}x + \frac{5}{4} = 0$$

$$\Rightarrow x^2 + \frac{3}{4}x + \left(\frac{3}{8}\right)^2 = \frac{-5}{4} + \frac{9}{64}$$

$$\Rightarrow \left(x + \frac{3}{8}\right)^2 = \frac{-71}{64}$$

$$\Rightarrow \left(x + \frac{3}{8}\right)^2 = \frac{-71}{64}$$

$$\Rightarrow x + \frac{3}{8} = \sqrt{\frac{-71}{64}}$$ not a real no.

Hence QE has no real roots.

3. The sum of areas of two squares is 468m^2 If the difference of their perimeters is 24cm, find the sides of the two squares.

Ans: Let the side of the larger square be x.
Let the side of the smaller square be y.

APQ $x^2+y^2 = 468$

Cond. II $4x-4y = 24$
$\Rightarrow x - y = 6$

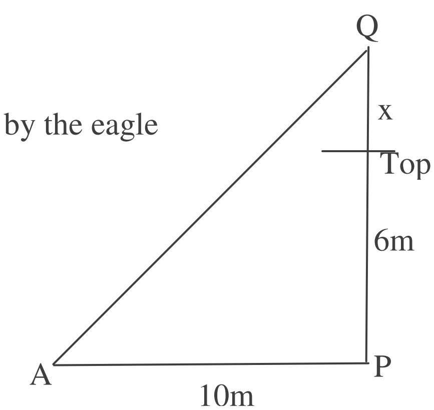
$\Rightarrow x = 6 + y$
$x^2 + y^2 = 468$
$\Rightarrow (6+y)^2 + y^2 = 468$
on solving we get $y = 12$
$\Rightarrow x = (12+6) = 18$ m
$\therefore$ sides are 18m & 12m.

4. A dealer sells a toy for Rs.24 and gains as much percent as the cost price of the toy. Find the cost price of the toy.

Ans: Let the C.P be x
$\therefore$ Gain = x%
$\Rightarrow$ Gain = $x.\dfrac{x}{100}$
S.P = C.P +Gain
SP = 24

$\Rightarrow x + \dfrac{x^2}{100} = 24$
On solving x=20 or -120 (rej)
$\therefore$ C.P of toy = Rs.20

5. A fox and an eagle lived at the top of a cliff of height 6m, whose base was at a distance of 10m from a point A on the ground. The fox descends the cliff and went straight to the point A. The eagle flew vertically up to a height x metres and then flew in a straight line to a point A, the distance traveled by each being the me. Find the value of x.



Ans: Distance traveled by the fox = distance traveled by the eagle
$(6+x)^2 + (10)^2 = (16 - x)^2$
on solving we get
$x = 2.72$m.

6. A lotus is 2m above the water in a pond. Due to wind the lotus slides on the side and only the stem completely submerges in the water at a distance of 10m from the original position. Find the depth of water in the pond.

Ans: $(x+2)^2 = x^2 + 10^2$

$x^2 + 4x + 4 = x^2 + 100$

$\Rightarrow 4x + 4 = 100$

$\Rightarrow x = 24$

Depth of the pond = 24m

7 Solve x = $\sqrt{6+\sqrt{6+\sqrt{6}}}$.........

Ans: $x = \sqrt{6+\sqrt{6+\sqrt{6+}}}$

$\Rightarrow x = \sqrt{6+x}$

$\Rightarrow x^2 = 6 + x$

$\Rightarrow x^2 - x - 6 = 0$

$\Rightarrow (x - 3)\,(x + 2) = 0$

$\Rightarrow x = 3$

8. The hypotenuse of a right triangle is 20m. If the difference between the length of the other sides is 4m. Find the sides.

Ans: APQ

$x^2 + y^2 = 20^2$

$x^2 + y^2 = 400$

also $x - y = 4$

$\Rightarrow x = 404 + y$

$(4 + y)^2 + y^2 = 400$

$\Rightarrow 2y^2 + 8y - 384 = 0$

$\Rightarrow (y + 16)\,(y - 12) = 0$

$\Rightarrow y = 12 \qquad y = -16$ (N.P)

$\therefore$ sides are 12cm & 16cm

9. The positive value of k for which $x^2 + Kx + 64 = 0$ & $x^2 - 8x + k = 0$ will have real roots .

Ans: $x^2 + Kx + 64 = 0$

$\Rightarrow b^2 - 4ac \geq 0$

$K^2 - 256 \geq 0$

$K \geq 16$ or $K \leq -16$ ……………(1)

$x^2 - 8x + K = 0$

$64 - 4K \geq 0$

$\Rightarrow 4K \le 64$

$K \le 16$ ……………(2)

From (1) & (2) $K = 16$

10. A teacher on attempting to arrange the students for mass drill in the form of a solid square found that 24 students were left over. When he increased the size of the square by one student he found he was short of 25 students. Find the number of students.

Ans: Let the side of the square be x.

No. of students = $x^2 + 24$

New side = $x + 1$

No. of students = $(x + 1)^2 - 25$

APQ $\Rightarrow x^2 + 24 = (x + 1)^2 - 25$

$\Rightarrow x^2 + 24 = x^2 + 2x + 1 - 25$

$\Rightarrow 2x = 48$

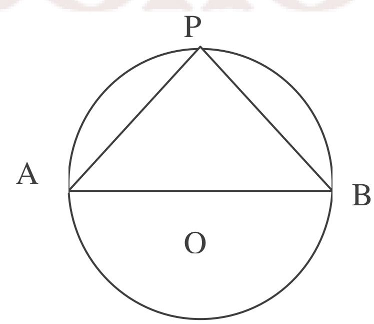
$\Rightarrow x = 24$

$\therefore$ side of square = 24

No. of students = 576 + 24

= 600

11. A pole has to be erected at a point on the boundary of a circular park of diameter 13m in such a way that the differences of its distances from two diametrically opposite fixed gates A & B on the boundary in 7m. Is it possible to do so? If answer is yes at what distances from the two gates should the pole be erected.



Ans: AB = 13 m

BP = x

$\Rightarrow$ AP – BP = 7

$\Rightarrow$ AP = $x + 7$

APQ

$\Rightarrow (13)^2 = (x + 7)^2 + x^2$

$\Rightarrow x^2 + 7x - 60 = 0$

$(x + 12)(x - 5) = 0$

$\Rightarrow x = -12$ N.P

$x = 5$

$\therefore$ Pole has to be erected at a distance of 5m from gate B & 12m from gate A.

12. If the roots of the equation $(a-b)x^2 + (b-c)x + (c - a) = 0$ are equal. Prove that 2a=b+c.

Ans: $(a-b)x^2 + (b-c)x + (c - a) = 0$

T.P $2a = b + c$

$B^2 - 4AC = 0$

$(b-c)^2 - [4(a-b)(c - a)] = 0$

$b^2 - 2bc + c^2 - [4(ac - a^2 - bc + ab)] = 0$

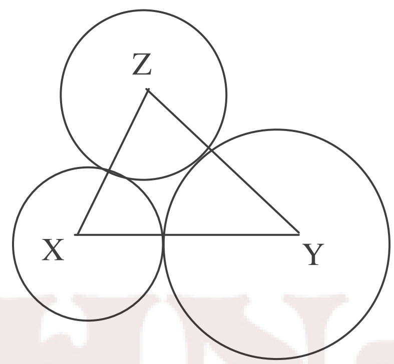
$\Rightarrow b^2 - 2bc + c^2 - 4ac + 4a^2 + 4bc - 4ab = 0$
$\Rightarrow b^2 + 2bc + c^2 + 4a^2 - 4ac - 4ab = 0$
$\Rightarrow (b + c - 2a)^2 = 0$
$\Rightarrow b + c = 2a$

13. X and Y are centers of circles of radius 9cm and 2cm and XY = 17cm. Z is the centre of a circle of radius 4 cm, which touches the above circles externally. Given that $\angle XZY = 90^o$, write an equation in r and solve it for r.

Ans: Let r be the radius of the third circle

XY = 17cm $\Rightarrow$ XZ = 9 + r YZ = 2

APQ
$(r + 9)^2 + (r + 2)^2 = (1 + r)^2$
$\Rightarrow r^2 + 18r + 81 + r^2 + 4r + 4 = 289S$
$\Rightarrow r^2 + 11r - 10r = 0$
$(r + 17)(r - 6) = 0$
$\Rightarrow r = -17$ (N.P)
$r = 6$ cm
$\therefore$ radius = 6cm.



UNIT-5

ARITHMETIC PROGRESSIONS

One of the endlessly alluring aspects of mathematics is that its thorniest paradoxes have a way of blooming into beautiful theories

1. The fourth term of an AP is 0. Prove that its 25^{th} term is triple its 11^{th} term.

Ans: $a_4 = 0$

$\Rightarrow a + 3d = 0$

T.P $a_{25} = 3\,(a_{11})$

$\Rightarrow a + 24d = 3\,(a + 10d)$

$\Rightarrow a + 24d = 3a + 30d$

RHS sub $a = -3d$

$-3d + 24d = 21d$

LHS $3a + 30d$

$-9d + 30d = 21d$

LHS = RHS

Hence proved

2. Find the 20^{th} term from the end of the AP 3, 8, 13……..253.

Ans: 3, 8, 13 ………….. 253

Last term = 253

a_{20} from end

$= l - (n-1)d$

$253 - (20-1)\,5$

$253 - 95$

$= 158$

3. If the p^{th}, q^{th} & r^{th} term of an AP is x, y and z respectively, show that $x(q-r) + y(r-p) + z(p-q) = 0$

Ans: p^{th} term $\Rightarrow x = A + (p-1)\,D$

q^{th} term $\Rightarrow y = A + (q-1)\,D$

r^{th} term $\Rightarrow z = A + (r-1)\,D$

T.P $x(q-r) + y(r-p) + z(p-q) = 0$

$= \{A+(p-1)D\}(q-r) + \{A + (q-1)D\}\,(r-p) + \{A+(r-1)D\}\,(p-q)$

$A\,\{(q-r) + (r-p) + (p-q)\} + D\,\{(p-1)(q-r) + (r-1)\,(r-p) + (r-1)\,(p-q)\}$

$\Rightarrow A.0 + D\{p(q-r) + q(r-p) + r\,(p-q) - (q-r) - (r-p) - (p-q)\}$

$= A.0 + D.0 = 0.$

Hence proved

4. Find the sum of first 40 positive integers divisible by 6 also find the sum of first 20 positive integers divisible by 5 or 6.

Ans: No's which are divisible by 6 are
6, 12 ……………. 240.
$S_{40} = \frac{40}{2}[6+240]$
= 20 x 246
= 4920
No's div by 5 or 6
30, 60 …………. 600
$\frac{20}{2}[30+600]$= 10 x 630
= 6300

5. A man arranges to pay a debt of Rs.3600 in 40 monthly instalments which are in a AP. When 30 instalments are paid he dies leaving one third of the debt unpaid. Find the value of the first instalment.

Ans: Let the value of I instalment be x S_{40} = 3600.
$\Rightarrow \frac{40}{2}[2a+39d]$=3600
$\Rightarrow$2a + 39d = 180 - 1
$S_{30} = \frac{30}{2}[2a+29d]$=2400
$\Rightarrow$30a + 435d = 2400
$\Rightarrow$2a + 29d = 160 - 2
Solve 1 & 2 to get
d = 2 a = 51.
$\therefore$ I instalment = Rs.51.

6. Find the sum of all 3 digit numbers which leave remainder 3 when divided by 5.

Ans: 103, 108……….998
a + (n-1)d = 998
$\Rightarrow$ 103 + (n-1)5 = 998
$\Rightarrow$ n = 180
$S_{180} = \frac{180}{2}[103+998]$
= 90 x 1101
S_{180} = 99090

7. Find the value of x if $2x + 1$, $x^2 + x + 1$, $3x^2 - 3x + 3$ are consecutive terms of an AP.

Ans: $a_2 - a_1 = a_3 - a_2$

$\Rightarrow x^2 + x + 1 - 2x - 1 = 3x^2 - 3x + 3 - x^2 - x - 1$

$x^2 - x = 2x^2 - 4x + 2$

$\Rightarrow x^2 - 3x + 2 = 0$

$\Rightarrow (x-1)(x-2) = 0$

$\Rightarrow x = 1$ or $x = 2$

8. Raghav buys a shop for Rs.1,20,000.He pays half the balance of the amount in cash and agrees to pay the balance in 12 annual instalments of Rs.5000 each. If the rate of interest is 12% and he pays with the instalment the interest due for the unpaid amount. Find the total cost of the shop.

Ans: Balance = Rs.60,000 in 12 instalment of Rs.5000 each.

Amount of I instalment $= 5000 + \frac{12}{100} 60{,}000$

II instalment = 5000 + (Interest on unpaid amount)

$= 5000 + 6600 \quad \left[\frac{12}{100} \text{x} 55000\right]$

= 11600

III instalment = 5000 + (Interest on unpaid amount of Rs.50,000)

$\therefore$ AP is 12200, 11600, 11000

D = is 600

Cost of shop = 60000 + [sum of 12 instalment]

$= 60{,}000 + \frac{12}{2}[24{,}400 - 6600]$

= 1,66,800

9. Prove that $a_{m+n} + a_{m-n} = 2a_m$

Ans: $a_{m+n} = a_1 + (m + n - 1)\, d$

$a_{m-n} = a_1 + (m - n - 1)\, d$

$a_m = a_1 + (m-1)\, d$

Add 1 & 2

$a_{m+n} + a_{m-n} = a_1 + (m+n-1)d + a_1 + (m-n-1)d$

$= 2a_1 + (m+n+m-n-1-1)d$

$= 2a_1 + 2(m-1)d$

$= \quad 2[a_1 + (m-1)d]$

$= \quad 2[a_1 + (m-1)d]$

$= \quad 2a_{m.}$ Hence proved.

10. If the roots of the equation $(b-c)x^2 + (c-a)x + (a-b) = 0$ are equal show that a, b, c are in AP.

Ans: Refer sum No.12 of Q.E.

If $(b-c)x^2 + (c-a)x + (a-b)x$ have equal root.

$B^2 - 4AC = 0$.

Proceed as in sum No.13 of Q.E to get c + a = 2b

$\Rightarrow$ b - a = c - b

$\Rightarrow$ a, b, c are in AP

11. Balls are arranged in rows to form an equilateral triangle .The first row consists of one ball, the second two balls and so on. If 669 more balls are added, then all the balls can be arranged in the shape of a square and each of its sides then contains 8 balls less than each side of the triangle. find the initial number of balls.

Ans: Let their be n balls in each side of the triangle

$\therefore$ No. of ball (in Δ) = 1 + 2+ 3………..= $\frac{n(n+1)}{2}$

No. of balls in each side square = n-8

No. of balls in square = $(n-8)^2$

APQ $\frac{n(n+1)}{2} + 660 = (n-8)^2$

On solving

$n^2 + n + 1320 = 2(n^2 - 16n + 64)$

$n^2 - 33n - 1210 = 0$

$\Rightarrow (n-55)(n+22) = 0$

n=-22 (N.P)

n=55

$\therefore$No. of balls = $\frac{n(n+1)}{2} = \frac{55x56}{2}$

= 1540

12. Find the sum of $(1-\frac{1}{n}) + (1-\frac{2}{n}) + (1-\frac{3}{n})$ ……. upto n terms.

Ans: $\left(1-\frac{1}{n}\right) + \left(1-\frac{2}{n}\right)$- upto n terms

$\Rightarrow$[1+1+…….+n terms] – [$\frac{1}{n} + \frac{2}{n}$ +….+ n terms]

n –[S_n up to n terms]

$S_n = \frac{n}{2}[2a + (n-1)d]$ $\quad (d = \frac{1}{n},\ a = \frac{1}{n})$

$= \frac{n}{2}\left[\frac{2}{n} + (n-1)\frac{1}{n}\right]$

$= \frac{n+1}{2}$ (on simplifying)

$n - \frac{n+1}{2} =$

$= \frac{n-1}{2}$ Ans

13. If the following terms form a AP. Find the common difference & write the next 3 terms3, 3+ √2, 3+2√2, 3+3√2……….

Ans: d= $\sqrt{2}$ next three terms $3 + 4\sqrt{\ }^{2}$, $3 + 5\sqrt{\ }^{2}$, $3 + 6\sqrt{\ }^{2}$……..

14. Find the sum of a+b, a-b, a-3b, …… to 22 terms.

Ans: a + b, a – b, a – 3b, up to 22 terms
d= a – b – a – b = 2b
$S_{22} = \frac{22}{2}$ [2(a+b)+21(-2b)]
11[2a + 2b – 42b]
= 22a – 440b Ans.

15. Write the next two terms √12, √27, √48, √75……………….

Ans: next two terms $\sqrt{108}$, $\sqrt{147}$ AP is $2\sqrt{3}$, $3\sqrt{3}$, $4\sqrt{3}$, $5\sqrt{3}$, $6\sqrt{3}$, $7\sqrt{3}$ ……

16. If the p^{th} term of an AP is q and the q^{th} term is p. P.T its n^{th} term is (p+q-n).

Ans: APQ
$a_p = q$
$a_q = p$
$a_n = ?$
a + (p-1) d = q
a + (q-1) d = p
d[p – q] = q – p Sub d = -1 to get ⇒ = -1 ⇒ a = q + p -1
a_n = a + (n – 1)d
= a + (n - 1)d
= (q + p – 1) + (n – 1) - 1
a_n = (q + p – n)

17. If $\frac{1}{x+2}, \frac{1}{x+3}, \frac{1}{x+5}$ are in AP find x.

Ans: $\frac{1}{x+2}, \frac{1}{x+3}, \frac{1}{x+5}$ are in AP find x.

$$\frac{1}{x+3} - \frac{1}{x+2} = \frac{1}{x+5} - \frac{1}{x+3}$$

$$\Rightarrow \frac{1}{x^2+5x+6} = \frac{2}{x^2+8x+15}$$

On solving we get $x = 1$

18. Find the middle term of the AP 1, 8, 15….505.

Ans: Middle terms

a + (n-1)d = 505

a + (n-1)7 = 505

$n - 1 = \frac{504}{7}$

n = 73

$\therefore$ 37^{th} term is middle term

$a_{37} = a + 36d$

$= 1 + 36(7)$

$= 1 + 252$

$= 253$

19. Find the common difference of an AP whose first term is 100 and sum of whose first 6 terms is 5 times the sum of next 6 terms.

Ans: a = 100

APQ $a_1 + a_2 + \ldots\ldots a_6 = 5\ (a_7 + \ldots\ldots + a_{12})$

$$6\left(\frac{a_1 + a_6}{2}\right) = 5 \text{ x } 6\left(\frac{a_7 + a_{12}}{2}\right)$$

$\Rightarrow$ a + a + 5d = 5[a + 6d + a + 11d]

$\Rightarrow$ 8a + 80d = 0 (a = 100)

$\Rightarrow$ d = - 10.

20. Find the sum of all natural no. between 101 & 304 which are divisible by 3 or 5. Find their sum.

Ans: No let 101 and 304, which are divisible by 3.

102, 105………….303 (68 terms)

No. which are divisible by 5 are 105, 110……300 (40 terms)

No. which are divisible by 15 (3 & 5) 105, 120…… (14 terms)

$\therefore$ There are 94 terms between 101 & 304 divisible by 3 or 5. (68 + 40 – 14)

$\therefore$ $S_{68} + S_{40} - S_{14}$

= 19035

21. The ratio of the sum of first n terms of two AP's is 7n+1:4n+27. Find the ratio of their 11[th] terms .

Ans: Let a_1, a_2… and d_1, d_2 be the I terms are Cd's of two AP's.

$$\frac{S_n \text{ of one AP}}{S_n \text{ of II AP}} = \frac{7n+1}{4n+27}$$

$$\frac{\frac{m}{2}\left[2a_1+(n-1)d_1\right]}{\frac{m}{2}\left[2a_2+(n-1)d_2\right]} = \frac{7n+1}{4n+27}$$

$$\Rightarrow \frac{2a_1+(n-1)d_1}{2a_2+(n-1)d_2} = \frac{7n+1}{4n+27}$$

We have sub. n = 21.

$$\frac{2a_1+20d_1}{2a_2+20d_2} = \frac{7x21+1}{4(21)+27}$$

$$\Rightarrow \frac{a_1+10d_1}{a_2+10d_2} = \frac{148}{111}$$

$$= \frac{4}{3}$$

$\therefore$ ratio of their 11^{th} terms = 4 :3.

22. If there are (2n+1)terms in an AP ,prove that the ratio of the sum of odd terms and the sum of even terms is (n+1):n

Ans: Let a, d be the I term & Cd of the AP.

$\therefore$ $a_k = a + (k-1)\, d$

s_1 = sum to odd terms

$s_1 = a_1 + a_3 + \ldots\ldots\ldots a_{2n+1}$

$$s_1 = \frac{n+1}{2}\left[a_1 + a_{2n+1}\right]$$

$$= \frac{n+1}{2}\left[2a_1 + 2nd\right]$$

$s_1 = (n + 1)(a + nd)$

s_2 = sum to even terms

$s_2 = a_2 + a_4 + \ldots\ldots a_{2n}$

$s_2 = \frac{n}{2}[a_2 + a_{2n}]$

$= \frac{n}{2}$ [a + d + a + (2n – 1)d]

=n [a + nd]

$\therefore\ s_1 : s_2 = \frac{(n+1)(a+nd)}{n(a+nd)}$

$= \frac{n+1}{n}$

23. Find the sum of all natural numbers amongst first one thound numbers which are neither divisible 2 or by 5

Ans: Sum of all natural numbers in first 1000 integers which are not divisible by 2 i.e. sum of odd integers.

1 + 3 + 5 + ………. + 999

n = 500

$S_{500} = \frac{500}{2}[1 + 999]$

= 2,50,000

No's which are divisible by 5

5 + 15 + 25 …….. + 995

n = 100

$S_n = \frac{100}{2}[5 + 995]$

= 50 x 1000 = 50000

$\therefore$ Required sum = 250000 – 50,000

= 200000

UNIT-6

TRIGONOMETRY

"The mathematician is scinated with the marvelous beauty of the forms he constructs, and in their beauty he finds everlasting truth."

1. If $x\cos\theta - y\sin\theta = a$, $x\sin\theta + y\cos\theta = b$, prove that $x^2+y^2=a^2+b^2$.

Ans: $x\cos\theta - y\sin\theta = a$

$x\sin\theta + y\cos\theta = b$

Squaring and adding

$x^2+y^2=a^2+b^2$.

2. Prove that $\sec^2\theta+\operatorname{cosec}^2\theta$ can never be less than 2.

Ans: S.T $\operatorname{Sec}^2\theta + \operatorname{Cosec}^2\theta$ can never be less than 2.

If possible let it be less than 2.

$1 + \operatorname{Tan}^2\theta + 1 + \operatorname{Cot}^2\theta < 2$.

$\Rightarrow \quad 2 + \operatorname{Tan}^2\theta + \operatorname{Cot}^2\theta$

$\Rightarrow \quad (\operatorname{Tan}\theta + \operatorname{Cot}\theta)^2 < 2$.

Which is not possible.

3. If $\sin\varphi = \frac{1}{2}$, show that $3\cos\varphi - 4\cos^3\varphi = 0$.

Ans: $\operatorname{Sin}\varphi = ½$

$\Rightarrow \varphi = 30^o$

Substituting in place of $\varphi = 30^o$. We get 0.

4. If $7\sin^2\varphi+3\cos^2\varphi = 4$, show that $\tan\varphi = \frac{1}{\sqrt{3}}$.

Ans: If $7\operatorname{Sin}^2\varphi + 3\operatorname{Cos}^2\varphi = 4$ S.T. $\operatorname{Tan}\varphi \frac{1}{\sqrt{3}}$

$7\operatorname{Sin}^2\varphi + 3\operatorname{Cos}^2\varphi = 4(\operatorname{Sin}^2\varphi + \operatorname{Cos}^2\varphi)$

$\Rightarrow 3\operatorname{Sin}^2\varphi = \operatorname{Cos}^2\varphi$

$\Rightarrow \frac{Sin^2\varphi}{Cos^2\varphi} = \frac{1}{3}$

$\Rightarrow \text{Tan}^2\varphi = \frac{1}{3}$

$\text{Tan}\varphi = \frac{1}{\sqrt{3}}$

5. If cosφ+sinφ = $\sqrt{2}$ cosφ, prove that cosφ - sinφ = $\sqrt{2}$ sin φ.

Ans: $\text{Cos}\varphi + \text{Sin}\varphi = \sqrt{2}\,\text{Cos}\varphi$

$\Rightarrow (\text{Cos}\varphi + \text{Sin}\varphi)^2 = 2\text{Cos}^2\varphi$

$\Rightarrow \text{Cos}^2\varphi + \text{Sin}^2\varphi + 2\text{Cos}\varphi\,\text{Sin}\varphi = 2\text{Cos}^2\varphi$

$\Rightarrow \text{Cos}^2\varphi - 2\text{Cos}\varphi\,\text{Sin}\varphi + \text{Sin}^2\varphi = 2\text{Sin}^2\varphi$ $\left[\begin{array}{l} \therefore 2\text{Sin}^2\varphi = 2 - 2\text{Cos}^2\varphi \\ 1 - \text{Cos}^2\varphi = \text{Sin}^2\varphi \ \& \ 1 - \text{Sin}^2\varphi = \end{array}\right]$

$\Rightarrow (\text{Cos}\varphi - \text{Sin}\varphi)^2 = 2\text{Sin}^2\varphi$

$\text{Cos}^2\varphi$

or $\text{Cos}\varphi - \text{Sin}\varphi = \sqrt{2}\ \text{Sin}\varphi$.

6. If tanA+sinA=m and tanA-sinA=n, show that $m^2 - n^2 = 4\sqrt{mn}$

Ans: TanA + SinA = m TanA – SinA = n.

$m^2 - n^2 = 4\sqrt{mn}$.

$m^2 - n^2 = (\text{TanA} + \text{SinA})^2 - (\text{TanA} - \text{SinA})^2$

$= 4\ \text{TanA SinA}$

RHS $4\sqrt{mn} = 4\sqrt{(TanA + SinA)(TanA - SinA)}$

$= 4\sqrt{Tan^2 A - Sin^2 A}$

$= 4\sqrt{\frac{Sin^2 A - Sin^2 A Cos^2 A}{Cos^2 A}}$

$= 4\sqrt{\frac{Sin^4 A}{Cos^2 A}}$

$= 4\,\frac{Sin^2 A}{Cos^2 A} = 4\ \text{TanA SinA}$

$\therefore m^2 - n^2 = 4\sqrt{mn}$

7. If secA= $x + \frac{1}{4x}$, prove that secA+tanA=2x or $\frac{1}{2x}$.

Ans: Secφ = x + $\frac{1}{4x}$

$\Rightarrow$ $Sec^2φ$ =(x + $\frac{1}{4x}$ $)^2$ ($Sec^2φ$= 1 + $Tan^2φ$)

$Tan^2φ$ = (x + $\frac{1}{4x}$ $)^2$-1

$Tan^2φ$ = (x - $\frac{1}{4x}$ $)^2$

Tanφ = $\pm$ x - $\frac{1}{4x}$

Secφ + Tanφ = x + $\frac{1}{4x}$ $\pm$ x - $\frac{1}{4x}$

= 2x or $\frac{1}{2x}$

8. If A, B are acute angles and sinA= cosB, then find the value of A+B.

Ans: A + B = 90^o

9. a)Solve for φ, if tan5φ = 1.

Ans: Tan 5φ = 1 $\Rightarrow$ φ = $\frac{45}{5}$ $\Rightarrow$ φ=9^o.

b)Solve for φ if $\frac{Sin\varphi}{1+Cos\varphi}+\frac{1+Cos\varphi}{Sin\varphi}=4$.

Ans: $\frac{Sin\varphi}{1+Cos\varphi}+\frac{1+Cos\varphi}{Sin\varphi}=4$

$$\frac{\mathrm{Sin}^2\varphi+1(\mathrm{Cos}\phi)^2}{\mathrm{Sin}\varphi(1+\mathrm{Cos}\varphi)}=4$$

$$\frac{\mathrm{Sin}^2\varphi+1+\mathrm{Cos}^2\varphi+2\mathrm{Cos}\varphi}{\mathrm{Sin}\varphi+\mathrm{Sin}\varphi\mathrm{Cos}\varphi}=4$$

$$\frac{2+2Cos\varphi}{Sin\varphi(1+Cos\varphi)}=4$$

$\Rightarrow \quad \frac{2+(1+Cos\varphi)}{Sin\varphi(1+Cos\varphi)} = 4$

$\Rightarrow \quad \frac{2}{Sin\varphi} = 4$

$\Rightarrow \quad$ Sinφ = $\frac{1}{2}$

$\Rightarrow \quad$ Sinφ = Sin30

φ = 30^o

10. If $\frac{cos\alpha}{cos\beta} = m$ and $\frac{cos\alpha}{sin\beta} = n, show\ that\ (m^2+n^2)cos^2\beta = n^2$

Ans: $\frac{Cos\alpha}{Cos\beta} = m \qquad \frac{Cos\alpha}{Sin\beta} = n$

$\Rightarrow m^{2=}\frac{Cos^2\alpha}{Cos^2\beta} \qquad n^{2=}\frac{Cos^2\alpha}{Sin^2\beta}$

LHS = (m^2+n^2) Cos$^2\beta$

$\left[\frac{Cos^2\alpha}{Cos^2\beta} + \frac{Cos^2\alpha}{Sin^2\beta}\right]Cos^2\beta$

$= Cos^2\alpha\left(\frac{1}{Cos^2\beta Sin^2\beta}\right)Cos^2\beta$

$= \frac{Cos^2\alpha}{Sin^2\beta} = n^2$

$\Rightarrow (m^2+n^2)\ Cos^2\beta = n^2$

11. If 7 cosecφ-3cotφ = 7, prove that 7cotφ - 3cosecφ = 3.

Ans: 7 Cosecφ-2Cotφ=7
P.T 7Cotφ - 3 Cosecφ=3
7 Cosecφ-3Cotφ=7
⇒7Cosecφ-7=3Cotφ
⇒7(Cosecφ-1)=3Cotφ

$\Rightarrow 7(\text{Cosec}\varphi-1)(\text{Cosec}\varphi+1)=3\text{Cot}\varphi(\text{Cosec}\varphi+1)$

$\Rightarrow 7(\text{Cosec}^2\varphi-1)=3\text{Cot}\varphi(\text{Cosec}\varphi+1)$

$\Rightarrow 7\text{Cot}^2\varphi=3\text{ Cot}\varphi\ (\text{Cosec}\varphi+1)$

$\Rightarrow 7\text{Cot}\varphi= 3(\text{Cosec}\varphi+1)$

$7\text{Cot}\varphi-3\text{ Cosec}\varphi=3$

12. $2(\sin^6\varphi+\cos^6\varphi) - 3(\sin^4\varphi+\cos^4\varphi)+1 = 0$

Ans: $(\text{Sin}^2\varphi)^3 + (\text{Cos}^2\varphi)^3-3\ (\text{Sin}^4\varphi+(\text{Cos}^4\varphi)+1=0$

Consider $(\text{Sin}^2\varphi)^3 +(\text{Cos}^2\varphi)^3$

$\Rightarrow(\text{Sin}^2\varphi+\text{Cos}^2\varphi)^3-3\text{ Sin}^2\varphi\text{Cos}^2\varphi(\text{ Sin}^2\varphi+\text{Cos}^2\varphi)$

$= 1- 3\text{Sin}^2\varphi\text{ Cos}^2\varphi$

$\text{Sin}^4\varphi+\text{Cos}^4\varphi(\text{Sin}^2\varphi)^2+(\text{Cos}^2\varphi)^2$

$= (\text{Sin}^2\varphi+\text{Cos}^2\varphi)^2-2\text{ Sin}^2\varphi\text{ Cos}^2\varphi$

$= 1- 2\text{ Sin}^2\varphi\text{ Cos}^2\varphi$

$= 2(\text{Sin}^6\varphi+\text{Cos}^6\varphi)-3(\text{Sin}^4\varphi+\text{Cos}^4\varphi) +1$

$= 2\ (1-3\text{ Sin}^2\varphi\text{ Cos}^2\varphi)-3\ (1-2\text{ Sin}^2\varphi+\text{Cos}^2\varphi)+1$

13. $5(\sin^8 A- \cos^8 A) = (2\sin^2 A - 1)\ (1- 2\sin^2 A\cos^2 A)$

Ans: Proceed as in Question No.12

14. If $\tan\theta = \frac{5}{6}$ & $\theta +\phi =90^o$ what is the value of $\cot\phi$.

Ans: $\text{Tan}\theta = 5/6$ i.e. $\text{Cot}\phi = 5/6$ Since $\varphi + \theta = 90^o$.

15. What is the value of $\tan\varphi$ in terms of $\sin\varphi$.

Ans: $\text{Tan }\varphi = \frac{Sin\varphi}{Cos\varphi}$

$\text{Tan }\varphi = \frac{Sin\varphi}{\sqrt{1-Sin^2\varphi}}$

16. If $\text{Sec}\varphi+\text{Tan}\varphi=4$ find $\sin\varphi$, $\cos\varphi$

Ans: $\text{Sec }\varphi + \text{Tan }\varphi = 4$

$$\frac{1}{Cos\varphi}+\frac{Sin\varphi}{Cos\varphi}=4$$

$$\frac{1+Sin\varphi}{Cos\varphi}=4$$

$$\Rightarrow \frac{(1+Sin\varphi)^2}{Cos^2\varphi} = 16$$

$\Rightarrow$ apply (C & D)

$$= \frac{(1+Sin\phi)^2 + Cos^2\phi}{(1+Sin\phi)^2 - Cos^2\phi} = \frac{16+1}{16-1}$$

$$\Rightarrow \frac{2(1+Sin\phi)}{2Sin\phi(1+Sin\phi)} = \frac{17}{15}$$

$$\Rightarrow \frac{1}{Sin\phi} = \frac{17}{15}$$

$$\Rightarrow Sin\varphi = \frac{15}{17}$$

$$Cos\varphi = \sqrt{1 - Sin^2\varphi}$$

$$\sqrt{1 - \left(\frac{15}{17}\right)^2} = \frac{8}{17}$$

17. Secφ+Tanφ=p, prove that sin$\varphi = \frac{p^2 - 1}{p^2 + 1}$

Ans: Secφ + Tanφ= P. P.T Sin$\varphi = \frac{P^2 - 1}{P^2 + 1}$

Proceed as in Question No.15

18. Prove geometrically the value of Sin 60^o

Ans: Exercise for practice.

19. If $\frac{1 - \tan\theta}{1 + \tan\theta} = \frac{\sqrt{3 - 1}}{\sqrt{3 + 1}}$,show that $\frac{\sin\theta}{\cos 2\theta}$ =1

Ans: Exercise for practice.

20. If 2x=secθ and $\frac{2}{x}$ = tanθ ,then find the value of $2\left(x^2 - \frac{1}{x^2}\right)$.

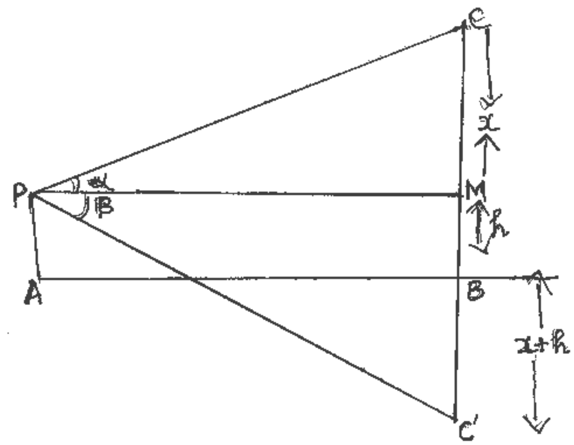
(Ans:1)

Ans: Exercise for practice.

HEIGHTS AND DISTANCES

1. If the angle of elevation of cloud from a point ‘h’ meters above a lake is α and the angle of depression of its reflection in the lake is β, prove that the height of the cloud is .

Ans :



If the angle of elevation of cloud from a point ‘n’ meters above a lake is ∝ and the angle of depression of its reflection in the lake is β, prove that the height of the cloud is $h\left(\frac{\tan\beta+\tan\alpha}{\tan\beta-\tan\alpha}\right)$

Let AB be the surce of the lake and

Let p be an point of observation such that AP = h meters. Let c be the position of the cloud and c’ be its reflection in the lake. Then $\angle CPM = \propto$ and $\angle MPC^1 = \beta$.

Let CM = x.

Then, CB = CM + MB = CM + PA = x + h

In Δ CPM, we have $\tan \propto = \frac{CM}{PM}$

$\Rightarrow \quad \tan \propto = \frac{x}{AB}$

[∴ PM = AB]

$\Rightarrow$ AB = x cot∝ ………..1

In Δ PMC’, we have

$\tan\beta = \frac{C'M}{PM}$

$\Rightarrow \quad \tan\beta = \frac{x+2h}{AB}$ [Θ C’M=C’B+BM = x + h + n]

$\Rightarrow \quad$ AB = (x + 2h) cot β

From 1 & 2

x cot ∝ = (x + 2h) cot β

x (cot ∝ - cot β) = 2h cot β (on equating the values of AB)

$$\Rightarrow x\left(\frac{1}{\tan\alpha}-\frac{1}{\tan\beta}\right)=\frac{2h}{\tan\beta} \Rightarrow x\left(\frac{\tan\beta-\tan\alpha}{\tan\alpha+\tan\beta}\right)=\frac{2h}{\tan\beta}$$

$$\Rightarrow x=\frac{2h\tan\alpha}{\tan\beta-\tan\alpha}$$

Hence, the height CB of the cloud is given by CB is given by CB = x + h

$$\Rightarrow \text{CB}=\frac{2h\tan\alpha}{\tan\beta-\tan\alpha}+\text{h}$$

$$\Rightarrow \text{CB-}\frac{2h\tan\alpha+h\tan\beta-h\tan\alpha}{\tan\beta-\tan\alpha}=\frac{h(\tan\alpha+h\tan\beta)}{\tan\beta-\tan\alpha}$$

2. From an aero plane vertically above a straight horizontal road, the angles of depression of two consecutive milestones on opposite sides of the aero plane are observed to be α and β. Show that the height of the aero plane above the road is $\frac{tan\alpha\ tan\beta}{tan\alpha+tan\beta}$.

Ans:

Let P Q be h

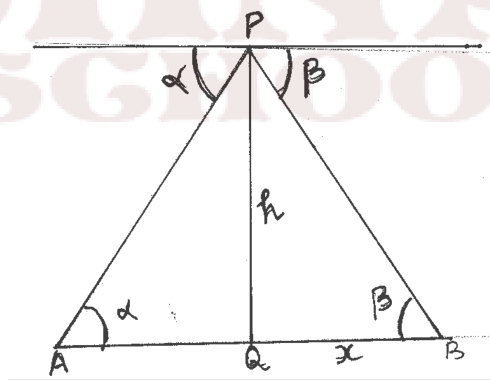
QB be x

Given : AB = 1 mile

QB = x

AQ = (1- x) mile

in ΔPAQ



Tan α = $\frac{PQ}{AQ}$

Tan α = $\frac{h}{1-x}$

$1-x=\frac{h}{Tan\alpha}$ …………1

In ΔPQB

Tan β = $\frac{h}{x}$

x = $\frac{h}{Tan\beta}$

Substitute for x in equation (1)

$1=\frac{h}{Tan\beta}+\frac{h}{Tan\alpha}$

$$1 = h\left\{\frac{1}{Tan\beta} + \frac{1}{Tan\alpha}\right\}$$

$$\frac{1}{h} = \frac{Tan\beta + Tan\alpha}{Tan\beta Tan\alpha}$$

$$\therefore h = \frac{tan\alpha\ tan\beta}{tan\alpha + tan\beta}$$

3. Two stations due south of a tower, which leans towards north are at distances 'a' and 'b' from its foot. If α and β be the elevations of the top of the tower from the situation, prove that its inclination 'θ' to the horizontal given by

$$cot\theta = \frac{bcot\alpha - acot\beta}{b-a}$$

Ans: Let AB be the leaning tower and C and D be the given stations. Draw BL ⊥ DA produced.

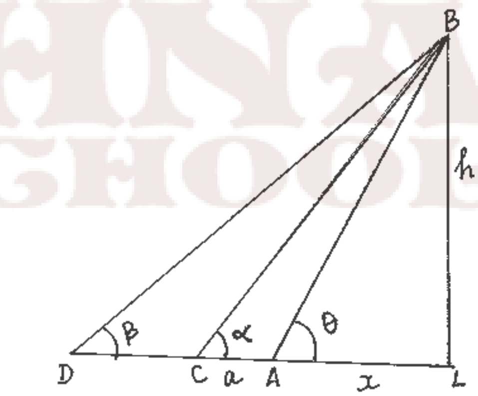
Then, ∠BAL = 0, ∠BCA = α, ∠BDC = a and DA = b.

Let AL = x and BL = h

In right ΔALB, we have :

$$\frac{AL}{BL} = \text{Cot } \theta \Rightarrow \frac{x}{h} = \text{Cot } \theta$$

$$\Rightarrow \frac{x}{h} = \text{Cot } \theta \Rightarrow x = \text{h cot } \theta \qquad \text{.....(i)}$$



In right ΔBCL, we have :

$$\frac{CL}{BL} = \text{Cot } \alpha \Rightarrow \text{a + x = h cot } \alpha$$

$$\Rightarrow \text{a = h (cot } \alpha \text{ - cot } \theta)$$

$$\Rightarrow \text{h} = \frac{a}{(\cot\alpha - \cot\theta)} \qquad \text{...(ii)}$$

In right ΔBDL, we have :

$$\frac{DL}{BL} = \cot\beta \Rightarrow \frac{DA + AL}{BL} = \cot\beta$$

$$\Rightarrow \frac{b+x}{h} = \cot\beta \Rightarrow \text{b + x = b cot } \beta$$

$$\Rightarrow \text{b = h ((cot } \beta \text{ - cot } \theta) \qquad \text{[using (i)]}$$

$$\Rightarrow \text{h} = \frac{b}{(\cot\beta - \cot\theta)} \qquad \text{..........(iii)}$$

equating the value of h in (ii) and (iii), we get:

$$\frac{a}{(\cot\alpha - \cot\theta)} = \frac{b}{(\cot\beta - \cot\theta)}$$

$\Rightarrow$ a cot β - a cot θ = b cot α - b cot θ
$\Rightarrow$ (b – a) cot θ = b cot α - a cotβ
$\Rightarrow$ cot θ = $\dfrac{b\cot\alpha - a\cot\beta\theta}{(b-a)}$

4. The angle of elevation of the top of a tower from a point on the me level as the foot of the tower is α. On advancing 'p' meters towards the foot of the tower, the angle of elevation becomes β. show that the height 'h' of the tower is given by h= $\dfrac{p(\tan\alpha\ \tan\beta)}{\tan\beta - \tan\alpha}$

5. A boy standing on a horizontal plane finds a bird flying at a distance of 100m from him at an elevation of 30^0. A girl standing on the roof of 20 meter high building finds the angle of elevation of the me bird to be 45^0. Both the boy and the girl are on opposite sides of the bird. Find the distance of the bird from the girl. (Ans: 42.42m)

Ans: In right Δ ACB

Sin 30 = $\dfrac{AC}{AB}$

$\dfrac{1}{2} = \dfrac{AC}{100}$

2 AC = 100

AC = 50m

$\Rightarrow$ AF = (50 – 20) = 30m

In right Δ AFE

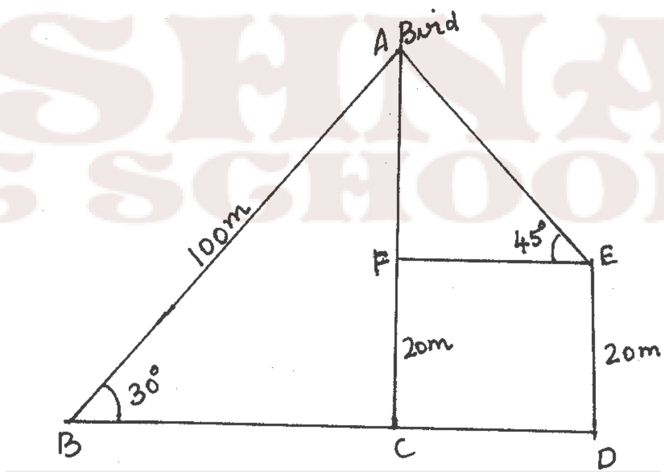
Sin 45 = $\dfrac{AF}{AE}$

$\Rightarrow \dfrac{1}{\sqrt{2}} = \dfrac{30}{AE}$

AE = 30 $\sqrt{2}$

= 30 x 1.414

= 42.42m

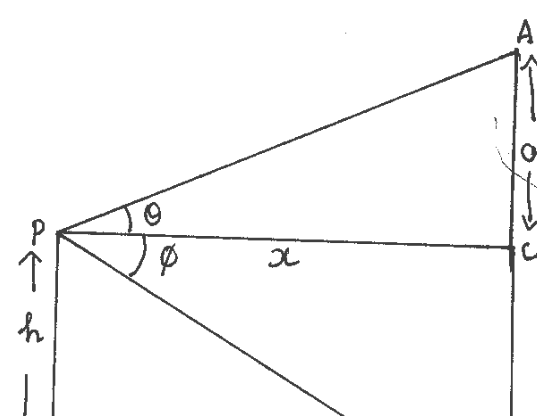


6. From a window x meters high above the ground in a street, the angles of elevation and depression of the top and the foot of the other house on the opposite side of the street are α and β respectively. Show that the height of the opposite house is $x(1 + \tan\alpha\ \cot\beta)$ Meters.

Ans: Let AB be the house and P be the window

Let BQ = x ∴ PC = x

Let AC = a



In Δ PQB, $\tan\theta = \dfrac{PQ}{QB}$ or $\tan\theta = \dfrac{h}{x}$

$\therefore\ x = \dfrac{h}{\tan\theta} = h\cot\theta$

In Δ PAC, $\tan\theta = \dfrac{AC}{PC}$ or $\tan\theta = \dfrac{a}{x}$

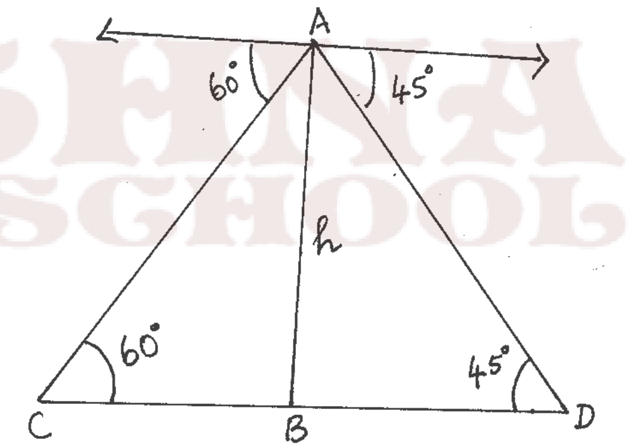
$\therefore\ a = x\tan\theta > (h\cot\theta)\tan\theta = h\tan\theta\cot\theta.$

$\therefore$ the height of the tower = AB = AC + BC

$= a + h = h\tan\theta\cot\theta + h = h(\tan\theta\cot\theta + 1)$

7. Two ships are iling in the sea on either side of a lighthouse; the angles of depression of two ships as observed from the top of the lighthouse are 60^0 and 45^0 respectively. If the distance between the ships is $200\left(\frac{1+\sqrt{3}}{\sqrt{3}}\right)$ meters, find the height of the lighthouse. (Ans:200m)

Ans: In right Δ ABC



$\tan 60 = \dfrac{h}{BC}$

$\sqrt{3} = \dfrac{h}{BC}$

$\Rightarrow H = \sqrt{3}\,BC$

In right Δ ABD

$\tan 45 = \dfrac{h}{BD}$

$h = BD$

$BC + BD = 200\left(\dfrac{1+\sqrt{3}}{\sqrt{3}}\right)$

$BC + \sqrt{3}\,BC = 200\left(\dfrac{1+\sqrt{3}}{\sqrt{3}}\right)$

$\Rightarrow BC = \dfrac{200(1+\sqrt{3})}{\sqrt{3}\sqrt{(1+\sqrt{3})}}$

$\therefore\ h = \sqrt{3}\,BC$

$= \sqrt{3}\,\dfrac{200}{\sqrt{3}}$

$= 200m$

$\therefore$ height of light house = 200m

8. A round balloon of radius ‘a’ subtends an angle θ at the eye of the observer while the angle of elevation of its centre is Φ. Prove that the height of the center of the balloon is a sin θ cosec Φ /2.

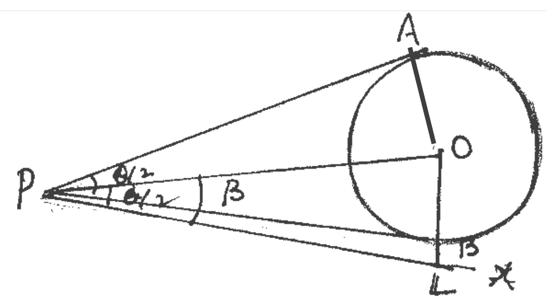
Ans: Let θ be the centre of the ballon of radius ‘r’ and ‘p’ the eye of the observer. Let PA, PB be tangents from P to ballong. Then $\angle APB = \theta$.

$\therefore \angle APO = \angle BPO = \dfrac{\theta}{2}$

Let OL be perpendicular from O on the horizontal PX. We are given that the angle of the elevation of the centre of the ballon is ϕ i.e.,

$\angle OPL = \phi$

In ΔOAP, we have $\sin\dfrac{\theta}{2} = \dfrac{OA}{OP}$



$\Rightarrow \sin\dfrac{\theta}{2} = \dfrac{a}{OP}$

$OP = a \operatorname{cosec}\dfrac{\theta}{2}$

In ΔOPL, we have $\sin\phi = \dfrac{OL}{OP}$

$\Rightarrow OL = OP \sin\phi = a \operatorname{cosec}\dfrac{\phi}{2} \sin\theta.$

Hence, the height of the center of the balloon is *a* sin θ cosec Φ /2.

9. The angle of elevation of a jet fighter from a point A on the ground is 60^0. After a flight of 15 seconds, the angle of elevation changes to 30^0. If the jet is flying at a speed of 720 km/hr, find the constant height at which the jet is flying.(Use $\sqrt{3}$ =1.732 (Ans: 2598m)

Ans: 36 km / hr = 10m / sec

720 km / h = $\dfrac{10 \times 720}{36}$

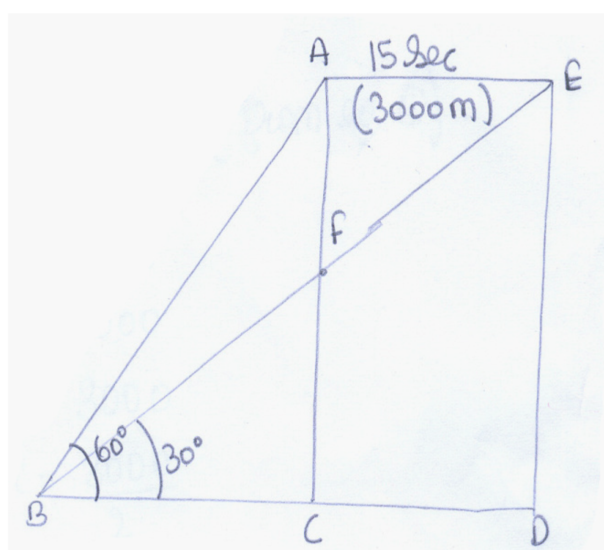
Speed = 200 m/s

Distance of jet from

AE = speed x time

= 200 x 15

= 3000 m



$\tan 60^o = \dfrac{AC}{BC}\left(\dfrac{opposite side}{adjacent side}\right)$

$\sqrt{3} = \dfrac{AC}{BC}$

$BC\sqrt{3} = AC$

AC = ED (constant height)

$\therefore$ BC $\sqrt{3}$ = ED ………………1

$\tan 30^{o} = \dfrac{ED}{BC+CD}\left(\dfrac{opposite side}{adjacent side}\right)$

$\dfrac{1}{\sqrt{3}} = \dfrac{ED}{BC+3000}$

$\dfrac{BC+3000}{\sqrt{3}}$ =ED

$\dfrac{BC+3000}{\sqrt{3}}$ =BC$\sqrt{3}$ (from equation 1)

BC + 3000 = 3BC

3BC – BC = 3000

2 BC = 3000

BC = $\dfrac{3000}{2}$

BC = 1500 m

ED = BC $\sqrt{3}$ (from equation 1)

= 1500 $\sqrt{3}$

= 1500 x 1.732

ED = 2598m

$\therefore$ The height of the jet fighter is 2598m.

10. A vertical post stands on a horizontal plane. The angle of elevation of the top is 60^{o} and that of a point x metre be the height of the post, then prove that $x = \dfrac{2h}{3}$.

Self Practice

11. A fire in a building B is reported on telephone to two fire stations P and Q, 10km apart from each other on a straight road. P observes that the fire is at an angle of 60^{o} to the road and Q observes that it is an angle of 45^{o} to the road. Which station should send its team and how much will this team have to travel? (Ans:7.32km)

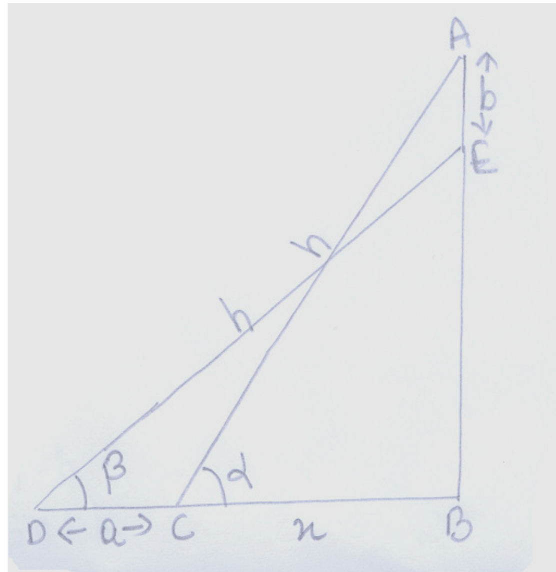
Self Practice

12. A ladder sets against a wall at an angle α to the horizontal. If the foot is pulled away from the wall through a distance of 'a', so that is slides a distance 'b' down the wall making an angle β with the horizontal. Show that $\dfrac{\cos\alpha - \cos\beta}{\sin\beta - \sin\alpha} = \dfrac{a}{b}$.

Ans: Let CB = x m. Length of ladder remains me

$\text{Cos } \alpha = \dfrac{CB}{CA}$ $\quad \therefore$ ED = AC Let Ed be

$\text{Cos } \alpha = \dfrac{x}{h}$ $\quad \therefore$ ED = AC = h



x = hcos α ………(1)

$\cos \beta = \dfrac{DC + CB}{ED}$

$\cos \beta = \dfrac{a + x}{h}$

a + x = hcos β

x = hcos β – a ………(2)

from (1) & (2)

hcos α = hcos β - a

hcos α - hcos β = - a

-a = h(cosα - cosβ) ……...(3)

$\text{Sin } \alpha = \dfrac{AE + EB}{AC}$

$\text{Sin } \alpha = \dfrac{b + EB}{h}$

hSin α – b = EB

EB = hSin α – b ……...(4)

$\text{Sin } \beta = \dfrac{EB}{DE}$

$\text{Sin } \beta = \dfrac{EB}{h}$

EB = hSin β ………(5)

From (4) & (5)

hSin β = hSin α – b

b = hsin α – hSin β

-b = h(Sin β - Sin α) ……...(6)

Divide equation (3) with equation (6)

$$\frac{-a}{-b} = \frac{h(\cos\alpha - \cos\beta)}{h(\sin\beta - Sin\alpha)}$$

$$\therefore \frac{a}{b} = \frac{Cos\alpha - Cos\beta}{Sin\beta - Sin\alpha}$$

13. Two stations due south of a leaning tower which leans towards the north are at distances a and b from its foot. If α, β be the elevations of the top of the tower from these stations, prove that its inclination φ is given by $\cot\varphi = \dfrac{b\cot\alpha - a\cot\beta}{b-a}$.

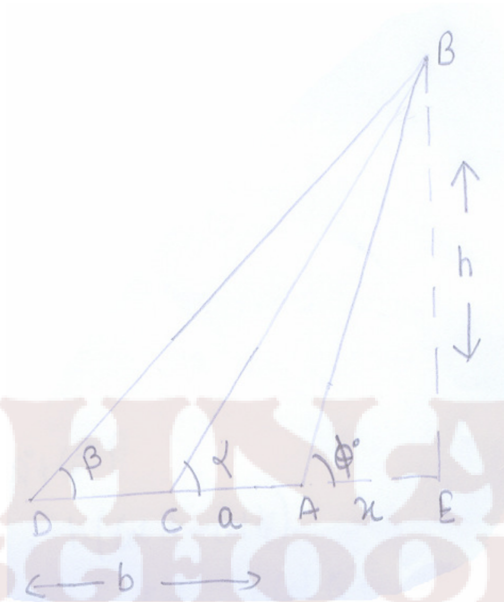
Ans:

Let AE = x, BE = h

Tan ϕ = $\dfrac{BE}{AE} = \dfrac{h}{x}$

x= h x $\dfrac{1}{\tan\phi}$

x = h cot ϕ ---------------1



tan α = $\dfrac{BE}{CE} = \dfrac{h}{a+x}$

a + x = h cot α

x= h cot α - a --------------2

tan β = $\dfrac{BE}{DE} = \dfrac{h}{b+x}$

b+x = h cot β

x = h cot β - b ---------------3

from 1 and 2

h cot ϕ = h cot α - a

h (cot ϕ + cot α) = a

h = $\dfrac{a}{-\cot\phi + \cot\alpha}$ -------------4

from 1 and 3

h cot ϕ = h cot β - b

h (cot ϕ - cot β) = b

h= $\dfrac{b}{-\cot\phi + \cot\beta}$

from 4 and 5

$$\frac{a}{-\cot\phi + \cot\alpha} = \frac{b}{-\cot\phi + \cot\beta}$$

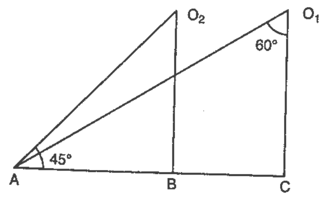
a (cot β - cot ϕ) = b (cot α - cot ϕ)

- a cot ϕ + b cot ϕ = b cot α - a cot β

(b – a) cot ϕ = b cot α - a cot β

$$\cot \phi = \frac{b \cot \alpha - a \cot \beta}{b-a}$$

14. In Figure, what are the angles of depression from the observing positions O_1 and O_2 of the object at A?



(Ans: 30^o ,45^o)

Self Practice

15. The angle of elevation of the top of a tower standing on a horizontal plane from a point A is α. After walking a distance d towards the foot of the tower the angle of elevation is found to be β. Find the height of the tower. (Ans: $\dfrac{d}{\cot\alpha - \cot\beta}$)

__Ans:__

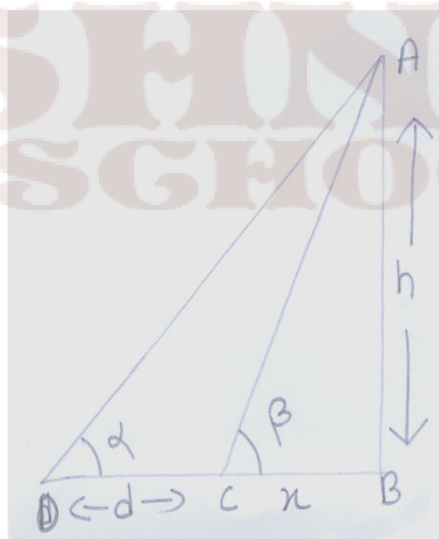
Let BC = x

$\tan \beta = \dfrac{AB}{CB}$

$\tan \beta = \dfrac{h}{x}$

$x = \dfrac{h}{\tan\beta}$

$x = h \cot \beta$ ---------(1)



$\tan \alpha = \dfrac{AB}{DC+CB}$

$\tan \alpha = \dfrac{h}{d+x}$

$d + x = \dfrac{h}{\tan\alpha} = h\cot\alpha$

$x = h \cot \alpha - d$ ----------(2)

from 1 and 2

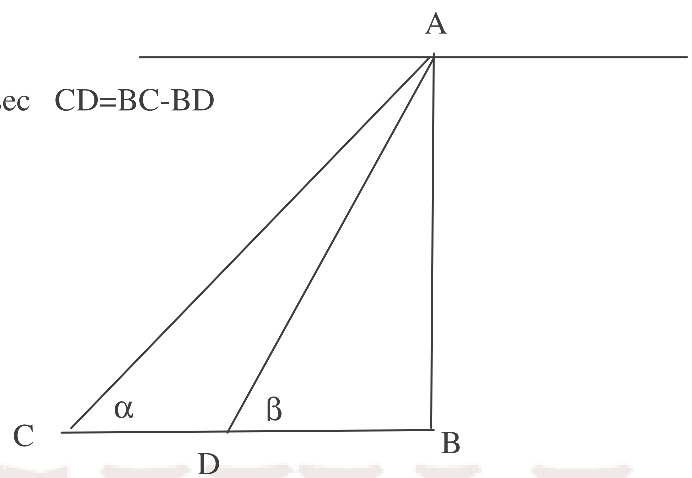
$h \cot \beta = h \cot \alpha - d$

$h (\cot \alpha - \cot \beta) = d$

$h = \dfrac{d}{\cot\alpha - \cot\beta}$

16. A man on a top of a tower observes a truck at an angle of depression α where $\tan\alpha = \frac{1}{\sqrt{5}}$ and sees that it is moving towards the base of the tower. Ten minutes later, the angle of depression of the truck is found to be β where $\tan\beta=\sqrt{5}$, if the truck is moving at a uniform speed, determine how much more time it will take to reach the base of the tower...

10 minutes=600sec



Ans: Let the speed of the truck be x m/sec CD=BC-BD

In right triangle ABC

$\tan\alpha = \frac{h}{BC}$ $\tan\alpha = \frac{1}{\sqrt{5}}$

$BC=h\sqrt{5}$………… 1

In right triangle ABD

$\tan\beta=\frac{h}{BD}$

$h=\sqrt{5}BD$ $(\tan\beta=\sqrt{5})$

CD=BC-BD (CD=600x)

600x =5BD-BD

BD=150x

Time taken $=\frac{150x}{x}$

=150 seconds

Time taken by the truck to reach the tower is 150 sec.

UNIT-7

CO-ORDINATE GEOMETRY

Mathematics is the tool specially suited for dealing with abstract concepts of any kind and there is no limit to its power in this field.

1. Find the points on the y axis whose distances from the points (6, 7) and (4,-3) are in the ratio 1:2. [Ans:(0, 9) , (0, $\frac{35}{3}$)]

Ans: Point on y-axis (0, y)

A(6, 7) B(4, -3) ratio 1:2

$$\frac{6^2+(7-y)^2}{4^2+(-3-y)^2}=\left(\frac{1}{2}\right)^2$$

On solving we get (0, 9) & (0, $\frac{35}{3}$)

2. Determine the ratio in which the line 2x + y -4 = 0 divide the line segment joining the points A (2,-2) and B (3, 7).Also find the coordinates of the point of division. [Ans:2 : 9 , ($\frac{24}{11}$,-$\frac{4}{11}$)]

Ans : Let the ratio be k:1

Let the co-ordinates of point of division be (x, y)

$$\therefore\ x=\frac{k(3)+1.2}{k+1}\qquad=\frac{3k+2}{k+1}$$

$$y=\frac{k(7)-1.2}{k+1}\qquad=\frac{7k-2}{k+1}$$

(x, y) lies on the line 2x + y – 4 = 0.

$$\therefore\ 2\left(\frac{3k+2}{k+1}\right)+\left(\frac{7k-2}{k+1}\right)-4=0$$

2(3k+2) + (7k-2) – 4 (k+1) = 0

6k + 4 + 7k - 2 – 4k – 4 = 0

9k – 2 = 0 k = $\frac{2}{9}$

Ratio is 2:9

$$\therefore x = \frac{3x\frac{2}{9}+2}{\frac{2}{9}+1} = \frac{\frac{2}{3}+2}{\frac{11}{9}} = \frac{\frac{2+6}{3}}{\frac{11}{9}} = \frac{8}{3}\text{ x }\frac{9}{11} = \frac{24}{11}$$

$$y = \frac{7\left(\frac{2}{9}\right)-2}{\frac{2}{9}+1} = \frac{\frac{14-18}{9}}{\frac{11}{9}} = \frac{-4}{9}\text{ x }\frac{9}{11} = \frac{-4}{11}$$

$$\therefore (x, y) = \left(\frac{24}{11}, \frac{-4}{11}\right)$$

3. Find the third vertex of a triangle if its two vertices are (-1, 4) and (5, 2) and mid point of one side is (0, 3). (Ans: (-5, 4) or (1, 2))

Ans : Let the third vertex be (x, y)

If (0,3) is mid point of BC then

$\frac{x+5}{2} = 0$ (or) $x = -5$

$\frac{y+2}{2} = 3$ $y = 4$. $(-5, 4)$

If (0,3) is mid point of AC then

$\frac{x-1}{2} = 0$ $x = 1$ $\frac{y+4}{2} = 3$ $y + 4 = 6$ $y = 2$ $(1, 2)$

$\therefore$ (-5. 4) or (1, 2) are possible answers.

4. If the vertices of a triangle are (1, k), (4, -3), (-9, 7) and its area is 15 sq units, find the value(s) of k.. $\left[\text{Ans: } -3, \frac{21}{13}\right]$

Ans: A(1, k) B(4, -3) C(-9, 7)

Area of Δ ABC = $\frac{1}{2}[x_1 (y_2-y_3)+x_2(y_3-y_1) + x_1(y_1-y_2)]$

$= \frac{1}{2}[1(-3-7)+4(7-k)+(-9)(k+3)] = 15$

$-10 + 28 - 4k - 9k - 27 = 30$

$-9 - 13k = 30 \quad \Rightarrow \quad k = -3$

$|-9 - 13k| = 30$

$9 + 13k = 30$

$k = \frac{21}{13}$

$k = -3, \frac{21}{13}$

5. The centre of a circle is (2x – 1, 3x + 1).Find x if the circle passes through (-3,-1) and the length of the diameter is 20 units. $\left[\text{Ans: x = 2 , - }\frac{46}{13}\right]$

Ans : D = 20 R = 10

$(2x - 1 + 3)^2 + (3x + 1 + 1)^2 = 10^2$

$(2x + 2)^2 + (3x + 2)^2 = 100$

$4x^2 + 8x + 4 + 9x^2 + 12x + 4 = 100$

$13x^2 + 20x + 8 = 100$

$13x^2 + 20x - 92 = 0$

$13x^2 + 46x - 26x - 92 = 0$

$(13x + 46) - 2(13x + 46) = 0$

$x = 2, \frac{-46}{13}$

6. If A & B are (-2,-2) and (2,-4) respectively, find the co ordinates of P such that

AP = $\frac{3}{7}$ AB and P lies on the line segment AB. $\left[\text{Ans: }(-\frac{2}{7}, -\frac{20}{7})\right]$

Ans : AP = $\frac{3}{7}$ AB

$\frac{AP}{AB} = \frac{3}{7}$ (i.e) $\frac{AP}{PB} = \frac{3}{4}$

AB = AP + PB

AP : PB = 3:4

Let P(x, y)

$x = \frac{3(2)+4(-2)}{7} = \frac{6-8}{7} = \frac{-2}{7}$

$y = \frac{3(-4)+4(-2)}{7} = \frac{-12-8}{7} = \frac{-20}{7}$

$(x, y) = (\frac{-2}{7}, \frac{-20}{7})$

7. Show that the points (3, 0), (4, 5), (-1, 4) and (-2, -1) taken in order are the vertices of a rhombus. Also find the area of the rhombus. (Ans: 24 sq units)

Ans : Let AC be d_1 & BD be d_2

Area = $\frac{1}{2}$ d1d_2

$d_1 = \sqrt{(3+1)^2 + (0-4)^2} = \sqrt{4^2 + 4^2} = \sqrt{32} = 4\sqrt{2}$

$d_2 = \sqrt{(-2-4)^2 + (-1-5)^2} = \sqrt{36+36} = \sqrt{72} = 6\sqrt{2}$

Area = $\frac{1}{2}$ d_1 d_2 = $\frac{1}{2}$ x 4 x 6 x 2 = 24sq units.

8. If A, B and P are the points (-4, 3), (0, -2) and (α,β) respectively and P is equidistant from A and B, show that $8\alpha - 10\beta + 21 = 0$.

Ans : $AP = PB \Rightarrow AP^2 = PB^2$
$(\propto + 4)^2 + (\beta - 3)^2 = \propto^2 + (\beta + 2)^2$
$\propto^2 + 8\propto + 16 + \beta^2 - 6\beta + 9 = \propto^2 + \beta^2 + 4\beta + 4$
$8\propto - 6\beta - 4\beta + 25 - 4 = 0$
$8\propto - 10\beta + 21 = 0$

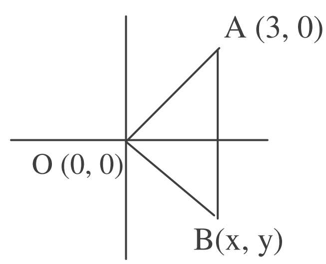
9. If the points (5, 4) and (x, y) are equidistant from the point (4, 5), prove that $x^2 + y^2 - 8x - 10y + 39 = 0$.

Ans : AP = PB
$AP^2 = PB^2$
$(5-4)^2 + (4-5)^2 = (x-4)^2 + (y-5)^2$
$1 + 1 = x^2 - 8x + 16 + y^2 - 10y + 25$
$x^2 + y^2 - 8x - 10y + 41 - 2 = 0$
$x^2 + y^2 - 8x - 10y + 39 = 0$

10. If two vertices of an equilateral triangle are (0, 0) and (3, 0), find the third vertex.
.

$\left[\text{Ans: } \frac{3}{2}, \frac{3\sqrt{3}}{2} \text{ or } \frac{3}{2}, -\frac{3\sqrt{3}}{2}\right]$

Ans: OA = OB = AB
$OA^2 = OB^2 = AB^2$
$OA^2 = (3-0)^2 + 0 = 9$
$OB^2 = x^2 + y^2$
$AB^2 = (x-3)^2 + y^2 = x^2 + y^2 - 6x + 9$
$OA^2 = OB^2 = AB^2$
$OA^2 = OB^2$ & $OB^2 = AB^2$
$9 = x^2 + y^2 \Rightarrow y^2 = 9 - x^2$
$x^2 + y^2 - 6x + 9 = 9$



$x^2 + 9 - x^2 - 6x + 9 = 9$
$6x = 9 \qquad x = \frac{3}{2}$

$y^2 = 9 - \left(\frac{3}{2}\right)^2 = 9 - \frac{9}{4} = \frac{36-9}{4} = \frac{27}{4}$

$y = \pm \frac{3\sqrt{3}}{2}$

$\therefore$ Third vertex is $\left(\frac{3}{2}, \frac{3\sqrt{3}}{2}\right)$ or $\left(\frac{3}{2}, \frac{-3\sqrt{3}}{2}\right)$

11. Find the centre of a circle passing through the points (6, -6), (3, -7) and (3, 3).Also find the radius. (Ans: (3, -2), 5 units)

Ans: OA=OB = OC = radius of the circle where O is the centre of the circle and let O be (x, y)

$OA^2 = OB^2 = OC^2$

$OA^2 = (x-6)^2 + (y+6)^2 = x^2 + y^2 - 12x + 36 + 12y + 36$

$OB^2 = (x-3)^2 + (y+7)^2 = x^2 + y^2 - 6x + 9 + 14y + 49$

$OC^2 = (x-3)^2 + (y-3)^2 = x^2 + y^2 - 6x + 9 - 6y + 9$

$OA^2 = OB^2$

$x^2 + y^2 - 12x + 12y + 72 = x^2 + y^2 - 6x + 14y + 58$

$- 12x + 12y + 6x - 14y + 72 - 58 = 0$

$- 6x - 2y + 14 = 0$

$- 3x - y + 7 = 0$ ....(1)

$x^2 + y^2 - 6x + 9 + 14y + 49 = x^2 + y^2 - 6x + 9 - 6y + 9$

$- 6x + 14y + 58 = -6x - 6y + 18$

$14y + 6y = 18 - 58$

$20y = -40$

$y = -2$ ....(2)

Substituting we get

$- 3x + 2 + 7 = 0$

$- 3x = - 9$

$x = 3$

$(x, y) = (3, -2)$

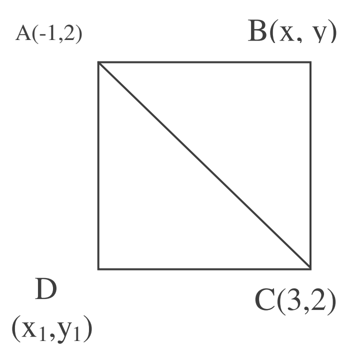
Diameter $= 3^2 + 2^2 - 6(3) + 18 - 6(-2)$

$= 9 + 4 - 18 + 18 + 12$

$= 13 + 12 = 25$

Radius $= \sqrt{25} = 5$ units

12. The two opposite vertices of a square are (-1, 2) and (3, 2). Find the coordinates of the other two vertices. (Ans: (1, 0), (1, 4))



Ans : $AB = BC \Rightarrow AD^2 = BC^2$

$(x + 1)^2 + (y-2)^2 = (x-3)^2 + (y-2)^2$

$x^2 + 2x + 1 + y^2 - 4y + 4 = x^2 - 6x + 9 + y^2 - 4y + 4$

$2x - 4y + 5 = - 6x - 4y + 13$

$8x = 13 - 5 \quad 8x = 8 \Rightarrow x = 1$

On substituting in $(x-3)^2 + (y-2)^2 + (x+1)^2 + (y-2)^2$

$= (-1 - 3)^2 + (2 - 2)^2$
We get y = 4 or 0.
$\therefore$ B (1, 4) or (1, 0)

AD = DC $\Rightarrow AD^2 = DC^2$
$(x_1 + 1)^2 + (y_1 - 2)^2 = (x_1 - 3)^2 + (y_1 - 2)^2$
$\therefore$ x = 1.
On substituting in $(x_1 + 1)^2 + (y_1 - 2)^2 + (x_1 - 3)^2 + (y_1 - 2)^2 = 16$
We get $y_1 = 0$ or 4.
$\therefore$ D (1, 4) or (1, 0)
$\therefore$ the opposite vertices are (1, 4) & (1, 0)

13. Find the coordinates of the point P which is three –fourth of the way from A (3, 1) to B (-2, 5). (Ans: $\left(-\frac{3}{4}, 4\right)$

Ans : Hint: Ratio AP:PB = 3 : 1

14. The midpoint of the line joining (2a, 4) and (-2, 3b) is (1, 2a +1).Find the values of a & b.

(Ans: a = 2, b = 2)

Ans : A(2a, 4) P(1, 2a + 1) B(-2, 3b)

$\frac{2a-2}{2} = 1$ & $\frac{4+3b}{2} = 2a+1$

We get a = 2 & b = 2.

15. Find the distance between the points (b + c, c + a) and (c + a, a + b) .

(Ans : $\sqrt{a^2 + 2b^2 + c^2 - 2ab - 2bc}$)

Ans : Use distance formula

16. Find the relation between x and y when the point (x,y) lies on the straight line joining the points (2,-3) and (1,4) [Hint: Use area of triangle is 0]

Ans : Hint: If the points are on straight line, area of the triangle is zero.

17. Find the distance between (cosθ, sinθ) and (sinθ, -cosθ). (Ans: √2)

Ans : $\sqrt{(\cos\theta - Sin\theta)^2 + (Sin\theta + \cos\theta)^2}$

On simplifying we get $\sqrt{2}$

18. Find the distance between (a cos35°, 0) (0, a cos65°). (Ans: a)

Ans : Proceed as in sum no.17.

19. The vertices of a $\Delta\, ABC$ are A(4, 6), B(1. 5) and C(7, 2). A line is drawn to intersect sides AB and AC at D and E respectively, such that $\frac{AD}{AB}=\frac{AE}{AC}=\frac{1}{4}$. Calculate the area of the Δ ADE and compare it with the area of Δ ABC. (Ans: $\frac{15}{32}$ sq units; 1:16)

Ans : Hint : $\frac{AD}{AB}=\frac{AE}{AC}=\frac{1}{4}$

$\therefore$ AD : DB = 1 : 3 & AE : EC = 1 : 3

Find D & E and find area of triangle ADE and triangle ABC and compare.

20. Plot the points A(2,0) and B (6,0) on a graph paper. Complete an equilateral triangle ABC such that the ordinate of C be a positive real number .Find the coordinates of C

(Ans: $(4,2\sqrt{3})$

Ans : Proceed by taking C(x, y)

AC = BC = AB

21. Find the ratio in which the line segment joining A(6,5) and B(4,-3) is divided by the line y=2 (Ans:3:5)

Ans : Let the ratio be k:1

$$x=\frac{4k+6}{k+1}$$

$$y=\frac{-3k+5}{k+1}$$

$$\frac{-3k+5}{k+1}=2$$

On solving we get k = 3 : 5

22. The base BC of an equilateral triangle ABC lies on the y-axis. The coordinates of C are (0,-3). If the origin is the midpoint of BC find the coordinates of points A and B.

Ans : Hint : The point A will lie on the x axis. Find A using AB = BC = AC.

Coordinates of B (0, 3)

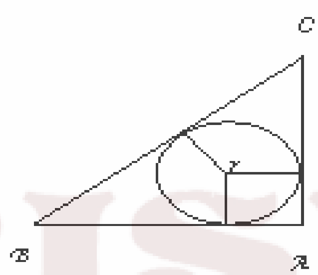
UNIT-8

SIMILAR TRIANGLES

Geometry is the right foundation of all painting, I have decided to teach its rudiments and principles to all youngsters eager for art.

1. ABC is a right-angled triangle, right-angled at A. A circle is inscribed in it. The lengths of the two sides containing the right angle are 6cm and 8 cm. Find the radius of the in circle.

(Ans: r=2)

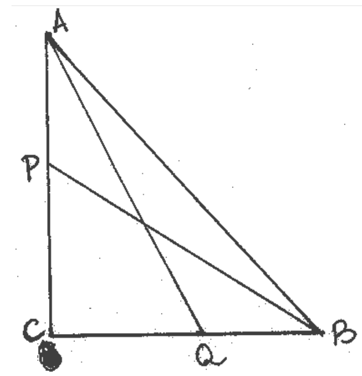


Ans: BC = 10cm
$y + z = 8cm$
$x + z = 6cm$
$x + y = 10$
$\Rightarrow x + y + z = 12$
$z = 12 - 10$
$z = 2$ cm
$\therefore$ radius = 2cm

2. ABC is a triangle. PQ is the line segment intersecting AB in P and AC in Q such that PQ parallel to BC and divides triangle ABC into two parts equal in area. Find BP: AB.

Ans: Refer example problem of text book.

3. In a right triangle ABC, right angled at C, P and Q are points of the sides CA and CB respectively, which divide these sides in the ratio 2: 1.
   Prove that $9AQ^2 = 9AC^2 + 4BC^2$
   $9BP^2 = 9BC^2 + 4AC^2$
   $9(AQ^2 + BP^2) = 13AB^2$



Ans: Since P divides AC in the ratio 2 : 1

$CP = \frac{2}{3}AC \qquad QC = \frac{2}{3}BC$

$AQ^2 = QC^2 + AC^2$

$AQ^2 = \frac{4}{9}BC^2 + AC^2$

$9AQ^2 = 4BC^2 + 9AC^2$ …………………. (1)

Similarly we get $9BP^2 = 9BC^2 + 4AC^2$ …….…………(2)

Adding (1) and (2) we get $9(AQ^2 + BP^2) = 13AB^2$

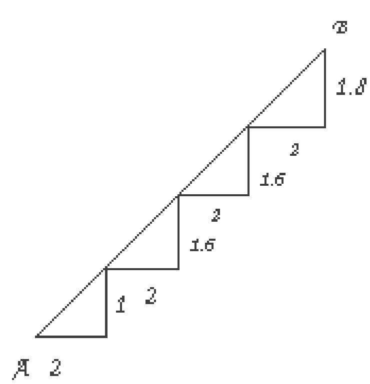
4. P and Q are the mid points on the sides CA and CB respectively of triangle ABC right angled at C. Prove that $4(AQ^2 + BP^2) = 5AB^2$

   Self Practice

5. In an equilateral triangle ABC, the side BC is trisected at D.
   Prove that $9AD^2 = 7AB^2$

   Self Practice

6. There is a staircase as shown in figure connecting points A and B. Measurements of steps are marked in the figure. Find the straight distance between A and B.
   (Ans:10)



Ans: Apply Pythagoras theorem for each right triangle add to get length of AB.

7. Find the length of the second diagonal of a rhombus, whose side is 5cm and one of the diagonals is 6cm. (Ans: 8cm)

Ans: Length of the other diagonal = 2(BO)
where BO = 4cm
$\therefore$ BD = 8cm.

8. Prove that three times the sum of the squares of the sides of a triangle is equal to four times the sum of the squares of the medians of the triangle.

Ans: To prove $3(AB^2 + BC^2 + CA^2) = 4(AD^2 + BE^2 + CF^2)$
In any triangle sum of squares of any two sides is equal to twice the square of half of third side, together with twice the square of median bisecting it.

If AD is the median

$AB^2 + AC^2 = 2\left\{AD^2 + \frac{BC^2}{4}\right\}$

$2(AB^2 + AC^2) = 4AD^2 + BC^2$

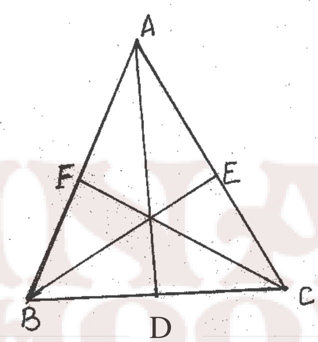
Similarly by taking BE & CF as medians we get

$\Rightarrow 2(AB^2 + BC^2) = 4BE^2 + AC^2$

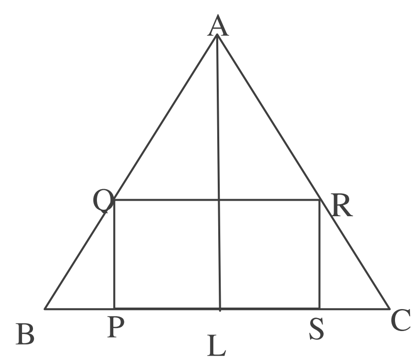
& $2(AC^2 + BC^2) = 4CF^2 + AB^2$

Adding we get

$\Rightarrow 3(AB^2 + BC^2 + AC^2) = 4(AD^2 + BE^2 + CF^2)$



9. ABC is an isosceles triangle is which AB=AC=10cm. BC=12. PQRS is a rectangle inside the isosceles triangle. Given PQ=SR= y cm, PS=QR=2x. Prove that $x = 6 - \frac{3y}{4}$.



Ans: AL = 8 cm
$\Delta BPQ \sim \Delta BAL$

$\Rightarrow \frac{BQ}{PQ} = \frac{BL}{AL}$

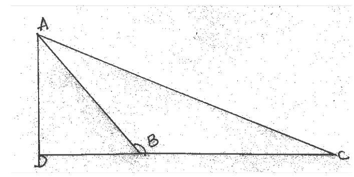
$\Rightarrow \frac{6-x}{y} = \frac{6}{8}$

$\Rightarrow$ x = 6 $-$ $\frac{3y}{4}$.

Hence proved

10. If ABC is an obtuse angled triangle, obtuse angled at B and if AD$\perp$CB

Prove that AC2 =AB^2 + BC^2+2BCxBD



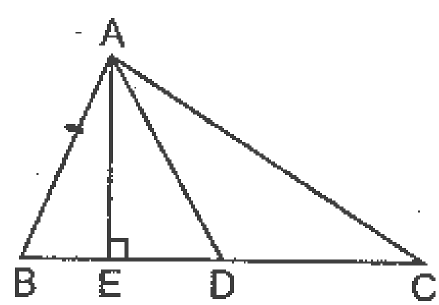
Ans: $AC^2 = AD^2 + CD^2$
$= AD^2 + (BC + BD)^2$
$= AD^2 + BC^2 + 2BC.BD+BD^2$
$= AB^2 + BC^2 + 2BC.BD$

11. If ABC is an acute angled triangle , acute angled at B and AD$\perp$BC

prove that $AC^2 = AB^2 + BC^2 - 2BCxBD$

Ans: Proceed as sum no. 10.

12. Prove that in any triangle the sum of the squares of any two sides is equal to twice the square of half of the third side together with twice the square of the median, which bisects the third side.



Ans: To prove $AB^2 + AC^2 = 2AD^2 + 2\left(\frac{1}{2}BC\right)^2$

Draw AE $\perp$ BC
Apply property of Q. No.10 & 11.
In Δ ABD since $\angle D > 90^0$
$\therefore AB^2 = AD^2 + BD^2 + 2BD$ x DE ….(1)
Δ ACD since $\angle$ D < 90^o
$AC^2 = AD^2 + DC^2 - 2DC$ x DE ….(2)
Adding (1) & (2)
$AB^2 + AC^2 = 2(AD^2 + BD^2)$

$= 2(AD^2 + \left(\frac{1}{2}BC\right)^2)$

Or $AB^2 + AC^2 = 2\ (AD^2 + BD^2)$
Hence proved

13. If A be the area of a right triangle and b one of the sides containing the right angle, prove that the length of the altitude on the hypotenuse is $\frac{2Ab}{\sqrt{b^4+4A^2}}$.

Ans: Let QR = b

A = Ar(ΔPQR)

A = $\frac{1}{2}$ x b x PQ

PQ = $\frac{2A}{b}$ ………………………(1)

Δ PNQ ~ ΔPQR (AA)

$\Rightarrow \frac{PQ}{PR} = \frac{NQ}{QR}$ …………………(2)

From Δ PQR

$PQ^2 + QR^2 = PR^2$

$\frac{4A^2}{b^2} + b^2 = PR^2$

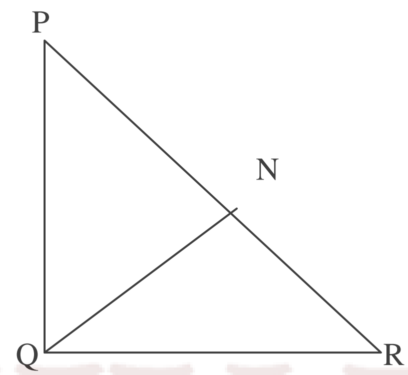
PR = $\sqrt{\frac{4A^2+b^4}{b^2}} = \frac{\sqrt{4A^2+b^4}}{b}$

Equation (2) becomes

$\frac{2A}{bxPR} = \frac{NQ}{b}$

NQ = $\frac{2A}{PR}$

NQ = $\frac{2Ab}{\sqrt{4A^2+b^4}}$ Ans



14. ABC is a right triangle right-angled at C and AC=$\sqrt{3}$ BC. Prove that ∠ABC=60°.

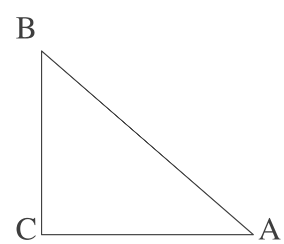
Ans: Tan B = $\frac{AC}{BC}$

Tan B = $\frac{\sqrt{3}BC}{BC}$

Tan B = $\sqrt{3}$

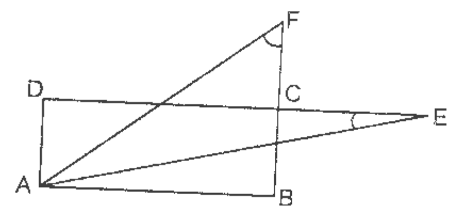
$\Rightarrow$ Tan B = Tan 60

$\Rightarrow$ B = 60°



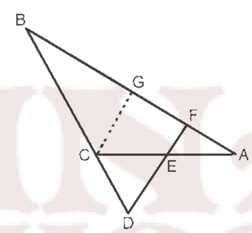
$\Rightarrow \angle ABC = 60^o$
Hence proved

15. ABCD is a rectangle. Δ ADE and Δ ABF are two triangles such that ∠E=∠F as shown in the figure. Prove that AD x AF=AE x AB.



Ans: Consider Δ ADE and Δ ABF
∠D = ∠B = 90^o
∠E = ∠F (given)
∴Δ ADE ≅ Δ ABF
$\frac{AD}{AB} = \frac{AE}{AF}$
⇒ AD x AF = AB x AE
Proved

16. In the given figure, ∠AEF=∠AFE and E is the mid-point of CA. Prove that $\frac{BD}{CD} = \frac{BF}{CE}$

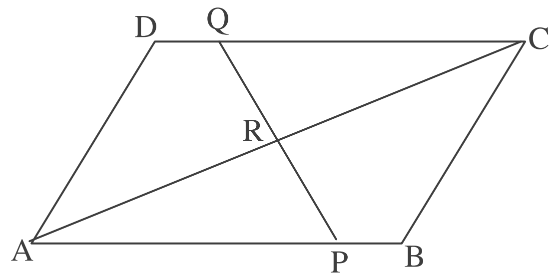


Ans: Draw CG ∥ DF
In Δ BDF
CG ∥ DF
$\therefore \frac{BD}{CD} = \frac{BF}{GF}$ ………….(1) BPT
In ΔAFE
∠AEF=∠AFE
⇒AF=AE
⇒AF=AE=CE…………..(2)

In Δ ACG
E is the mid point of AC
⇒ FG = AF

∴ From (1) & (2)
$\frac{BD}{CD} = \frac{BF}{CE}$
Hence proved

17. ABCD is a parallelogram in the given figure, AB is divided at P and CD and Q so that AP:PB=3:2 and CQ:QD=4:1. If PQ meets AC at R, prove that AR=$\frac{3}{7}$AC.



Ans: $\Delta APR \sim \Delta CQR$ (AA)

$\Rightarrow \frac{AP}{CQ}=\frac{PR}{QR}=\frac{AR}{CR}$

$\Rightarrow \frac{AP}{CQ}=\frac{AR}{CR}$ & AP=$\frac{3}{5}$AB

$\Rightarrow \frac{3AB}{5CQ}=\frac{AR}{CR}$ & CQ=$\frac{4}{5}$CD = $\frac{4}{5}$ AB

$\Rightarrow \frac{AR}{CR}=\frac{3}{4}$

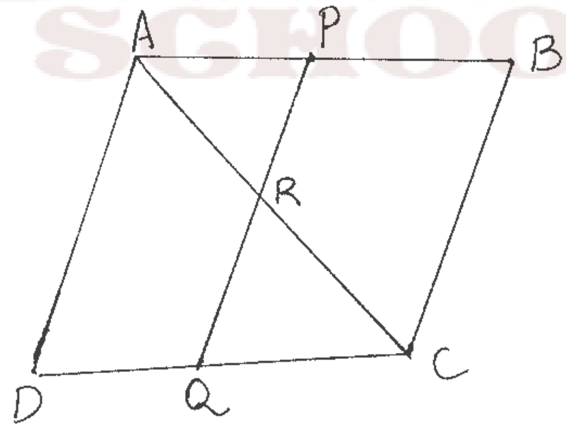
$\Rightarrow \frac{CR}{AR}=\frac{4}{3}$

$\frac{CR+AR}{AR}=\frac{4}{3}+1$

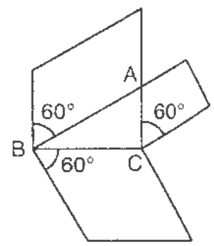
$\Rightarrow \frac{AC}{AR}=\frac{7}{3}$

$\Rightarrow$ AR =$\frac{3}{7}$AC

Hence proved



18. Prove that the area of a rhombus on the hypotenuse of a right-angled triangle, with one of the angles as 60^o, is equal to the sum of the areas of rhombuses with one of their angles as 60^o drawn on the other two sides.



Ans: Hint: Area of Rhombus of side a & one angle of 60^o

$= \frac{\sqrt{3}}{2} \times a \times a = \frac{\sqrt{3}}{2} a^2$

19. An aeroplane leaves an airport and flies due north at a speed of 1000 km/h. At the me time, another plane leaves the me airport and flies due west at a speed of 1200 km/h. How r apart will be the two planes after 1½ hours. (Ans:300√61Km)

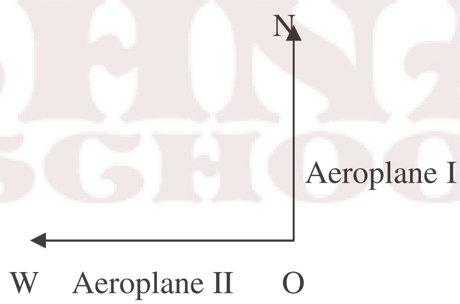
Ans: ON = 1500km (dist = s x t)

OW = 1800 km

$NW = \sqrt{1500^2 + 1800^2}$

$= \sqrt{5490000}$

$= 300\sqrt{61}$ km



20. ABC is a right-angled isosceles triangle, right-angled at B. AP, the bisector of $\angle BAC$, intersects BC at P. Prove that $AC^2 = AP^2 + 2(1+\sqrt{2})BP^2$

Ans: $AC = \sqrt{2}\, AB$ (Since AB = BC)

$\frac{AB}{AC} = \frac{BP}{CP}$ (Bisector Theorem)

$\Rightarrow CP = \sqrt{2}\, BP$

$AC^2 - AP^2 = AC^2 - (AB^2 + BP^2)$

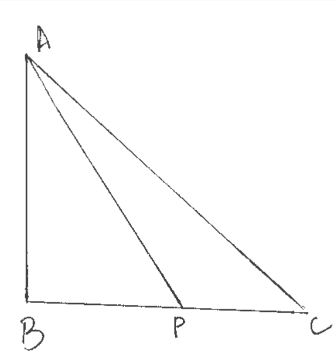
$= AC^2 - AB^2 - BP^2$

$= BC^2 - BP^2$

$= (BP + PC)^2 - BP^2$

$= (BP + \sqrt{2}\, BP)^2 - BP^2$

$= 2BP^2 + 2\sqrt{2}\, BP^2$



$= 2(\sqrt{2}+1)BP^2 \Rightarrow AC^2 = AP^2 + 2(1+\sqrt{2})BP^2$
Proved

UNIT-9

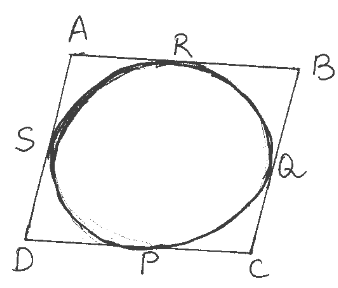
CIRCLES

1. Prove that the parallelogram circumscribing a circle is rhombus.

Ans Given : ABCD is a parallelogram circumscribing a circle.
To prove : - ABCD is a rhombus
or
AB=BC=CD=DA

Proof: Since the length of tangents from external are equal in length
$\therefore AS = AR$ …..(1)
$BQ = BR$ …..(2)
$QC = PC$ …..(3)
$SD = DP$ …..(4)

Adding (1), (2), (3) & (4).
AS + SD + BQ + QC = AR + BR + PC + DP
AD + BC = AB + DC
AD + AD = AB + AB
Since BC = AD & DC = AB (opposite sides of a parallelogram are equal)
2AD = 2AB
$\therefore AD = AB$ …..(5)
$\left.\begin{array}{l} BC = AD \\ DC = AB \end{array}\right\}$ (opposite sides of a parallelogram) …..(6)
From (5) and (6)
AB = BC = CD = DA
Hence proved



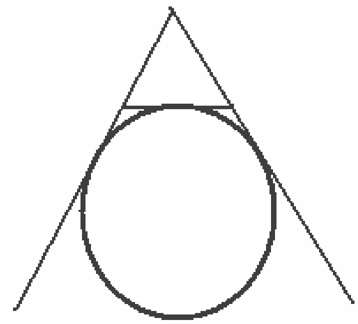
2. A circle touches the side BC of a triangle ABC at P and touches AB and AC when produced at Q and R respectively as shown in figure.

Show that AQ= $\frac{1}{2}$ (perimeter of triangle ABC)

A

P
B C

Q R



Ans: Since the length of tangents from external point to a circle are equal.

AQ = AR

BQ = BP

PC = CR

Since AQ = AR

AB + BQ = AC + CR

$\therefore$ AB + BP = AC + PC (Since BQ = BP & PC = CR)

Perimeter of Δ ABC = AB + AC + BC

= AB + BP + PC + AC

= AQ + PC + AC (Since AB + BP = AQ)

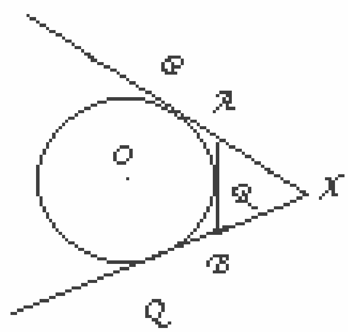
= AQ + AB + BP (Since PC + AC = AB + BP)

= AQ + AQ (Since AB + BP = AQ)

Perimeter of Δ ABC = 2AQ

$\therefore$ AQ = $\frac{1}{2}$ (perimeter of triangle ABC)

3. In figure, XP and XQ are tangents from X to the circle with centre O. R is a point on the circle. Prove that XA+AR=XB+BR



Ans: Since the length of tangents from external point to a circle are equal

XP = XQ

PA = RA

BQ = BR

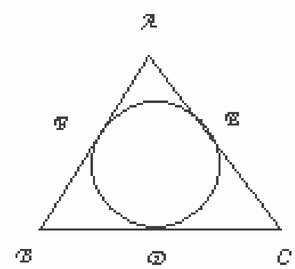
XP = XQ

$\Rightarrow$XA + PA = XB + BQ

$\Rightarrow$XA + AR = XB + BR $\left(\Theta\ PA = AR \ \&\ BQ = BR\right)$

Hence proved

4. In figure, the incircle of triangle ABC touches the sides BC, CA, and AB at D, E, and F respectively. Show that AF+BD+CE=AE+BF+CD= $\frac{1}{2}$ (perimeter of triangle ABC),



Ans: Since the length of tangents from an external point to are equal

$\therefore$ AF = AE
FB = BD
EC = CD

Perimeter of ΔABC = AB + BC+ AC
= AF + FB + BD + DC + AE + EC
= AF + BD + BD + CE + AF + CE
(Θ AF=AE, FB=BD, EC=CD)
= AF + AF + BD + BD + CE + CE

Perimeter of ΔABC = 2(AF + BD+ CE)

$\therefore$ AF + BD + CE = $\frac{1}{2}$ (perimeter of ΔABC) ……..(1)

Perimeter of ΔABC = AB + BC + AC
= AF + FB + BD + DC + AE + EC
= AE + BF + BF + CD + AE + CD
(Θ AF = AE, FB = BD, EC = CD)
= AE + AE + BF + BF + CD + CD

Perimeter of ΔABC = 2(AE + BF + CD)

$\therefore$ AE + BF + CD = $\frac{1}{2}$ (perimeter of ΔABC) ……..(2)

From (1) and (2)

AF + BD + CE = AE + BF + CD = $\frac{1}{2}$ (perimeter of ΔABC)

5. A circle touches the sides of a quadrilateral ABCD at P, Q, R and S respectively. Show that the angles subtended at the centre by a pair of opposite sides are supplementary.

Ans: To prove :- $\angle AOB + \angle DOC = 180^o$

$\angle BOC + \angle AOD = 180^o$

Proof : - Since the two tangents drawn from an external point to a circle subtend equal angles at centre.

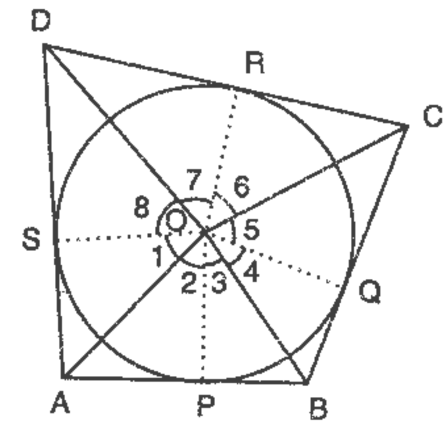
$\therefore \angle 1 = \angle 2, \angle 3 = \angle 4, \angle 5 = \angle 6, \angle 7 = \angle 8$

but $\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8 = 360^o$

$2(\angle 2 + \angle 3 + \angle 6 + \angle 7) = 360^o$

$\angle 2 + \angle 3 + \angle 6 + \angle 7 = 360^o$

$\therefore \angle AOB + \angle DOC = 180^o$



Similarly

$\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8 = 360^o$

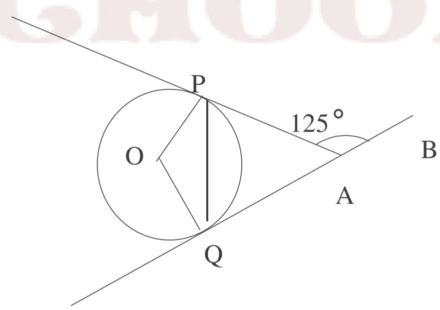
$2(\angle 1 + \angle 8 + \angle 4 + \angle 5) = 360^o$

$\angle 1 + \angle 8 + \angle 5 = 180^o$

$\therefore \angle BOC + \angle AOD = 180^o$

Hence proved

6. In figure, O is the centre of the Circle .AP and AQ two tangents drawn to the circle. B is a point on the tangent QA and $\angle PAB = 125°$, Find $\angle POQ$.
(Ans: 125^o)



Ans: Given $\angle PAB = 125^o$

To find : - $\angle POQ = ?$

Construction : - Join PQ

Proof : - $\angle PAB + \angle PAQ = 180^o$ (Linear pair)

$\angle PAQ + 125^o = 180^o$

$\angle PAQ = 180^o - 125^o$

$\angle PAQ = 55^o$

Since the length of tangent from an external point to a circle are equal.

PA = QA

$\therefore$ From ΔPAQ

$\angle APQ = \angle AQP$

In ΔAPQ

$\angle APQ + \angle AQP + \angle PAQ = 180^o$ (angle sum property)

$\angle APQ + \angle AQP + 55^o = 180^o$

$2\angle APQ = 180^o - 55^o$ ($\Theta\ \angle APQ = \angle AQP$)

$\angle APQ = \dfrac{125^o}{2}$

$\therefore \angle APQ = \angle AQP = \dfrac{125^o}{2}$

OQ and OP are radii

QA and PA are tangents

$\therefore \angle OQA = 90^o$

& $\angle OPA = 90^o$

$\angle OPQ + \angle QPA = \angle OPA = 90^o$ (Linear Pair)

$\angle OPQ + \dfrac{125^o}{2} = 90^o$

$\angle OPQ = 90^o - \dfrac{125^o}{2}$

$= \dfrac{180^o - 125^o}{2}$

$\angle OPQ = \dfrac{55^o}{2}$

Similarly $\angle OQP + \angle PQA = \angle OQA$

$\angle OQP + \dfrac{125^o}{2} = 90^o$

$\angle OQP = 90^o - \dfrac{125^o}{2}$

$\angle OQP = \dfrac{55^o}{2}$

In ΔPOQ

$\angle OQP + \angle OPQ + \angle POQ = 180^o$ (angle sum property)

$\dfrac{55^o}{2} + \dfrac{55^o}{2} + \angle POQ = 180^o$

$\angle POQ + \dfrac{110}{2} = 180^o$

$\angle POQ = 180^o - \dfrac{110}{2}$

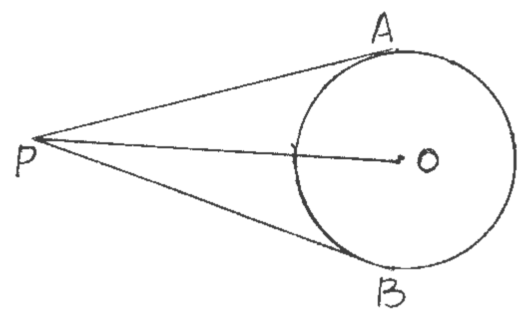
$\angle POQ = \dfrac{360^o - 110^o}{2}$

$\angle POQ = \dfrac{250^o}{2}$

$\angle POQ = 125^o$
$\therefore \angle POQ = 125^o$

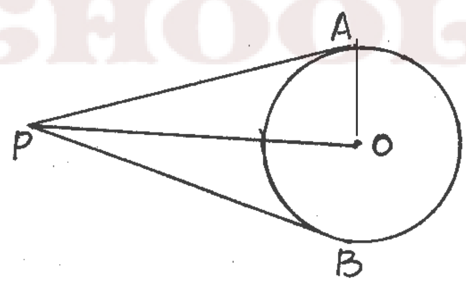
7. Two tangents PA and PB are drawn to the circle with center O, such that $\angle APB = 120^o$. Prove that OP=2AP.

Ans: Given : - $\angle APB = 120^o$
Construction : -Join OP
To prove : -OP = 2AP
Proof :- $\angle APB = 120^o$
$\therefore \angle APO = \angle OPB = 60^o$
$\cos 60^o = \frac{AP}{OP}$
$\frac{1}{2} = \frac{AP}{OP}$
$\therefore OP = 2AP$
Hence proved



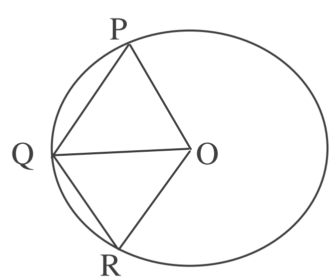
8. From a point P, two tangents PA are drawn to a circle with center O. If OP=diameter of the circle show that triangle APB is equilateral.

Ans: PA=PB (length of tangents from an external point
From ΔOAP,
$\sin\angle APO = \frac{OA}{OP} = \frac{1}{2}$
Since OP = 2OA (Since OP=Diameter)
$\therefore \angle APO = 30^o$
since $\Delta\ APO \cong \Delta BPO$
$\angle APO = \angle BPO = 30^o$
$\therefore \angle APB = 60^o$
ΔAPB is equilateral



9. In the given fig OPQR is a rhombus, three of its vertices lie on a circle with centre O If the area of the rhombus is $32\sqrt{3}\ cm^2$. Find the radius of the circle.

Ans: QP = OR
OP = OQ
$\therefore \Delta OPQ$ is a equilateral Δ.
area of rhombus = 2 (ar of ΔOPQ)
$32\ \sqrt{3} = 2\left(\frac{\sqrt{3}r^2}{4}\right)$



$32\sqrt{3} = \frac{\sqrt{3}r^2}{2}$

$r^2 = 32 \times 2 = 64$

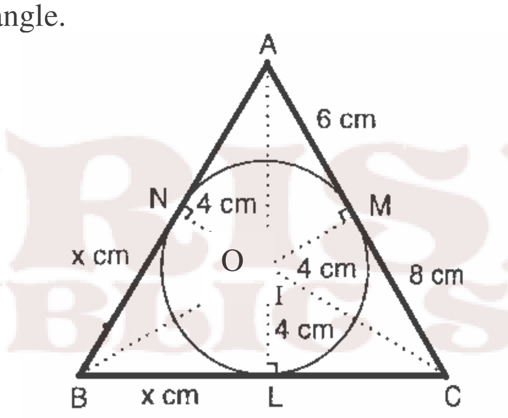
$\Rightarrow r = 8cm$

$\therefore$ Radius = 8cm

10. If PA and PB are tangents to a circle from an outside point P, such that PA=10cm and $\angle APB = 60^o$. Find the length of chord AB.

Self Practice

11. The radius of the in circle of a triangle is 4cm and the segments into which one side is divided by the point of contact are 6cm and 8cm. Determine the other two sides of the triangle.



(Ans: 15, 13)

Ans: a = BC = $x + 8$

b = AC = 6 + 8 = 14cm

c = AB = $x + 6$

Semi – perimeter = $\frac{a+b+c}{2}$

$= \frac{BC + AC + AB}{2}$

$= \frac{x + 8 + 14 + x + 6}{2}$

$= \frac{2x + 28}{2}$

$= x + 14$

Area of $\Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$ on substituting we get

$= \sqrt{(x+14)(6)(x)(8)}$

$= \sqrt{(x+14)(48x)}$ ……………..(1)

Area of ΔABC = area ΔAOB + area BOC + area Δ AOC

area ΔAOC = $\left(\frac{1}{2} b.h\right) = \frac{1}{2}$ x 4 x 14

= 28

On substituting we get

∴ area ΔABC = area ΔAOC + area ΔBOC + area ΔAOB

= 4x + 56 ……………(2)

From (1) and (2)

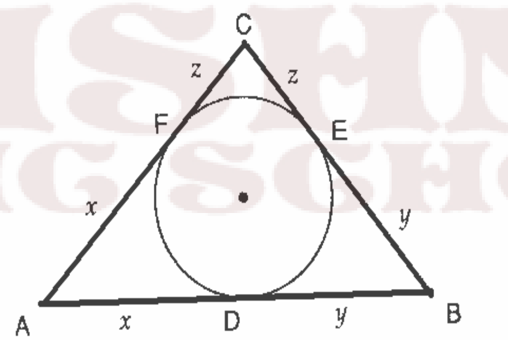
4x + 56 = $\sqrt{(x+14)(48x)}$

Simplify we get $x = 7$

∴AB = x + 6 = 7 + 6 = 13cm

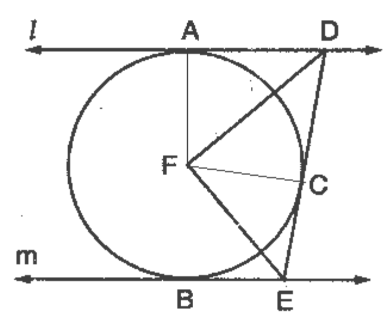
∴BC = x + 8 = 7 + 8 = 15cm

12. A circle is inscribed in a triangle ABC having sides 8cm, 10cm and 12cm as shown in the figure. Find AD, BE and CF. (Ans :7cm ,5cm,3cm)



Self Practice

13. Prove that the intercept of a tangent between two parallel tangents to a circle subtends a right angle at the centre.



Since Δ ADF ≅ Δ DFC

∠ADF = ∠CDF

∴ ∠ADC = 2 ∠CDF

Similarly we can prove ∠CEB = 2∠CEF

Since $l \parallel$ m

∠ADC + ∠CEB = 180°

⇒2∠CDF + 2∠CEF = 180°

$\Rightarrow \angle CDF + \angle CEF = 90^o$

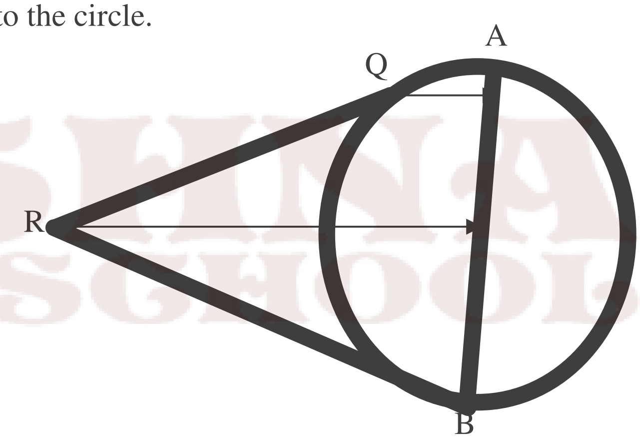
In Δ DFE

$\angle DFE = 90^o$

14. Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.

Ans: me as question No.5

15. QR is the tangent to the circle whose centre is P. If QA || RP and AB is the diameter, prove that RB is a tangent to the circle.



Self Practice

UNIT-10

CONSTRUCTIONS

Questions for self practice

1. Draw a line segment AB of length 4.4cm. Taking A as centre, draw a circle of radius 2cm and taking B as centre, draw another circle of radius 2.2cm. Construct tangents to each circle from the centre of the other circle.

2. Draw a pair of tangents to a circle of radius 2cm that are inclined to each other at an angle of 90^0.

3. Construct a tangent to a circle of radius 2cm from a point on the concentric circle of radius 2.6cm and measure its length. Also, verify the measurements by actual calculations. (length of tangent =2.1cm)

4. Construct an isosceles triangle whose base is 7cm and altitude 4cm and then construct another similar triangle whose sides are $1\frac{1}{2}$ times the corresponding sides of the isosceles triangle.

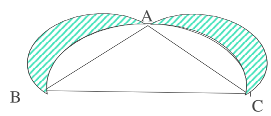
5. Draw a line segment AB of length 8cm. taking A as center, draw a circle of radius 4cm and taking B as centre, draw another circle of radius 3cm. Construct tangents to each circle from the center of the other circle.

MENSURATION

AREAS RELATED TO CIRCLES

The mathematical sciences particularly exhibit order, symmetry, and limitation; and these are the greatest forms of the beautiful.

1. In the adjoining figure ΔABC right angled triangle right angled at A. Semi circles are drawn on the sides of the triangle ΔABC. Prove that area of the Shaded region is equal to area of ΔABC



Ans: Refer CBSE paper 2008

2. The sum of the diameters of two circles is 2.8 m and their difference of circumferences is 0.88m. Find the radii of the two circles (Ans: 77, 63)

Ans: $d_1 + d_2 = 2.8$ m= 280cm

$r_1+r_2 = 140$

$2\ \Pi\ (r_1 - r_2) = 0.88\text{m} = 88\text{cm}$

$r_1 - r_2 = \dfrac{88}{2\Pi} = \dfrac{88x7}{44} = 2 \text{ x } 7 = 14$

$r_1+r_2 = 140$

$r_1-r_2 = 14$

$2r_1 = 154$

$r_1=77$

$r_2 = 140 - 77 = 63$

$r_1 = 77$ cm, $r_2= 63$cm

3 Find the circumference of a circle whose area is 16 times the area of the circle with diameter 7cm (Ans: 88cm)

Ans: $\Pi R^2 = 16\ \Pi r^2$

$R^2 = 16\ r^2$

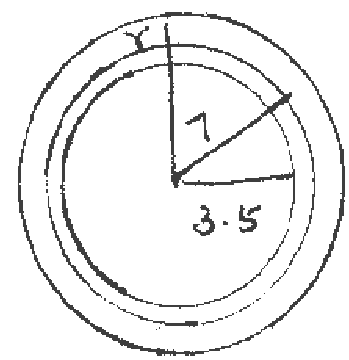
$R^2 = 16 \text{ x } \dfrac{7}{2} \text{ x } \dfrac{7}{2}$

$= 49 \text{ x } 4 \quad \Rightarrow R = 7 \text{ x } 2 = 14\text{cm}$

Circumference $= 2 \text{ x } \dfrac{22}{7} \text{ x } 14 = 2 \text{ x } 22 \text{ x } 2 = 88$ cm

4. Find the area enclosed between two concentric circles of radii 3.5cm, 7cm. A third concentric circle is drawn outside the 7cm circle so that the area enclosed between it and the 7cm circle is me as that between two inner circles. Find the radius of the third circle (Ans: 115.5 cm^2 $r = \sqrt{343}/2$)

Ans: Area between first two circles = $\Pi \times 7^2 - \Pi \times 3.5^2$



$= 49\Pi - 12.25\Pi$ -------------(1)

Area between next two circles = $\Pi r^2 - \Pi \times 7^2$

$= \Pi r^2 - 49\Pi$ -----------------(2)

(1) & (2) are equal

$49\Pi - 12.25\Pi = \Pi r^2 - 49\Pi$

$\Pi r^2 = 49\Pi + 49\Pi - 12.25\Pi$

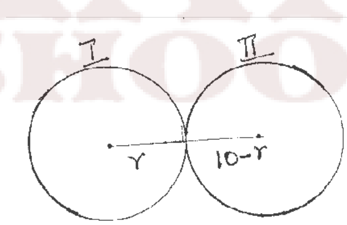
$\therefore r^2 = 98 - 12.25 = 85.75$

$r^2 = \frac{8575}{100} = \frac{343}{4}$

$r = \frac{\sqrt{343}}{2}$ cm.

5. Two circles touch externally. The sum of their areas is 58π cm^2 and the distance between their centres is 10 cm. Find the radii of the two circles. (Ans:7cm, 3cm)

Ans: Sum of areas = $\Pi r^2 + \Pi(10 - r)^2 = 58\Pi$



$\Pi r^2 + \Pi(100 - 20r + r^2) = 58\Pi$

$r^2 + 100 - 20r + r^2 = 58$

$2r^2 - 20r + 100 - 58 = 0$

$2r^2 - 20r + 42 = 0$

$r^2 - 10r + 21 = 0$

$(r-7), (r-3) = 0$

r=7cm,3cm

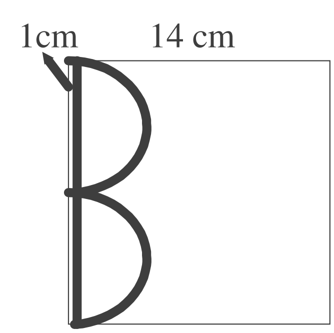
6. From a sheet of cardboard in the shape of a square of side 14 cm, a piece in the shape of letter B is cut off. The curved side of the letter consists of two equal semicircles & the breadth of the rectangular piece is 1 cm. Find the area of the remaining part of cardboard. (Ans: 143.5 cm^2)

Ans: Area of remaining portion = Area of square – Area of 2 semi circles – Area of rectangle

$= 14 \times 14 - \Pi \times 3.5^2 - 14 \times 1$

$=196 - \frac{22}{7} \times 3.5 \times 3.5 - 14$

$=196 - 38.5 - 14 = 143.5\ cm^2$



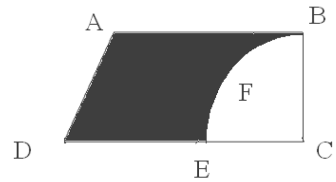
7. A piece of cardboard in the shape of a trapezium ABCD & AB || DE, $\angle BCD = 90^0$, quarter circle BFEC is removed. Given AB = BC = 3.5 cm, DE = 2 cm. Calculate the area of remaining piece of cardboard. (Ans:6.125 cm^2)

Ans: Area of remaining portion = Area of trap – Area of quadrant

$$= \frac{1}{2} \text{ x } 3.5\ (5.5 + 3.5) - \frac{1}{4} \text{ x } \frac{22}{7} \text{ x}3.5\text{x}3.5$$

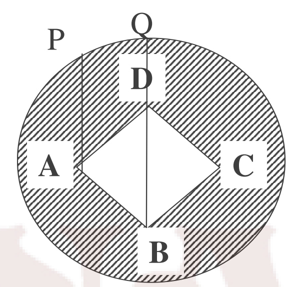
$$= 15.75 - \frac{19.25}{2} = 15.75 - 9.625$$

$$= 6.125 \text{ cm}^2$$



8. In the figure, ABCD is a square inside a circle with centre O. The Centre of the square coincides with O & the diagonal AC is horizontal of AP, DQ are vertical & AP = 45 cm, DQ = 25 cm. Find a) the radius of the circle b) si

c) area of shaded region (use π =3.14 , $\sqrt{2}$ = 1.41)



Ans: a) 53cm
b) 39.48cm
c) 7252.26 cm^2

Self Practice

9. The area enclosed between two concentric circles is 770cm^2. If the radius of the outer circle is 21cm, find the radius of the inner circle. (Ans :14cm)

Ans:

$$\Pi R^2 - \Pi r^2 = 770$$

$$\Pi (21^2 - r^2) = 770$$

$$21^2 - r^2 = \frac{770}{22} \text{ x } 7 = \frac{70}{2} \text{ x } 7$$

$$r^2 = 441 - \frac{490}{2} = 441 - 245 = 196$$

$$r = \pm 14$$

$$r = 14\text{cm}$$

10. A circular disc of 6 cm radius is divided into three sectors with central angles 120^0, 150^0,90^0. What part of the circle is the sector with central angles 120^0. Also give the ratio of the areas of three sectors. (Ans: $\frac{1}{3}$ (Area of the circle) 4 : 5 : 3)

Ans: Ratio of areas $= \frac{120}{360} \Pi \text{ x } 6^2 : \frac{150}{360} \Pi \text{ x } 6^2 : \frac{90}{360} \Pi \text{ x } 6^2$

$= 12\ \Pi : 15\Pi : 9\Pi$
$= 4 : 5 : 3$

Area of sector of central angle $120^o = \frac{120^o}{360^o} \text{x} \Pi\, r^2$

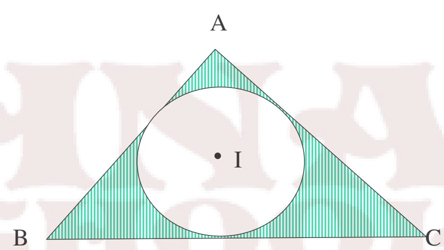
(i.e.) $\frac{1}{3}$ of area of the circle.

11. If the minute hand of a big clock is 1.05 m long, find the rate at which its tip is moving in cm per minute. (Ans:11cm/min)

Ans: Self Practice

12. ABC is a right angled triangle in which $\angle A = 90^0$. Find the area of the shaded region if AB = 6 cm, BC=10cm & I is the centre of the Incircle of ΔABC. (Ans: $\frac{80}{7}$ sq.cm)

Ans: $\angle A = 90^0$



BC = 10cm; AB = 6cm;
$\therefore$ AC = 6cm

Area of the $\Delta = \frac{1}{2}$ x 6 x 8= 24 cm^2

Let the Radius of the Incircle be r

$\therefore \frac{1}{2}$ x 10 x r + $\frac{1}{2}$ x 8 x r + $\frac{1}{2}$ x 6 x r = 24

$\frac{1}{2}$ r [10 + 8 + 6] = 24

r= 2 cm

$\therefore$ Area of circle = $\Pi r^2 = \frac{22}{7}$ x 2 x 2 = $\frac{88}{7} cm^2$

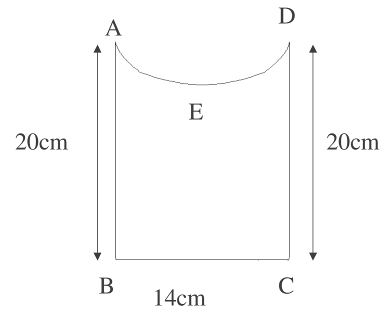
Area of shaded region = 24 - $\frac{88}{7} = \frac{168-88}{7} = \frac{80}{7} cm^2$

13. Find the perimeter of the figure, where AED is a semi-circle and ABCD is a rectangle. (Ans : 76cm)

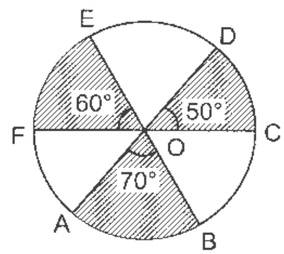
Ans: Perimeter of the fig = 20 + 14 + 20 + length of the arc (AED)

Length of Arc = $(\Pi \text{ x } r) = \frac{22}{7}$ x7 = 22cm

$\therefore$Perimeter of the figure = 76 cm



14. Find the area of shaded region of circle of radius =7cm, if $\angle AOB=70^o$, $\angle COD=50^o$ and $\angle EOF=60^o$.



(Ans:77cm^2)

Ans: Ar(Sector AOB + Sector COD + Sector OEF)

$$= \frac{70}{360}\,\Pi \text{ x } 7^2 + \frac{50}{360}\,\Pi \text{ x } 7^2 + \frac{60}{360}\,\Pi \text{ x } 7^2$$

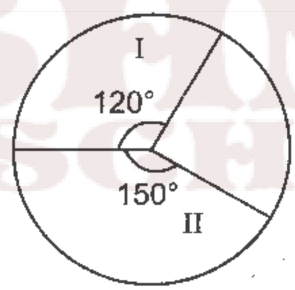
$$49\,\Pi\,\left(\frac{7}{36} + \frac{5}{36} + \frac{6}{36}\right) = 49\,\Pi \text{ x } \frac{18}{36} = \frac{49}{2} \text{ x } \frac{22}{7} = 77 \text{ cm}^2$$

15. What is the ratio of the areas of sectors I and II ? (Ans:4:5)

Ans: Ratio will be

$$\frac{120}{360}\,\Pi r^2 : \frac{150}{360}\,\Pi r^2$$

$$\frac{4}{12} : \frac{5}{12} = 4{:}5$$



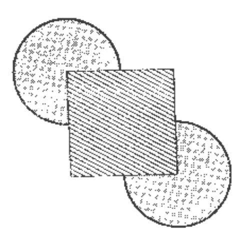
16. Find the area of shaded region, if the side of square is 28cm and radius of the sector is ½ the length of side of square. (Ans:1708cm)

Ans: Area of shaded region is

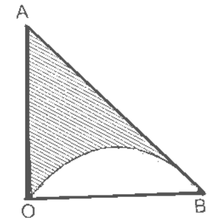
$$2\left(\frac{270}{360}\right)\Pi \text{ x } 14 \text{ x } 14 + 28 \text{ x } 28$$

$$2 \text{ x } \frac{3}{4} \text{ x } \frac{22}{7} \text{ x } 14 \text{ x} 14 + 784$$

$$924 + 784 = 1708 \text{ cm}^2$$



17. If OA = OB = 14cm, $\angle AOB = 90^o$, find the area of shaded region. (Ans:21cm^2)



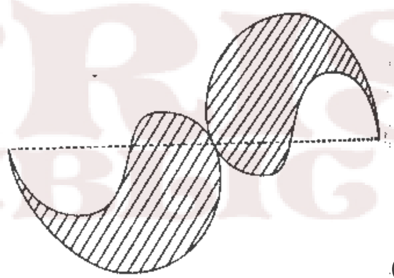
Ans: Area of the shaded region

= Area of Δ AOB – Area of Semi Circle

$= \frac{1}{2} \times 14 \times 14 - \frac{1}{2} \times \frac{22}{7} \times 7 \times 7$

$98 - 77 = 21 \text{ cm}^2$

18. The given figure consists of four small semicircles and two big semicircles. If the smaller semicircles are equal in radii and the bigger semicircles are also equal in radii, find the perimeter and the area of the shaded portion of the figure. Given that radius of each bigger semicircle is 42cm.



(Ans:528cm, 5544 sq cm)

Ans: Perimeter of the shaded region

= 2 [Perimeter (Bigger semi circle) + Perimeter (smaller semi circle) + Perimeter (small semi circle)]

$= 2 (42 \Pi + 21 \Pi + 21 \Pi)$

$= 84 \Pi$

$= 2 \times 84 \times \frac{22}{7} = 24 \times 22 = 528$ cm

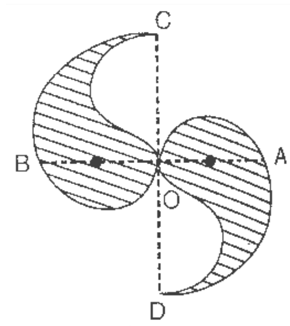
Area of shaded region

= [Area(big semi circle)]

$= 2 \times \Pi \times 42 \times 42 \times \frac{1}{2} = \frac{22}{7} \times 42 \times 42 = 5544 \text{ cm}^2$

19. The boundary of the shaded portion in the adjoining figure consists of our half-circles and two quarter-circles. Find the length of the boundary and the area of the shaded portion, if OA=OB=OC=OD=14cm. (Ans:132 cm, 308 sq cm)

Ans: Proceed as in sum no 18.



20. The adjoining figure shows the cross-section of a railway tunnel.

The radius of the tunnel is 3.5m (i.e., OA=3.5m) and $\angle AOB=90^o$.

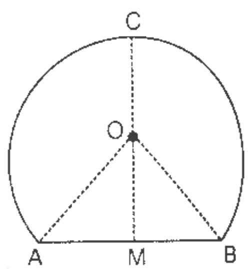
Calculate :

i. the height of the tunnel.

ii. the perimeter of its cross section, including base.

iii. the area of the cross-section

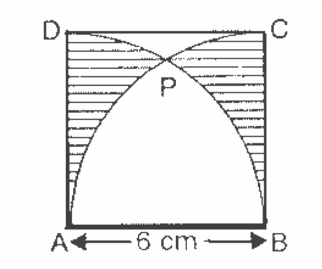
iv. the internal surce area of the tunnel, excluding base, if its length is 50m.



(Ans: (i) 5.97m (ii) 21.44m (iii) 28.875 sq m (iv) 825 sq m)

Ans: Self Practice

21. In the adjoining figure, ABCD is a square of side 6cm. Find the area of the shaded region.



(Ans: 34.428 sq cm)

Ans: From P draw PQ $\perp$ AB

AQ = QB = 3cm

Join PB. Since arc APC is described by a circle with center B,

so BA = BP=BC =6cm.

In Δ PQB $\cos\theta = \frac{QB}{PB} = \frac{1}{2}$

$\therefore \theta = 60^o$

Area of sector BPA = $\frac{60}{360}\Pi\,(6^2) = 18.84$cm

Area of Δ BPQ = $\frac{1}{2}$(QB) (PQ) = $\frac{1}{2}$(3)(6 Sin 60) = 7.794Sq.cm

→ Area of portion APQ = Area of sector BPA – Area of Δ BPQ

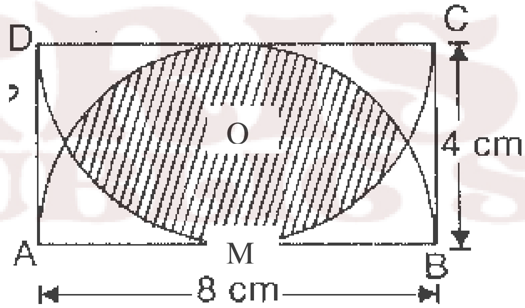
= 18.84 – 7.794 = 11.046 Sq.cm

Area of shaded portion = 2 x Area of Quadrant ABC – 2 Area APQ

= [2 x $\frac{\Pi}{4}(6)^2$ - 2 x 11.046]

=34.428 Sq.cm

22. In the adjoining figure, ABCD is a rectangle with sides 4cm and 8cm. Taking 8cm as the diameter, two semicircles are drawn. Find the area overlapped by the two semicircles.



(Ans:19.66 sq cm)

Ans: In Δ OMB

Cos∠ BOM= $\frac{OM}{OB} = \frac{2}{4} = \frac{1}{2}$

$\therefore \angle$ BOM = 60^0

$\angle$AOB = 120^0

Area. Overlapped by semi circles

= 2 ($\frac{120}{360}$ x Π (4^2) - $\frac{1}{2}$ AB x OM)

= 2 ($\frac{\Pi}{3}$ x 16 - $\frac{1}{2}$ (2 x AM Sin 60^0) x 2)

= 2 ($\frac{22}{7}$ x $\frac{1}{3}$ x 16 – 2 x 4 x $\frac{\sqrt{3}}{2}$)

$= 2\ (16.76 - 6.93) = 19.66$ Sq. cm

UNIT-12

PROBLEMS BASED ON CONVERSION OF SOLIDS

1. A solid is in the form of a right circular cone mounted on a hemisphere. The radius of the hemisphere is 3.5 cm and the height of the cone is 4 cm. The solid is placed in a cylindrical tub, full of water, in such a way that the whole solid is submerged in water. If the radius of the cylindrical tub is 5 cm and its height is 10.5 cm, find the volume of water left in the cylindrical tub (use $\pi = \frac{22}{7}$]

(Ans: 683.83 cm³)

Ans: No. of solid = vol of cone + vol of hemisphere

$= \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3$

$= \frac{1}{3}\pi r^2 [h + 2r]$

On substituting we get,

$= 141.17\ cm^3$

vol of cylinder $= \pi r^2 h$

On substituting we get,

$= 825\ cm^3$

volume of H_2O left in the cylinder $= 825 - 141.17$

$= 683.83\ cm^3$

2. A bucket of height 8 cm and made up of copper sheet is in the form of frustum of right circular cone with radii of its lower and upper ends as 3 cm and 9 cm respectively. Calculate
   i) the height of the cone of which the bucket is a part
   ii) the volume of water which can be filled in the bucket
   iii) the area of copper sheet required to make the bucket (Leave the answer in terms of π (Ans: 129 π cm^2)

Ans: Let total height be h

$\Rightarrow \frac{h}{h+8} = \frac{3}{9}$ (similar Δ's)

$\Rightarrow h = 4$ cm

$\therefore$ ht. of cone which bucket is a part = 4 cm

Substitute to get Ans.: for ii) iii)

3. A sphere and a cube have equal surce areas. Show that the ratio of the volume of the

sphere to that of the cube is $\sqrt{6} : \sqrt{\pi}$.

Ans: S.A. of sphere = S.A of cube

$\Rightarrow$ $4\pi r^2 = 6a^2$

$\Rightarrow$ $r = \sqrt{\frac{6 \ \ ^2}{4\Pi}}$

$\therefore$ ratio of their volume $\frac{v_1}{v_2} = \frac{\frac{4}{3}\Pi\gamma^3}{a^3}$

On simplifying & substituting, we get √6 : √π

4. A right triangle whose sides are 15 cm and 20 cm is made to revolve about its hypotenuse. Find the volume and surce area of the double cone so formed.

(Ans : 3768cu.cm,1318.8 Sq.cm)

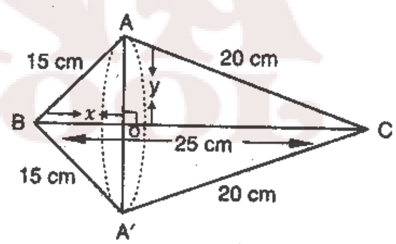
Ans: $BC = \sqrt{15^2 + 20^2}$ = 25 cm

Apply Py. Th to right Δ OAB & OAC to get OB = 9cm OA = 12cm

Vol of double cone = vol of CAA^1 + vol of BAA^1

$= \frac{1}{3}\pi \times 12^2 \times 16 + \frac{1}{3}\pi \times 12^2 \times 9$

$= 3768 \text{ cm}^3$



of double cone = C of CAA^1 + C of BAA^1

$= \pi \times 12 \times 20 + \pi \times 12 \times 15$

$= 1318.8 \text{ cm}^3$

5. Water in a canal 30 dm wide and 12 dm deep is flowing with a velocity of 10 km/h. How much area will it irrigate in 30 minutes if 8 cm of standing water is required for irrigation? (Ans: 225000 cu. m)

Ans: Width of canal = 30 dm = 3m

Depth of canal = 1.2 m

Velocity = 10 km / h = 10000 m/h

Length of water column is formed in 30 min = 10000 x $\frac{1}{2}$ = 5000 m

Let xm^2 of area be irrigated $\Rightarrow x \times \frac{8}{100} = 5000 \times 1.2 \text{ x } 3$

$\Rightarrow x = 225000 \text{ m}^2$

6. A cylindrical vessel of diameter 14 cm and height 42 cm is fixed symmetrically inside a similar vessel of diameter 16 cm and height 42 cm. The total space between two vessels is filled with cork dust for heat insulation purposes. How many cubic centimetres of cork dust will be required?

(Ans:1980 cu.cm)

Ans: volume of cork dust required $= \pi R^2 h - \pi r^2 h$

$= \pi\, 42\, [\,64 - 49]$

$= 1980\ cm^3$

7. An ice-cream cone has a hemispherical top. If the height of the cone is 9 cm and base radius is 2.5 cm, find the volume of ice cream cone. (Ans: $91\frac{2}{3}$ cu.cm)

Ans: Do yourself

8. A building is in the form of a cylinder surrounded by a hemispherical vaulted dome and contains $41\ \frac{19}{21}$ cu m of air. If the internal diameter of the building is equal to its total height above the floor, find the height of the building.

(Ans : 4m)

Ans: Volume of building $= 41\ \frac{19}{21}\ m^3$

$\Rightarrow \pi.r^2.r + \frac{2}{3}\pi r^3 = 41\ \frac{19}{21}$

$\Rightarrow \pi \times r^3 \times \frac{5}{3} = \frac{880}{21}$

$\Rightarrow r^3 = \frac{880}{21} x \frac{7}{22} x \frac{3}{5}$

$\Rightarrow r^3 = 8$

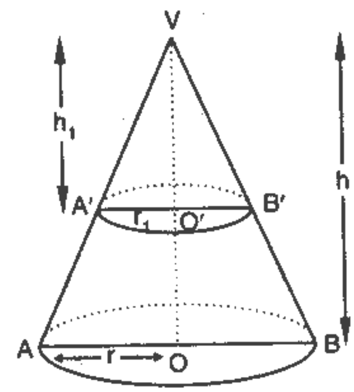
$\Rightarrow r = 2\ m$

$\therefore$ height of building $= 4$ cm

9. The height of the Cone is 30 cm A small cone is cut of f at the top by a plane parallel to its base if its volume be $\frac{1}{27}$ of the volume of the given cone at what height above the base is the section cut

(Ans:20 cm)

Ans: $\Delta\ VO^1B \sim \Delta VOB$



$\therefore\ \frac{H}{h} = \frac{R}{r} = \frac{30}{h} = \frac{R}{r}$ -----(1)

APQ: vol of cone $VA^1B^1 = \frac{1}{27}$ (vol of cone VAB)

$\Rightarrow \frac{1}{3}\pi r^2 h = \frac{1}{27}\left(\frac{1}{3}\pi R^2 H\right)$

$\Rightarrow h^3 = 1000$ (using (1)

h = 10 cm

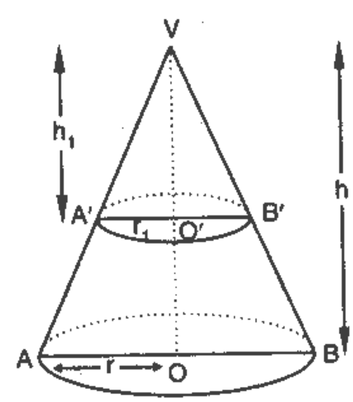
$\therefore$ height at which section is made (30 – 10) = 20 cm

10. A hollow cone is cut by a plane parallel to the base and the upper portion is removed. If the curved surce of the remainder is $\frac{8}{9}$ th of the curved surce of the whole cone, find the ratio of the line segments into which the cone's altitude is divided by the plane.

(Ans:1:2)

We know that $\Delta\ VO^1B \sim \Delta VOB$

$\frac{h}{H} = \frac{r}{R} = \frac{l}{L}$



C. of frustum = $\frac{8}{9}$ (C of the cone)

$\Pi\ (R + r)\ (L - l) = \frac{8}{9}\Pi\ RL$

$\Rightarrow \left(\frac{R+r}{R}\right)\left(\frac{L-l}{L}\right) = \frac{8}{9}$

$\Rightarrow \left(1+\frac{r}{R}\right)\left(1-\frac{l}{L}\right) = \frac{8}{9}$

$\Rightarrow \left(1+\frac{h}{H}\right)\left(1-\frac{h}{H}\right) = \frac{8}{9}$

On simplifying we get $\frac{h^2}{H^2} = \frac{1}{9}$

$$\frac{h}{H} = \frac{1}{3}$$

$\Rightarrow$ H = 3h

required ratios = $\frac{h}{H-h} = \frac{1}{2}$

11. Two right circular cones X and Y are made X having 3 times the radius of Y and Y having half the Volume of X. Calculate the ratio of heights of X and Y. (Ans: 9 : 2)

Ans: Let radius of cone X = r

Radius of Cone Y = 3r

V of Y = $\frac{1}{2}$ volume of X

$\frac{1}{3}\pi r^2{}_1 h_1 = \frac{1}{2}\left(\frac{1}{3}\pi r^2{}_2 h_2\right)$

=> $r^2 h_1 = \frac{1}{2} 9 r^2 h_2$

$\frac{h_1}{h_2} = \frac{9r^2}{2r^2}$

$\frac{h_1}{h_2} = \frac{9}{2}$

12. If the areas of three adjacent ces of cuboid are x, y, z respectively, Find the volume of the cuboids.

Ans: lb = x , bh = y, hl = z

Volume of cuboid = lbh

$V^2 = l^2 b^2 h^2 = xyz$

$V = \sqrt{xyz}$

13. A shuttlecock used for playing badminton has the shape of a frustum of a Cone mounted on a hemisphere. The external diameters of the frustum are 5 cm and 2 cm, and the height of the entire shuttlecock is 7cm. Find the external surce area.

(Ans: $74.26 cm^2$)

Ans: r_1 = radius of lower end of frustum = 1 cm

r_2 = radius of upper end = 2.5 cm

h = ht of frustum = 6cm

$l = \sqrt{h^2 + (r_2 - r_1)^2} = 6.18$ cm

External surce area of shuttlecock = $\pi (r_1 + r_2) l + 2\pi r^2{}_1$

On substituting we get, $= 74.26 \text{ cm}^2$

14. A Solid toy in the form of a hemisphere surmounted by the right circular cone of height 2cm and diameter of the base 4 cm .If a right circular cylinder circumscribes the toy, find how much more space than the toy it will cover.

(Ans: 8π)

Ans : Self practice

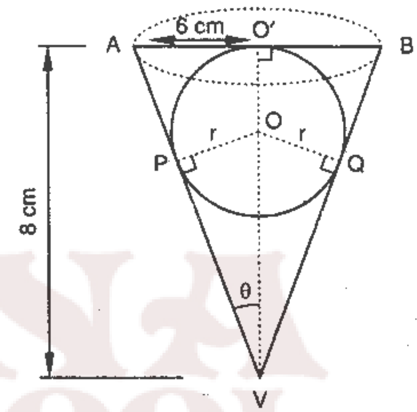
15. A conical vessel of radius 6cm and height 8cm is completely filled with water. A sphere is lowered into the water and its size is such that when it touches the sides, it is just immersed as shown in the figure. What fraction of water flows out.

[Ans: $\frac{3}{8}$]

Ans: This problem can be done in many ways

Let "r" be the radius of sphere

In right triangle



Tan θ = $\frac{6}{8} = \frac{3}{4}$

=> Sin θ = $\frac{3}{5}$

in rt Δ

Sin θ = $\frac{r}{VO} = \frac{3}{5} = \frac{r}{8-r}$

r = 3 cm

Volume of H_2O that flows out of cone = volume of sphere

fraction of water Overflows = $\frac{\text{volume f sphere}}{\text{Volume of cone}}$

$= \frac{36\pi}{96\pi} = \frac{3}{8}$

16. A golf ball has a diameter equal to 4.1cm. Its surce has 150 dimples each of radius 2mm. Calculate the total surce area which is exposed to the surroundings assuming that the dimples are hemispherical.

(Ans: 71.68)

Ans: of ball = $4\pi \times (\frac{4.1}{2})^2 = 16.8\,\pi\ cm^2$

T exposed to surroundings

= of ball – 150 $\times \pi r^2 + 150 \times 2\pi r^2$

$= 16.8\pi + 150\pi r^2$

$= 71.68\ cm^2$

17. A solid metallic circular cone 20cm height with vertical angle 60 is cut into two parts at the middle point of its height by a plane parallel to the base. If the frustum, so obtained be drawn into a wire of diameter $\frac{1}{16}$cm Find the length of the wire. (Ans:7964.4m)

Ans: Let r_2 & r_2 be the two ends of the frustum $\frac{r_1}{20}$ = tan 30

$$r_1 = \frac{20}{\sqrt{3}}; r_2 \frac{10}{\sqrt{3}} cm$$

volume of frustum = $\frac{1}{3}\pi$ h ($r^2{}_1 + r^2{}_2 + r_1 r_2$)

$$= \frac{1}{3}\pi \times 10 \left(\frac{400}{3}+\frac{100}{3}+\frac{200}{3}\right)\text{cm}$$

Since the frustum is drawn into a wire of length x

Volume of frustum = volume of cylinder

$$\frac{1}{3}\pi \times 10 \times \frac{700}{3} = \pi \left(\frac{1}{32}\right)^2 \times x$$

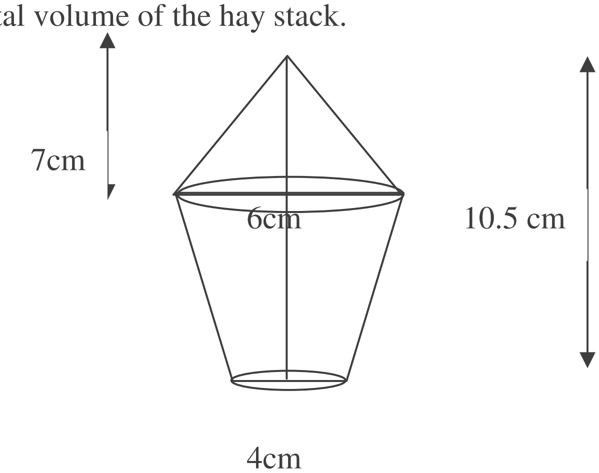
$$\Rightarrow x = \frac{7168000}{9} \text{ cm}$$

x = 7964.4m

18. If the areas of the circular bases of a frustum of a cone are $4cm^2$ and $9cm^2$ respectively and the height of the frustum is 12cm. What is the volume of the frustum. (Ans:$44cm^2$).

Ans: Self practice

19. The lower portion of a hay stack is an inverted cone frustum and the upper part is a cone find the total volume of the hay stack. (Ans: 135.67cu cm)



Ans: Self practice

20. A vessel in shape of a inverted cone is surmounted by a cylinder has a common radius of 7cm this was filled with liquid till it covered one third the height of the cylinder. If the height of each part is 9cm and the vessel is turned upside down. Find the volume of the liquid and to what height will it reach in the cylindrical part. (Ans:924 π cu cm, 6cm)

Ans: Volume of liquid in the vessel = $\frac{1}{3}\pi\ (7)^2\ (9) + \pi\ (7)^2\ (3)$

$= 924$ cu cm

height of cylindrical part = $\dfrac{924}{\frac{22}{7}\times 49}$ = 6 cm

UNIT 13

STATISTICS AND PROBABILITY

Statistics are the only tools by which an opening can be cut through the formidable thicket of difficulties that bars the path of those who pursue the Science of Man.

1. Marks obtained by 70 students are given below:

Marks	20	70	50	60	75	90	40
No. of Students	8	12	18	6	9	5	12

Find the median. (Ans:50)

Ans:

Marks	No . of students	c.f
20	8	8
40	12	20
50	18	38
60	6	44

70	12	53
75	9	58
90	5	70

N = 70

$$\frac{N}{2} = \frac{70}{2} = 35$$

The corresponding value of marks for 35 is 50

2. The sum of deviations of a set of values $x_1, x_2, x_3, \ldots\ldots\ldots\ldots x_n$, measured from 50 is -10 and the sum of deviations of the values from 46 is 70.
Find the value of n and the mean. (Ans:20,.49.5)

Ans: We have

$$\sum_{i=1}^{n}(X_i - 50) = -10 \text{ and } \sum_{i=1}^{n}(X_i - 46) = 70$$

$$\sum_{i=1}^{n} X_i - 50n = -10 \quad \ldots\ldots\ldots\ldots (1)$$

and $\sum_{i=1}^{n} X_i - 46_{\text{m}} = 70 \ldots\ldots\ldots\ldots..(2)$

subtracting (2) from (1) , we get

- 4 n = - 80 we get n = 20

$$\sum_{i=1}^{n} X_i - 50 \times 20 = -10$$

$$\sum_{i=1}^{n} X_i = 990$$

$$\text{Mean} = \frac{1}{n}\left(\sum_{i=1}^{n} X_i\right) = \frac{990}{20} = 49.5$$

hence n = 20 and mean = 49.5

3. Prove that $\sum(x_i - \bar{x}) = 0$

Ans: To prove $\sum_{i=1}^{n}(X_i - \bar{X}) = 0$ algebraic sum of deviation from mean is zero

We have, $\bar{X} = \frac{1}{n}\left(\sum_{i=1}^{n} X_i\right)$

$n\bar{X} = \sum_{i=1}^{n} X_i$

Now, $\sum_{i=1}^{n}(X_i - \bar{X}) = (X_1 - \bar{X}) + (X_2 - \bar{X}) + \ldots\ldots\ldots + (X_n - \bar{X})$

$$\sum_{i=1}^{n}(X_i - \bar{X}) = (X_1 + X_2 + \ldots\ldots\ldots + X_n) - n\bar{X}$$

$$\sum_{i=1}^{n}(X_i - \bar{X}) = \sum_{i=1}^{n} X_i - n\bar{X}$$

$$\sum_{i=1}^{n}(X_i - \overline{X}) = n\overline{X} - n\overline{X}$$

$$\sum_{i=1}^{n}(X_i - \overline{X}) = 0$$

Hence, $\sum_{i=1}^{n}(X_i - \overline{X}) = 0$

4.Compute the median from the following data

Mid value	115	125	135	145	155	165	175	185	195
Frequency	6	25	48	72	116	60	38	22	3

(Ans:135.8)

Ans: Here , we are given the mid values. So, we should first find the upper and lower limits of the various classes. The difference between two consecutive values is h = 125 – 115 = 10

$\therefore$ Lower limit of a class = Midvalue - h /2

Upper limit = Midvalue + h / 2

Calculate of Median

Mid – value	Class Groups	Frequency	Cumulative frequency
115	110-120	6	6
125	120-130	25	31
135	130-140	48	79
145	140-150	72	151
155	150-160	116	267
165	160-170	60	327
175	170-180	38	365
185	180-190	22	387
195	190-200	3	390
			$N = \Sigma f_i = 390$

We have,

N = 390 $\therefore$ N / 2 = 390 / 2 = 195

The cumulative frequency first greater than N i.e. 195 is 267 and the corresponding class is 150 – 160, so, 150 – 160 is the median class.

L = 150, f = 116, h = 10, f = 151

Now,

$$\text{Median} = L + \frac{\frac{n}{2} - f}{f} \times h$$

$$\text{Median} = 150 + \frac{195-151}{116} \times 10 = 153.8$$

5. The mean of 'n' observation is $\bar{x}$, if the first term is increased by 1, second by 2 and so on. What will be the new mean. (Ans: $\bar{x} + \frac{n+1}{2}$)

Ans: I term + 1
II term +2
III term + 3
.
.
n term + n

The Mean of the new numbers is $\bar{X} + \frac{\frac{n(n+1)}{2}}{n} = \bar{X} + \frac{(n+1)}{2}$

6. In a frequency distribution mode is 7.88, mean is 8.32 find the median. (Ans: 8.17)

Ans: Mode = 3 median – 2 mean
7.88 = 3 median – 2 x 8.32
7.88 +16.64 = 3 median
$\frac{24.52}{3}$ = median

$\therefore$ median = 8.17

7. The mode of a distribution is 55 & the modal class is 45-60 and the frequency preceding the modal class is 5 and the frequency after the modal class is 10.Find the frequency of the modal class. (Ans:15)

Ans: mode = 55
Modal class = 45 – 60
Modal class preceding $f_1 = 5$
After the modal class = $f_2 = 10$

$$\text{Mode} = L + \frac{f - f_1}{2f - f_1 - f_2} \times h$$

$$55 = 45 + \frac{f-5}{2f-5-10} \text{ x } 15$$

$$10 = (\frac{f-5}{2f-15}) \text{ x } 15$$

$$\frac{10}{15} = \frac{f-5}{2f-15}$$

20 f - 150 = 15 f – 75

5 f = 75

$$f = \frac{75}{5} = 15$$

8. The mean of 30 numbers is 18, what will be the new mean, if each observation is increased by 2? (Ans:20)

Ans: Let x_1 , x_2, x_3 …… x_{30} be 30 number with then mean equal to 18 then

$$\bar{X} = \frac{1}{n}(\sum x_i)$$

$$18 = \frac{x_1 + x_2 + x_3 ...... + x_{30}}{30}$$

$x_1 + x_2 + x_{3+}$ …… $+ x_{30}$ = 18 x 30 = 540

New numbers are x_1 + 2, x_2, + 2 x_3 + 3 …… x_{30} + 2

Let $\bar{X}$ be the mean of new numbers

then $\bar{X} = \frac{(x_1+2)+(x_2+2)+......+(x_{30}+2)}{30}$

$$\bar{X} = \frac{\frac{n(n+1)}{2}}{n}$$

$$\bar{X} = \frac{n+1}{2}$$

$$\frac{(x_1 + x_2 + ... + x_{30}) + 2X30}{30} = \frac{540+60}{30}$$

Mean of new numbers = $\frac{600}{30}$ = 20

9. In the graphical representation of a frequency distribution if the distance between mode and mean is k times the distance between median and mean then find the value of k. (Ans:k=3)

Self Practice

10. Find the mean of 30 numbers given mean of ten of them is 12 and the mean of remaining 20 is 9. (Ans:10)

Ans: Total number of mean = 30

Mean of 10 is = 12

$$12 = \frac{\sum_{i=1}^{n} X_i}{10}$$

$\sum X_i = 12 \times 10 = 120$ ---(1)

Mean of 20 numbers is = 9

$$9 = \frac{\sum X_i}{20}$$

$9 \times 20 = \sum_{i=1}^{n} X_i$ ----- (2)

$180 = = \sum X_i$

(1) + (2)

$$\text{Mean of 20 numbers} = \frac{120 + 180}{30}$$

$$= \frac{300}{30} = 10$$

PROBABILITY

Life is a school of probability.

1. **An** integer is chosen at random from the first two hundreds digit. What is the probability that the integer chosen is divisible by 6 or 8. (Ans : $\frac{1}{4}$)

Ans: Multiples of 6 first 200 integers
6, 12, 18, 24, 30, 36, 42, 48, 54, 60, 66, 72, 78, 84, 90, 96, 102, 108, 114, 120, 126, 132, 138, 144, 150, 156, 162, 168, 174, 180, 186, 192, 198

Multiples of 8 first 200 integers
8,16,24,32,40,48,56,64,72,80,88,96,104,112,120,128,136,144,152,160, 168, 176,184,192,200

Number of Multiples of 6 or 8 = 50
P(Multiples of 6 or 8) = 50 / 200 = 1/4

2. A box contains 12 balls out of which x are black .if one ball is drawn at random from the box what is the probability that it will be a black ball ? If 6 more black balls are put in the box ,the probability of drawing a black ball is now double of what it was before. Find x. (Ans: x = 3)

Ans: Random drawing of balls ensures equally likely outcomes

Total number of balls = 12

Total number pf possible outcomes = 12

Number of black balls = x

(1) out of total 12 outcomes, vourable outcomes = x

$$P\text{ (black ball)} = \frac{\text{Number of vourable outcomes}}{\text{Total number of possible outcomes}} = \frac{x}{12}$$

(2) if 6 more black balls are put in the bag, then

The total number of black balls = x + 6

Total number of balls in the bag = 12 + 6 = 18

According to the question

Probability of drawing black ball is second case

= 2 X probability drawing of black ball in first case

$$\frac{x+6}{18} = 2\left(\frac{x}{12}\right)$$

$$\frac{x+6}{18} = \frac{x}{6}$$

$6x + 36 = 18x$

$x = 3$

hence number of black balls = 3

3. A bag contains 8 red balls and x blue balls, the odd against drawing a blue ball are 2: 5. What is the value of x? (Ans:20)

Ans: No. of blue balls be x
No. of red balls be 8
Total no. of balls = x + 8

Probability of drawing blue balls $= \frac{x}{8+x}$

Probability of drawing red balls $= \frac{8}{8+x}$

$$\frac{8}{8+x} : \frac{x}{8+x} = 2 : 5$$

$$2\left(\frac{x}{8+x}\right) = 5\left(\frac{8}{8+x}\right)$$

$2x = 40$

$\therefore\ x = 20$

4. A card is drawn from a well shuffled deck of cards
   (i) What are the odds in vour of getting spade? (Ans: 1:3, 3:1, 3:10, 1:25)
   (ii) What are the odds against getting a spade?
   (iii) What are the odds in vour of getting a ce card?
   (iv) What are the odds in vour of getting a red king

Ans: Total cards 52
Spade = 13
Remaining cards 39

i) The odds in vour of getting spade 13
The odds is not in vour of getting spade 39

$$= \frac{13}{52} : \frac{39}{52} = 1 : 3$$

ii) The odds against getting a spade 39

The odds not against getting a spade 13

$$= \frac{39}{52} : \frac{13}{52} = 3 : 1$$

iii) The odds in vour of getting a ce card 12
The odds not in vour of getting a ce card 40

$$= \frac{12}{52} : \frac{40}{52} = 3 : 10$$

iv) The odds in vour of getting a red king 2
The odds not in vour of getting a red king 50

$$= \frac{2}{52} : \frac{50}{52} = 1 : 25$$

5 A die is thrown repeatedly until a six comes up. What is the mple space for this experiment? HINT ;A= {6} B={1,2,3,4,5,}

Ans: The mple space is = {A, BA, BBA, BBBA, BBBBA……..})

6. Why is tossing a coin considered to be a ir way of deciding which team should get the ball at the beginning of a foot ball match?

Ans: equally likely because they are mutually exclusive events .

7. A bag contains 5 red balls and some blue balls. If the probability of drawing a blue ball is double that of a red ball , determine the number of blue balls in the bag. (Ans:10)

Ans: Let the number of blue balls is the bag be x

Then total number of balls is the bag = 5 + x

$\therefore$ Number of all possible outcomes = 5 + x

Number of outcomes vourable to the event of drawing a blue ball = x

(Q there are x blue balls)

$\therefore$ Probability of drawing a blue ball $\frac{x}{5+x}$

Similarly, probability of drawing a red ball = $\frac{5}{5+x}$

According to the answer

$$\frac{x}{5+x} = 2\left(\frac{5}{5+x}\right)$$

$x = 10$

8. A box contains 12 balls out of which x are black. If one ball is drawn at random from the box, what is the probability that it will be a black ball? If 6 more black balls are put in the box the probability of drawing a black ball is now double of what it was before. Find x? (Ans: 3)

Ans: Number of all possible outcomes = 12

Number of outcomes vourable to the event of drawing black ball = x

Required probability = $\frac{x}{12}$

Now when 6 more black balls are put in the box,

Number of all possible outcomes = 12 + 6 = 18

Number of outcomes vourable to the event of drawing a black ball = x + 6

$\therefore$ Probability of drawing a black ball = $\frac{x+6}{18}$

According to the question,

$\frac{x+6}{18} = 2\left(\frac{x}{12}\right)$

$\therefore \quad x = 3$

9. If 65% of the populations have black eyes, 25% have brown eyes and the remaining have blue eyes. What is the probability that a person selected at random has (i) Blue eyes (ii) Brown or black eyes (iii) Blue or black eyes (iv) neither blue nor brown eyes (Ans: $\frac{1}{10}, \frac{9}{10}, \frac{3}{4}, \frac{13}{20}$)

Ans: No. of black eyes = 65
No. of Brown eyes = 25
No. of blue eyes = 10
Total no. of eyes = 180

i) P (Blue eyes) = $\frac{10}{100} = \frac{1}{10}$

ii) P (Brown or black eyes) = $\frac{90}{100} = \frac{9}{10}$

iii) P(Blue or black eyes) = $\frac{75}{100} = \frac{3}{4}$

iv) P(neither blue nor brown eyes) = $\frac{65}{100} = \frac{13}{20}$

10. Find the probability of having 53 Sundays in

(i) a leap year (ii) a non leap year (Ans: $\frac{2}{7},\frac{1}{7}$)

Ans: An ordinary year has 365 days i.e. 52 weeks and 1 day
This day can be any one of the 7 days of the week.

$\therefore$ P(that this day is Sunday) = $\frac{1}{7}$

Hence, P(an ordinary year has 53 Sunday) = $\frac{1}{7}$

A leap year 366 days i.e. 52 weeks and 2 days
This day can be any one of the 7 days of the week

$\therefore$ P (that this day is Sunday) = $\frac{2}{7}$

Hence, P(a leap year has 53 Sunday) = $\frac{2}{7}$

11. Find the probability that the month June may have 5 Mondays in

(i) a leap year (ii) a non leap year (Ans: $\frac{2}{7},\frac{2}{7}$)

Self Practice

12. Find the probability that the month February may have 5 Wednesdays in

(i) a leap year (ii) a non leap year (Ans: $\frac{1}{7}$,0)

Self Practice

13. Five cards – the ten, jack, queen, king and ace, are well shuffled with their ce downwards. One card is then picked up at random.
(i) What is the probability that the card is a queen?
(ii) If the queen is drawn and put aside, what is the probability that the second card picked up is a (a) an ace (b) a queen (Ans: $\frac{1}{5},\frac{1}{4}$,0)

Ans : Here, the total number of elementary events = 5

(i) Since, there is only one queen

$\therefore$ vourable number of elementary events = 1

$\therefore$ Probability of getting the card of queen = $\frac{1}{5}$

(ii) Now, the total number of elementary events = 4

(a) Since, there is only one ace

$\therefore$ vourable number of elementary events = 1

$\therefore$ Probability of getting an ace card $= \frac{1}{4}$

(b) Since, there is no queen (as queen is put aside)

$\therefore$ vourable number of elementary events = 0

$\therefore$ Probability of getting a queen $= \frac{0}{4} = 0$

14. A number x is chosen at random from the numbers -3, -2, -1, 0 1, 2, 3. What is the probability that $|x| < 2$ (Ans: $\frac{3}{7}$)

Ans : x can take 7 values

To get $|x| < 2$ take $-1, 0, 1$

Probability $(|x| < 2) = \frac{3}{7}$

15. A number x is selected from the numbers 1,2,3 and then a second number y is randomly selected from the numbers 1,4,9. What is the probability that the product xy of the two numbers will be less than 9? (Ans: $\frac{5}{9}$)

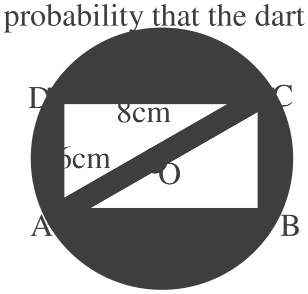
Ans : Number X can be selected in three ways and corresponding to each such way there are three ways of selecting number y . Therefore , two numbers can be selected in 9 ways as listed below:

(1,1), (1,4), (2,1), (2,4), (3,1)

$\therefore$ vourable number of elementary events = 5

Hence, required probability $= \frac{5}{9}$

16. In the adjoining figure a dart is thrown at the dart board and lands in the interior of the circle. What is the probability that the dart will land in the shaded region.



$\left(\text{Ans: } \frac{25\pi - 48}{25\pi}\right)$

Ans: We have

$AB = CD = 8$ and $AD = BC = 6$

using Pythagoras Theorem is ΔABC, we have

$AC^2 = AB^2 + BC^2$

$AC^2 = 8^2 + 6^2 = 100$

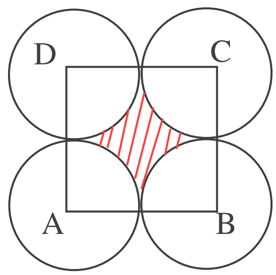
$AC = 10$

$OA = OC = 5$ [Q O is the midpoint of AC]

$\therefore$ Area of the circle = π $(OA)^2 = 25$ π sq units [Q Area = π r^2]
Area of rectangle ABCD = AB x BC = 8 x 6 = 48 sq units
Area of shaded region = Area of the circle – Area of rectangle ABCD
Area of shaded region = 25 π - 48 sq unit.
Hence

P (Dart lands in the shaded region) = $\frac{\text{Area of shaded region}}{\text{Area of circle}} = \frac{25\pi - 48}{25\pi}$

17. In the fig points A ,B ,C and D are the centres of four circles ,each having a radius of 1 unit . If a point is chosen at random from the interior of a square ABCD ,what is the probability that the point will be chosen from the shaded region .

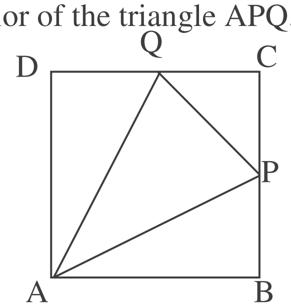


(Ans: $\frac{4-\pi}{4}$)

Ans: Radius of the circle is 1 unit
Area of the circle = Area of 4 sector
$\pi r^2 = \pi \text{ x } 1^2 = \pi$
Side of the square ABCD = 2 units
Area of square = 2 x 2 = 4 units
Area shaded region is
= Area of square – 4 x Area of sectors
= 4 - π

Probability = $\left(\frac{4-\Pi}{4}\right)$

18. In the adjoining figure ABCD is a square with sides of length 6 units points P & Q are the mid points of the sides BC & CD respectively. If a point is selected at random from the interior of the square what is the probability that the point will be chosen from the interior of the triangle APQ.



(Ans: $\frac{3}{8}$)

Ans: Area of triangle PQC = $\frac{1}{2}$ x 3 x 3 = $\frac{9}{2}$ = 4. 5 units

Area of triangle ABP = $\frac{1}{2}$ x 6 x 3 = 9

Area of triangle ADQ = $\frac{1}{2}$ x 6 x 3 = 9

Area of triangle APQ = Area of a square – (Area of a triangle PQC + Area of triangle

ABP + Area of triangle ABP)

= 36 – (18+4.5)
= 36 – 22.5
= 13.5

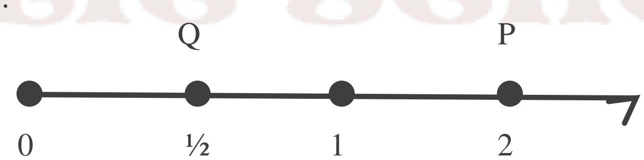
Probability that the point will be chosen from the interior of the triangle APQ = $\frac{13.5}{36}$

$= \frac{135}{360} = \frac{3}{8}$

19. In a musical chair game the person playing the music has been advised to stop playing the music at any time within 2 minutes after she starts playing. What is the probability that the music will stop within the half minute after starting.

(Ans: $\frac{1}{4}$)

Ans: Here the possible outcomes are all the numbers between 0 and 2.
This is the portion of the number line from 0 to 2 as shown in figure.
Let A be the event that 'the music is stopped within the first half minute.' Then, outcomes vorable to event A are all points on the number line from O to Q i.e., from 0 to 1/2 .



The total number of outcomes are the points on the number line from O to P i.e., 0 to 2.

$\therefore$ P (A) = $\frac{\text{Length of OQ}}{\text{Length of OP}} = \frac{1/2}{2} = \frac{1}{4}$

20. A jar contains 54 marbles each of which is blue , green or white. The probability of selecting a blue marble at random from the jar is $\frac{1}{3}$ and the probability of selecting a green marble at random is $\frac{4}{9}$. How many white marbles does the jar contain? (Ans:12)

Ans: Let there be b blue, g green and w white marbles in the marbles in the jar. Then,

$$b + g + w = 54$$

$\therefore$ P (Selecting a blue marble) = $\frac{b}{54}$

It is given that the probability of selecting a blue marble is $\frac{1}{3}$.

$\therefore \ \frac{1}{3} = \frac{b}{54}$ => b = 18

We have,

P(Selecting a green marble) = $\frac{4}{9}$

=> $\frac{g}{54} = \frac{4}{9}$ [Q P (Selecting a green marble) = $\frac{4}{9}$ (Given)]

=> g = 24

Substituting the values of b and g in (i), we get

$$18 + 24 + w = 54 \Rightarrow w = 12$$

CHAPTER – 1

ALGEBRA (A P + QUADRATIC EQUATION)

1. Ram asks the labour to dig a well upto a depth of 10 metre. Labour charges Rs. 150 for first metre and Rs. 50 for each subsequent metres. As labour was uneducated, he claims Rs. 550 for the whole work. What should be the actual amount to be paid to the labours? What value of Ram is depicted in the question if he pays Rs. 600 to the labour?

2. Nidhi ves Rs. 2 on first day of the month, Rs. 4 on second day, Rs. 6 on third day and so on. What will be her ving in the month of Feb. 2012? What value is depicted by Nidhi?

3. 200 logs are stacked such that 20 logs are in the bottom row, 19 in the next row, 18 in the row next to it and so on. In how many rows are the 200 logs placed? What value is depicted in the pattern of logs?

4. How many two digit numbers are there in between 6 and 102 which are divisible by 6. Ram calculated it by using A.P. while Shyam calculated it directly. Which value is depicted by Ram?

5. In a school, students thought of planting trees in an around the school to reduce air pollution. It was decided that the number of trees, that each section of each class will plant, will be the me as the class, in which they are studying e.g. a section of class-I will plant 1 tree, a section of class II will plant 2 trees and so on till class XII. There are three sections of each class. How many trees will be planted by the students? What value can you infer from the planting the trees?

6. Rs. 9000 were divided equally among a certain number of students. Amit was given the responsibility of dividing this amount among the students but 20 more students admitted to the school. Now each students got Rs. 160 less. Find the original number of students? What value of Amit is depicted in the question?

7. By a reduction of Rs. 2 per kg in the price of sugar, Anita can purchase 2 Kg sugar more for Rs. 224. Find the original price of sugar per kg. What value of Anita is depicted in the question?

8. Due to some technical problems, an aeroplane started late by one hour from it starting point. The pilot decided to increase the speed of the aeroplane by 100 km/hr. from its usual speed, to cover a journey of 1200 km in time. Find the usual speed of the aeroplane? What value (Quality) of the pilot is represented in the question?

9. A motor boat whose speed is 9 km/hr. in still water goes 12 km down stream and comes back in a total time 3 hours. Find the speed of the stream? Explain the situation when speed of stream is more than the speed of boat in still water.

10. A peacock is sitting on the top of the pillar which is 9 metre high. From the point 27 metre away from the bottom of the pillar, a snake is coming to its hole at the base of the pillar. Seeing the snake, the peacock pounces on it. If their speeds are equal, at what distance from the hole is the snake caught? What value is experienced by the peacock after catching snake?

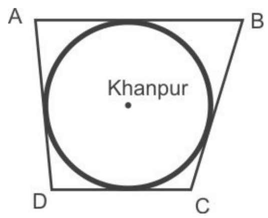
CHAPTER – 2

CIRCLE

1. There are 3 villages A, B and C such that the distance from A to B is 7 km, from B to C is 5 km and from C to A is 8 km. The gram pradhan wants to dig a well in such a way that the distance from each villages are equal. What should be the location of well? Which value is depicted by gram pradhan?

2. People of village wants to construct a road nearest to a circular village Rampur. The road cannot pass through the village. But the people wants that road should be at the shortest distance from the center of the village (i) which road will be the nearest to the center of village? (ii) which value is depicted by the people of village?

3. Four roads have to be constructed by touching village Khanpur in circular shape of radius 1700 m in the following manner.

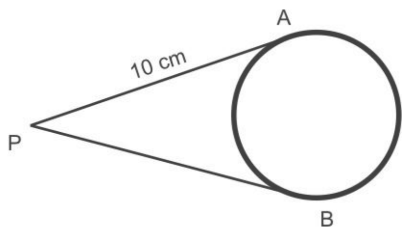


vita got contract to construct the roads AB and CD while Vijay got contract to construct AD and BC.
Prove that AB + CD = AD + BC.

Which value is depicted by the contractor?

4. Two roads starting from P are touching a circular path at A and B.

rita ran from P to A 10 km and Ramesh ran from P to B. (i) If rita wins the race than how much distance Ramesh ran? (ii) Which value is depicted?



5. A rmer wants to divide a sugarcane of 7 ft length between his son and daughter equally. Divide it Geometrically, considering sugarcane as a line of 7 cm, using construction.

(i) Find the length of each part.

(ii) Which value is depicted?

CHAPTER – 3

APPLICATIONS OF TRIGONOMETRY

1. A tree breaks due to storm and the broken part bends so that the top of the tree touches the ground making an angle 30° with it. The distance between the foot of the tree to the point where the top touches the ground is 8 m. The teacher asked the students to find the height of the tree. All the students iled but Neeraj took initiative and calculated it correctly using trigonometry. What height Neeraj calculated? What quality of Neeraj is depicted here?

2. A person, standing on the bank of a river, observes that the angle subtended by a tree on the opposite bank is 60°. When he retreates 20 m from the bank, he finds the angle to be 30°. Find the height of the tree and the breadth of the river? What skill is used by the person.

3. Anand is watching a circus artist climbing a 20 m long rope which is tightly stretched and tied from the top of a vertical pole to the ground. Find the height of the pole if the angle made by the rope with the ground level is 30°. What value is experienced by Anand?

4. A pilot is flying an areoplane at an altitude of 1800 m observes that two ships are iling towards it in the opposite directions. The angles of depressions of the ships as observed from the aeroplane are 60° & 30° respectively. Find the distance between the two ships? What value of the pilot is shown?

5. The angle of elevation of a bird observed by a hunter who is 12 m above a lake is 30° and the angle of depression of bird's reflection in the lake is 60°. Find the distance between the bird and the hunter. What value is used by the hunter if he want to hit the bird?

CHAPTER – 4

PROBABILITY

1. In a class discussion, Himanshu ys that the probability of an event can't be – 1.3. He shows

   (a) Truth value (b) Economical value

   (c) Leadership (d) Environmental value

2. $P(E) + P(\bar{E}) = 1$

   The statement depicts _____________ value

3. E be an event associated with a random experiment and $0 \le P(E) \le x$. Find the max value of x ? Which value is depicted by this statement?

4. If E_1, E_2, E_3 are the possible elementary events of a random experiment and

   $P(E_1) + P(E_2) = \frac{2}{3}$ Show that $P(E_3) = \frac{1}{3}$

   Which value is depicted by this?

5. A selection committee interviewed some people for the post of les Manager. The committee wanted that the female candidates should also be given the ir chance. So they called male and female candidates in 3:4 ratio.

   a. What is the probability of a female candidate being selected?

   b. Which value is shown by the selection committee?

6. 12 defective ball pens are accidentally mixed with 156 good one. It is not possible to just look at pen and tell whether or not it is defective. The shopkeeper draws one pen at random.

   a. Determine the probability that the pen taken out is a good one.

   b. Suppose the pen drawn is defective. The shopkeeper did not sell out and kept the pen aside. He again draws one more pen at random from the rest. What is the probability that pen is not defective.

   c. Which value is shown by the shopkeeper?

7. During Van Mahotv a group of students, planted a number of plants in 20 houses of a locality.

No. of Plants	0-2	2-4	4-6	6-8	8-10	10-12	12-14
No. of Houses	x	$2x$	x	$2x+3$	$6x$	$2x$	$2x+1$

a. What is the probability of houses who has plants ≤ 8?

b. Which value is depicted by students?

8. Ramesh has got ₹24000/- as Puja Bonus. He donated ₹5000/- to temple. He gave ₹12000/- to his wife, ₹2000/- to his servant and gave rest of the amount to his daughter.

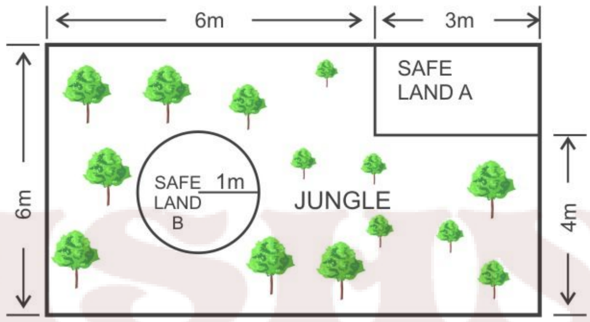
a. What is the probability of wife's share?

b. Calculate the probability of servant's share?

c. What is the probability of daughter's share?

d. Which value are depicted by Ramesh from the way the amount is distributed?

9. Due to some deult in the engine of a helicopter, a pilot has to make an emergency landing in an area as shown in the given figure.



a. What is the probability of fe landing?

b. What is the probability of landing in jungle?

c. Due to bigger area, the pilot decided to land on fe land A rather than fe land B.

Which value is shown by the pilot?

10. In a co-operative society, 60 people go to me office. They all use their conveyance. 10 people use their scooters, 10 go by their cars and the rest use their motorcycles.

a. What is the probability of people going by motorcycle?

b. One day they all decided to go by cars but a car can accommodate only 5 people. What is the probability of people going by car now?

c. What is the probability of people not going by cars now?

d. Which value is shown in 'b'?

11. 240 students reside in a Hostel. Out of which 50% go for the yoga classes early in the morning 25% have joined the Gym club and 15% of them go for the morning walk. Rest of the students have joined the laughing club.

a. What is the probability of students who have joined the laughing club?

b. What is the probability of students who have not joined any class or club?

c. Which value is depicted by students?

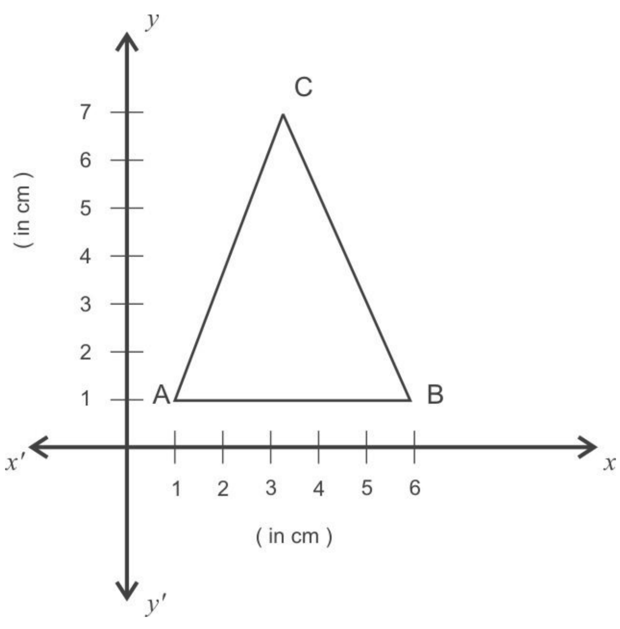
CHAPTER – 5

COORDINATE GEOMETRY

1. There are two routes to travel from source A to destination B by bus. First bus reaches at B via point C and second bus reaches from A to B directly. If coordinates of A, B and C are (-2, -3), (2, 3) and (3, 2) respectively then by which bus do you want to travel from A to B (Assume that both buses have me speed.) Which value is depicted in the question?

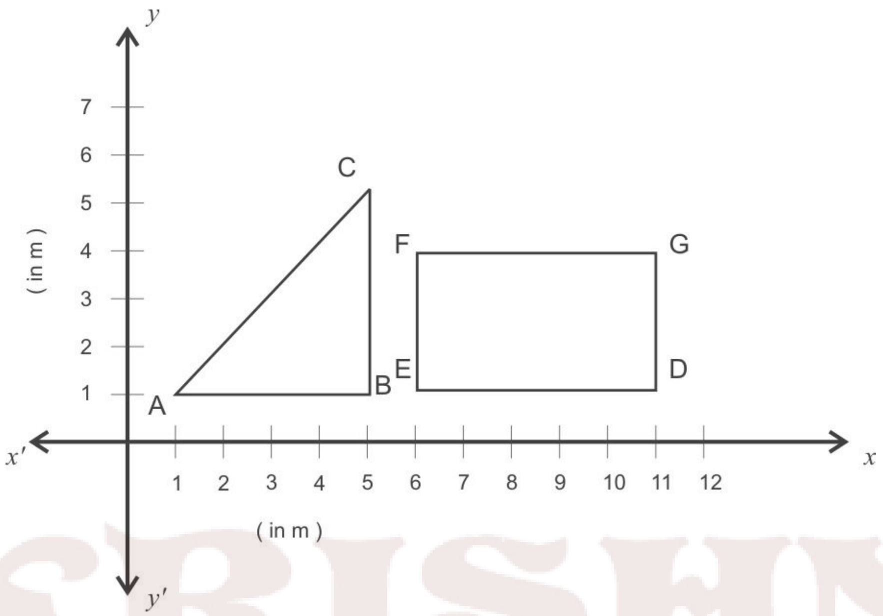
2. In a sports day celebration, Ram and Shyam are standing at positions A and B whose coordinates are (2, -2) and (4, 8) respectively. The teacher asked Geeta to fix the country flag at the mid point of the line joining points A and B. Find the coordinates of the mid point? Which type of value would you infer from the question?

3. To raise social awareness about hazards of smoking, a school decided to start "No Smoking" campaign. 10 students are asked to prepare campaign banners in the shape of triangle as shown in the fig. If cost of 1 cm^2 of banner is Rs.2 then find the overall cost incurred on such campaign. Which value is depicted in the question?



4. The coordinates of the houses of meer and Rahim are (7, 3) and (4, -3) whereas the coordinates of their school is (2, 2). If both leaves their houses at the me time in the morning and also reaches school on time then who travel ster? Which value is depicted in the question?

5. There are two types of fields are available as shown in the fig. which type of field will you purchase if both have me cost? Which value is depicted in the question?

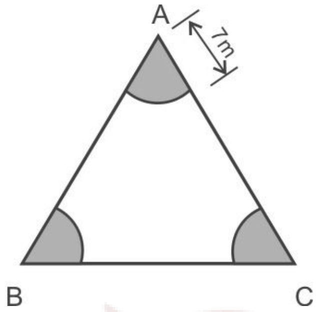


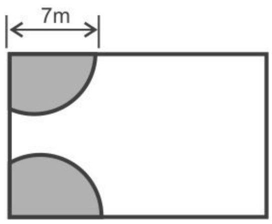
CHAPTER – 6

MENSURATION

1. An ice cream seller has two types of ice cream container in the form of cylindrical shape and a cone with hemi-spherical base. Both have me height of 7 cm and me diameter of 7 cm. The cost of container are me but the seller decide to sell ice cream in cylindrical containers. (i) Calculate the volume of the both containers. (ii) Which value is depicted by the seller?

2. Two types of water tankers are available in a shop. One is in a cubic form of dimensions 1 m x 1 m x 1 m and another is in the form of cylindrical form of diameter 1 m and height is also 1 m. Calculate the volume of both tankers. The shopkeeper advise to purchase cuboid tank. Which value is depicted?





3. rmer has two types of field in the form of triangle and rectangle. Geeta is allowed to cut the grass of triangular field (shaded position) and Vijay is allowed to cut the grass of rectangular field (shaded portion) in the following manner. Calculate the areas of both shaded portions? Which value is depicted?

4. A rmer has two types of fields, one is in the form of a squared area 144 m^2 and another is in the form of a rectangle of sides 16 m and 8 m. rmer wants to fence his field. So he gave this work to Ramesh for Square field and rita for rectangular field. Find the Length of fencing of both the fields. Which value is depicted?

5. A rmer wants to dig a well either in the form of cuboid of dimensions (1m x 1m x 7m) or in the form of cylinder of diameter 1 meter and radius 7m. The rate to dig the well is Rs. 50/m^3. Find the cost to dig both wells. The rmer decides to dig the cylindrical well. By his decision which value is depicted?

ANSWER SHEET

ALGEBRA (A.P. + QUADRATIC EQUATIONS)

1. Rs. 600, Honesty, Sincerity.
2. Rs. 870, Economy, ving
3. 16, Space ving, Creative, Reasoning, Balancing
4. 15, Time ving, Seasoning
5. 234, Environmental, Social
6. 25, Logical, Sincerity, Leadership
7. Rs. 16, Economical, ving, Leadership
8. 300 km/hr, Leadership, Punctuality
9. 3 km/hr., Logically, if speed of stream is more than speed of boat in still water then the boat will not il.
10. 12 m.

CIRCLE

1. A, B, C will lie on the circumference of the circle and location of well will be at the centre of the circle. Social, Honesty, Equality.
2. (i) Tangent of the circle (ii) Economical
3. Gender equality
4. 10 km, Gender equality
5. 3.5 ft, Gender equality.

APPLICATION OF TRIGNOMETRY

1. $8\sqrt{3}$ m, Leadership Logical, Reasoning.
2. $10\sqrt{3}$ m, 10 m, Logical
3. 10 m, 30 y, Fun, Entertainment
4. $2400\sqrt{3}$ m, Reasoning, Logical
5. 24 m, Accuracy, Concentration, Confidence, Focus, Anticipation, Reasoning.

PROBABILITY

1. Truth value.
2. Righteous value.
3. 1, Righteous value.
4. Righteous value.
5. (a) $\frac{4}{7}$ (b) Gender equlaity.
6. (a) $\frac{13}{14}$ (b) $\frac{156}{167}$ (c) Honesty

7. (a) $\frac{9}{20}$ (b) Environmental value

8. (a) $\frac{1}{2}$ (b) $\frac{1}{12}$ (c) $\frac{5}{24}$ (d) Social value, religious value.

9. (a) $\frac{6+\pi}{54}$ (b) $\frac{48-\pi}{54}$ (c) Leadership

10. (a) $\frac{2}{3}$ (b) $\frac{5}{6}$ (c) $\frac{1}{6}$

11. (a) $\frac{1}{10}$ (b) $\frac{3}{20}$ (c) Physical Fitness

CO-ORDINATE GEOMETRY

1. By direct route from A to B. Reasoning, Time ving, Economical
2. (3, 3), Enjoyment, Reasoning.
3. Rs. 300, Social awareness
4. mir, Punctuality, Sincerity.
5. Rectangular, Economical

MENSURATION

1. $\frac{343}{4}\pi cm^3$, $\frac{343}{8}\pi cm^3$, Honesty.
2. 1 m^3, 0.785 m^3, Honesty
3. $245\pi m^2, 24.5\ m^2$, Gender equality.
4. 48m, 48m, Gender equality.
5. Rs. 350, Rs. 275, Economical